

Modeling and Feasibility Study of Nonlinear Suspension Components in Multibody Systems using Absolute Nodal Coordinate Formulation Based Beam Elements -Application to Stabilizer Bar

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ABSTRACT

This study concerns the modeling of the stabilizer bar in a car suspension. This is a crucial and difficult task if its non-linear behavior should be captured correctly. In this study, the modeling of a stabilizer bar is done using beam finite elements based on the absolute nodal coordinate formulation (ANCF).

An ANCF beam element is reviewed and its implementation in a multibody dynamics framework is explained. The specific element is chosen since it is assumed to be appropriate for modeling stabilizer bars. To test the feasibility of using the chosen ANCF beam element for modeling stabilizer bars, several numerical studies have been performed. These include eigenfrequency and static analyses where results obtained using ANCF beam elements are compared with results obtained using other methods. Furthermore, the ANCF beam element has been implemented in a multibody dynamics model together with rigid bodies, springs and dampers to generate a representative model of a vehicle front suspension. This model is then subjected to a transient analysis to assess the influence of including a stabilizer bar in a front suspension.

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INTRODUCTION

The modeling of a vehicle suspension system is one of the crucial tasks when designing and optimizing modern vehicles. One of the preferred tools to assist the designers in this task is multibody dynamics. When using multibody dynamics, it is possible to model arbitrary systems with high mechanical complexity, include non-linear characteristics of e.g. springs and shock absorbers, and simulate the system behavior in the time domain to assess its performance in various scenarios.

A non-trivial job when building a vehicle model, is estimating the characteristics of the many suspension components that exhibits nonlinear behavior. Examples of such components are bushings, springs, shock absorbers, and stabilizer bars. Especially a stabilizer bar can be difficult to characterize using an idealized spring with a non-linear force-deflection curve, since the roll moment is carried by different deformations depending on where on the stabilizer bar the reaction forces are calculated. For instance, the roll moment can be transferred to the stabilizer bar as point forces, which may cause bending in the first section. As the load travels through the stabilizer bar, the load may be carried as bending in one part, and torsion in another depending on the design of the stabilizer bar. This means that the stiffness depends highly on its geometry and deformation level. The stabilizer bar is a key component in the suspension of many

vehicle types where it limits roll and enables adjustment in overall vehicle understeer characteristics. Therefore, it is important to include an accurate and representative model of the stabilizer bar in the multibody dynamics model to assess and optimize its effect on the overall vehicle behavior.

The high modeling flexibility that multibody dynamics offers makes the designer capable of making models with different levels of complexity based on rigid bodies, idealized joints and spring-damper elements [1]. An initial low-level model of the stabilizer bar can be modeled using two rigid bodies attached to each side of the wheel attachments and connected with a torsional spring at the centerline of the vehicle. The attachment to the car body can be done either with kinematic joints or bushing elements. As the need of a more sophisticated model grows, the stabilizer bar model can be expanded with more rigid elements and springs, for a better representation of the actual geometry and load path. This modeling approach is very attractive since most basic multibody dynamic codes have a library of rigid bodies, idealized joints and spring-damper elements. The stiffness coefficients of the idealized springs can be estimated using structural mechanics models or the finite element method.

A higher modeling fidelity of the stabilizer bar can be obtained using a flexible body generated using the finite element method in combination with the component mode synthesis (CMS) and the floating frame of reference (FFR) technique [2, 3]. Using this approach, a better structural representation is obtained while maintaining an acceptable computational effort. However, when using the FFR technique the model loses validity when the deformation of the stabilizer bar exceeds the linear range, which is not uncommon at limit maneuvers. A method to overcome this is to divide the stabilizer bar into smaller flexible bodies assembled using fixed joints. Hereby, the valid assumption of small deformations within each flexible body is preserved. However, this makes the modeling task more complex, adds more degrees of freedom and constraints to the model, and increases time consumption in both setting up the model and in computational time.

The absolute nodal coordinate formulation (ANCF) is a novel method developed specifically for the modeling of flexible components that undergo large deformations and rotations [4]. The ANCF is a finite element method that applies slope vectors instead of rotational degrees of freedom. Using these slope vectors, no assumptions are made regarding the rotation magnitude of the nodes, which means that models with ANCF finite elements can be solved without applying an incremental solution technique. This makes it very desirable for use in a multibody dynamics framework. The library of ANCF elements have been under development during the last decades such that it now consists of a wide variety of elements. Much research attention has been devoted to the development of ANCF beam elements such the library includes validated and proven beam elements that are considered locking free, robust and versatile. Some of the more advanced elements are capable of describing non-uniform cross section along the element span, cross section deformation, and material non-linearity [5], which makes them applicable for the modeling of stabilizer bars.

This study investigates the feasibility of using ANCF beam finite elements to model key components in a suspension system. A simple structural member is modeled using both the FFR and ANCF elements and several aspects such as necessary modeling complexity and computational effort are discussed. Finally, the implementation of the ANCF in the multibody dynamics framework is demonstrated using a rigid body model of the front suspension of a car including a stabilizer bar modeled using ANCF beam elements. The study concludes that the use of ANCF beam elements is feasible and may be used in a design phase, leading to acceptable precision with less modeling effort.

ABSOLUTE NODAL COORDINATE FORMULATION

A variety of different ANCF beam elements are available in the literature and many of them are mentioned in a recent review paper [4]. This present study aims to give a brief introduction to the general concept of the ANCF and then a more in-depth review of an appropriate element chosen to model the stabilizer bar.

Kinematics of ANCF Beam Elements

A general fully parameterized ANCF beam element is shown in Figure 1 with its global reference frame XYZ . The beam element is spanned by two end nodes and a set of slope vectors are defined at each node. The slope vectors are defined as the partial derivatives of the position field spanned by the volume of the beam with respect to the local element coordinates xyz as $\partial \mathbf{r} / \partial x$, $\partial \mathbf{r} / \partial y$, and $\partial \mathbf{r} / \partial z$, respectively. Here, $\mathbf{r}$ is the position of an arbitrary point inside the volume of the beam. An ANCF element is said to be fully parameterized when it uses slope coordinates with respect to all available spatial coordinates. It is important to stress that these slope vectors does not need to be orthogonal or to have constant lengths. This leads to a kinematic description capable of describing complex deformation modes, including transverse shear and cross section deformation [6].

