

Effects of Non-Associated Flow on Residual Stress Distributions in Crankshaft Sections Modeled as Pressure-Sensitive Materials under Fillet Rolling

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ABSTRACT

In this paper, the evolution equation for the active yield surface during the unloading/reloading process based on the pressure-sensitive Drucker-Prager yield function and a recently developed anisotropic hardening rule with a non-associated flow rule is first presented. A user material subroutine based on the anisotropic hardening rule and the constitutive relation was written and implemented into the commercial finite element program ABAQUS. A two-dimensional plane strain finite element analysis of a crankshaft section under fillet rolling was conducted. After the release of the roller, the magnitude of the compressive residual hoop stress for the material with consideration of pressure sensitivity typically for cast irons is smaller than that without consideration of pressure sensitivity. In addition, the magnitude of the compressive residual hoop stress for the pressure-sensitive material with the non-associated flow rule is smaller than that with the associated flow rule. The computational results indicate that without consideration of pressure sensitivity and the non-associated flow rule, the residual stresses in crankshaft sections under fillet rolling are overestimated.

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INTRODUCTION

Crankshafts in automotive engines can experience a significant number of service cyclic loads. Since fatigue cracks initiated near the fillet can lead to one of the primary failure mechanisms of automotive crankshafts, fillet rolling processes have been used to improve the fatigue lives of crankshafts for many years. Based on the results of the recent research works [1, 2, 3, 4, 5], accurate predictions of the distributions of residual stresses up to a few millimeters from the fillet surface are very important to determine the fatigue failure process and the fatigue limit of crankshaft sections.

Cast irons are commonly used for gasoline engine crankshafts for good mechanical property and, above all, the ability to be cast into complex shape at low cost. It is well known that the tensile and compressive yield behaviors of cast irons are different due to the existence of graphite particles. The different yield behaviors under tension and compression can be modeled by a pressure-sensitive yield function. The material elements near the fillets under rolling are usually subjected to cyclic loading/unloading conditions. Current commercial finite element codes cannot be used to model the materials with the different tensile and compressive yield behaviors under cyclic loading conditions as discussed in Choi and Pan [4].

The yield functions to describe pressure-sensitive yielding behavior usually involve the hydrostatic stress and the second invariant of the deviatoric stress [6, 7, 8, 9]. However, with the associated flow rule,

the pressure-sensitive yield functions usually lead to a plastic dilatancy which is at least an order of magnitude larger than that observed in experiments [8,9]. Therefore, non-associated flow rules based on pressure-sensitive yield functions were adopted to properly model the plastic behaviors of pressure-sensitive materials under monotonic loading conditions [10, 11, 12].

Recently, Choi [13] formulated a generalized anisotropic hardening rule based on a general yield function to model cyclic plastic behavior of materials based on the concepts of the Mroz model [14,15]. The closed-form solutions for one-dimensional stress-plastic strain curves were also derived and plotted for materials under cyclic loading conditions based on the Mises yield function, the Hill quadratic anisotropic yield function and the Drucker-Prager yield function. For materials based on the Drucker-Prager yield function with the associated flow rule, the stress-plastic strain curves do not close and show an unusually large ratcheting effects under uniaxial cyclic loading conditions. The unusually large ratcheting effect contradicts the nearly closed hysteresis loops of cast irons under cyclic loading conditions observed in experiments. Therefore, Choi [13] proposed an anisotropic hardening rule with a conventional and a non-conventional non-associated flow rules in order to take away the large ratcheting effects. The anisotropic hardening rule of Choi [13] with the conventional non-associated flow rule is adopted here to characterize the pressure-sensitive cyclic plastic behavior of cast

iron crankshaft section during the fillet rolling process since the current commercial finite element codes do not provide a cyclic plasticity model for pressure-sensitive materials such as cast irons.

Choi and Pan [5] adopted the anisotropic hardening rule of Choi [13] based on the Drucker-Prager function with the non-conventional non-associated flow rule to investigate the stress distributions near the fillet of a crankshaft section under fillet rolling and subsequent bending. Their constitutive relation based on the tangent modulus procedure for rate-sensitive materials was formulated and implemented into a user material subroutine for ABAQUS. A two-dimensional plane-strain finite element analysis of a crankshaft section under fillet rolling and subsequent bending was conducted for different values of the pressure sensitivity in order to investigate the effect of the pressure-sensitive yielding and the non-conventional non-associated flow rule on the stress distributions near the fillet. The computational results showed that, as the pressure sensitivity increases, the maximum tensile stress at the fillet surface increases and the size of the tensile stress region increases after the release of roller and under subsequent bending.

In this paper, the evolution equation for the active yield surface during the unloading/reloading process based on the anisotropic hardening rule of Choi [13] and the Drucker-Prager yield function is first presented. A user material subroutine based on the anisotropic hardening rule and the constitutive relation for rate-insensitive materials was written and implemented into the commercial finite element program ABAQUS. A two-dimensional plane strain finite element analysis of a crankshaft section under fillet rolling was conducted for different values of the pressure sensitivity and different flow rules in order to investigate the effect of the pressure-sensitive yielding and the non-associated flow rule on the stress distributions near the fillet. The residual stress distributions after the release of the roller are obtained and compared for pressure-insensitive and pressure-sensitive materials with both the associated and the conventional non-associated flow rule. Finally, some conclusions based on the computational results are made.

AN ANISOTROPIC HARDENING RULE OF CHOI [13]

Choi [13] developed a cyclic plasticity theory for materials based on a general yield function. Fig. 1 shows the evolution of the active yield surface during the unloading/reloading process based on the anisotropic hardening rule of Choi [13]. As shown in Fig. 1, the active yield surface Φ_A continuously expands with the unloading stress point $\sigma_{ij}^{M,t}$ being kept as the tangential contact point and the center α_{ij}^A also continuously moves along the line joining the unloading stress point $\sigma_{ij}^{M,t}$ to the center α_{ij}^M of the largest yield surface previously reached. The yield surface Φ_M is a memory yield surface which is used to determine the subsequent plastic behavior during the unloading/reloading process. The unloading stress point $\sigma_{ij}^{M,t}$ is defined whenever the unloading takes place.

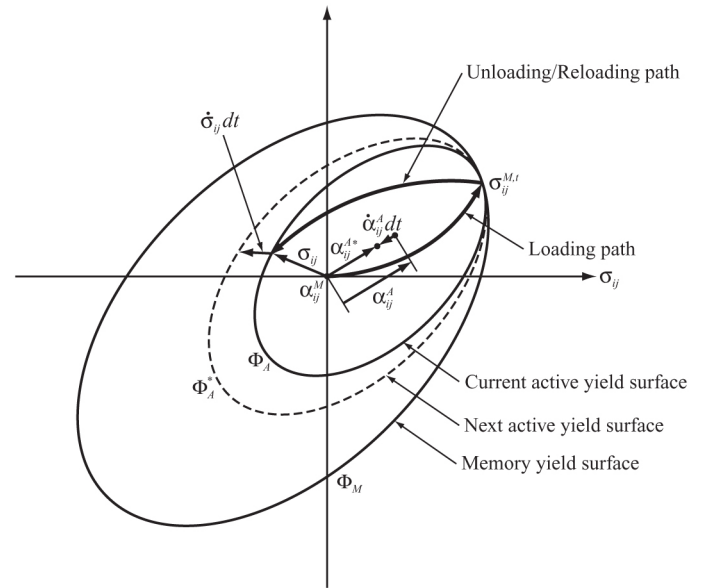


Figure 1. A schematic plot of the evolution of the active yield surface during the unloading/reloading process based on the anisotropic hardening rule of Choi [13].

In Fig. 1, the similarity condition for the memory yield surface and the active yield surface can be expressed as

$$\frac{\sigma_{ij}^{M,t} - \alpha_{ij}^M}{\bar{\sigma}_M} = \frac{\sigma_{ij}^{M,t} - \alpha_{ij}^A}{\bar{\sigma}_A} \quad (1)$$

where $\bar{\sigma}_M$ and $\bar{\sigma}_A$ represent the sizes of the memory yield surface and the active yield surface, respectively, and α_{ij}^M and α_{ij}^A represent the centers of these yield surfaces, respectively. The evolution equation for α_{ij}^A along the line joining the centers of the active yield surface and the memory yield surface can be obtained from the differentiation of Eq. (1) as

$$\dot{\alpha}_{ij}^A = - \left(\frac{\sigma_{ij}^{M,t} - \alpha_{ij}^M}{\bar{\sigma}_M} \right) \dot{\bar{\sigma}}_A \quad (2)$$

The consistency condition for the active yield surface Φ_A as a function of σ_{ij} , α_{ij}^A and $\bar{\sigma}_A$ during the unloading/reloading process can be expressed as

$$\dot{\Phi}_A(\sigma_{ij}, \alpha_{ij}^A, \bar{\sigma}_A) = \frac{\partial \Phi_A}{\partial \sigma_{ij}} \dot{\sigma}_{ij} + \frac{\partial \Phi_A}{\partial \alpha_{ij}^A} \dot{\alpha}_{ij}^A + \frac{\partial \Phi_A}{\partial \bar{\sigma}_A} \dot{\bar{\sigma}}_A = 0 \quad (3)$$

where $\bar{\sigma}_A$ is not a constant but increases during the unloading/reloading process. Substituting Eq. (2) into Eq. (3) gives the size increase rate $\dot{\bar{\sigma}}_A$ of the active yield surface. Then, substituting $\dot{\bar{\sigma}}_A$ into Eq. (2) gives the evolution equation for α_{ij}^A of the active yield surface Φ_A .

Drucker and Prager [6] proposed a phenomenological yield function for pressure-sensitive materials. For the unloading/reloading process, the Drucker-Prager yield function can be expressed with respect to the center of the active yield surface at α_{ij}^A as

$$\Phi_A(\sigma_{ij}, \alpha_{ij}^A, \bar{\sigma}_A) = \sigma_{e,A} + \sqrt{3}\mu\sigma_{m,A} - \bar{\sigma}_A = 0 \quad (4)$$

where μ is the pressure sensitivity, $\sigma_{e,A}$ is the relative effective stress based on the Mises yield function, $\sigma_{m,A}$ is the relative mean stress, and $\bar{\sigma}_A$ is the generalized effective stress representing the size of the active yield surface. Here, $\sigma_{e,A}$ and $\sigma_{m,A}$ are defined, respectively, as

$$\sigma_{e,A} = \left[\frac{3}{2} (\sigma'_{ij} - \alpha'_{ij}{}^A) (\sigma'_{ij} - \alpha'_{ij}{}^A) \right]^{\frac{1}{2}} \quad (5a)$$

$$\sigma_{m,A} = \frac{\sigma_{kk} - \alpha_{kk}^A}{3} \quad (5b)$$

where σ'_{ij} is the deviatoric stress tensor and $\alpha'_{ij}{}^A$ is the deviatoric part of the yield surface center.

A NON-ASSOCIATED FLOW RULE AND CONSTITUTIVE RELATION

A non-associated flow rule based on the Drucker-Prager yield function is adopted here in order to characterize the cyclic plastic behavior of cast iron crankshafts during the fillet rolling process. The Mises plastic potential function $\Phi_{p,A}$ is employed for the non-associated flow rule and it can be expressed as

$$\Phi_{p,A}(\sigma_{ij}, \alpha_{ij}^A, \bar{\sigma}_{p,A}) = \sigma_{e,A} - \bar{\sigma}_{p,A} = 0 \quad (6)$$

where $\sigma_{e,A}$ is defined in Eq. (5a) and $\bar{\sigma}_A$ represents a fictitious generalized effective stress. The plastic potential function in Eq. (6) is assumed to pass through the current stress point and have the same center as that of the active yield surface in Eq. (4). For metals, the pressure sensitivity μ is usually small in the range of a few percents and the plastic dilatancy is one order of magnitude smaller than that predicted by the associated flow rule [8,9]. For cast irons under compression dominant loading conditions due to the fillet rolling process, the volume change due to graphite particles and the void formation are assumed to be small under large hydrostatic compressive stresses as shown in Chien et al. [1] and Choi and Pan [4]. Consequently, plastic deformation is assumed to mainly come from the nearly plastic incompressible iron matrices. Therefore, the Mises plastic potential function $\Phi_{p,A}$ that leads to plastic incompressibility is adopted here.

Based on the non-associated flow rule, the plastic strain rate $\dot{\epsilon}_{ij}^p$ can be written as [16]

$$\dot{\epsilon}_{ij}^p = \frac{1}{\bar{E}_p} \left(\frac{\partial \Phi_{p,A}}{\partial \sigma_{ij}} \right) \left(\frac{\partial \Phi_A}{\partial \sigma_{kl}} \right) \dot{\sigma}_{kl} \quad (7)$$

where $\bar{E}_p$ represents the generalized plastic modulus from a generalized effective stress-strain curve and is considered as a function of the generalized effective stress which represents the size of the active yield surface. In Eq. (7), $\bar{E}_p$ is defined as

$$\bar{E}_p = \frac{\bar{\sigma}}{\bar{\epsilon}^p} \quad (8)$$

where $\bar{\sigma}$ and $\bar{\epsilon}^p$ represent the generalized effective stress and the generalized effective plastic strain, respectively.

The generalized plastic modulus $\bar{E}_p$ in Eq. (8) can be related to the plastic modulus E_p of a uniaxial tensile or compressive stress-strain curve. Note here that the plastic moduli $\bar{E}_p$ and E_p are functions of $\bar{\sigma}_A$ and the uniaxial compressive stress σ_{11} , respectively (Choi [13]). In this study, a uniaxial compressive stress-strain curve, where σ_{11} is the only nonzero stress component, is considered as a reference stress-strain curve in view of compression dominant cyclic loading conditions during the fillet rolling process. Writing Eq. (7) under uniaxial compressive loading conditions gives

$$\begin{aligned} \dot{\epsilon}_{11}^p &= \frac{1}{\bar{E}_p} \left(\frac{3\sigma'_{11}}{2\sigma_{e,A}} \right) \left(\frac{3\sigma'_{11}}{2\sigma_{e,A}} + \frac{\mu}{\sqrt{3}} \right) \dot{\sigma}_{11} \\ &= \frac{1}{\bar{E}_p} \left(\frac{3\sigma'_{11}}{-2\sigma_{11}} \right) \left(\frac{3\sigma'_{11}}{-2\sigma_{11}} + \frac{\mu}{\sqrt{3}} \right) \dot{\sigma}_{11} = \frac{1}{E_p} \left(1 - \frac{\mu}{\sqrt{3}} \right) \dot{\sigma}_{11} \end{aligned} \quad (9)$$

Note that $\sigma_{e,A} = -\sigma_{11}$ and $\sigma'_{11} = 2\sigma_{11}/3$ under uniaxial compressive loading conditions. Since $E_p = \dot{\sigma}_{11}/\dot{\epsilon}_{11}^p$, the relation between $\bar{E}_p$ and E_p is given from Eq. (9) as

$$\bar{E}_p = \left(1 - \frac{\mu}{\sqrt{3}} \right) E_p \quad (10)$$

Under uniaxial compressive loading conditions, $\bar{\sigma}_A$ is expressed as

$$\bar{\sigma}_A = - \left(1 - \frac{\mu}{\sqrt{3}} \right) \sigma_{11} \quad (11)$$

Therefore, when the plastic modulus E_p is first determined as a function of σ_{11} , the plastic modulus $\bar{E}_p$ can be obtained as a function of $\bar{\sigma}_A$ based on Eqs. (10) and (11). The detailed description of the non-associated flow rule can be found in Choi [13].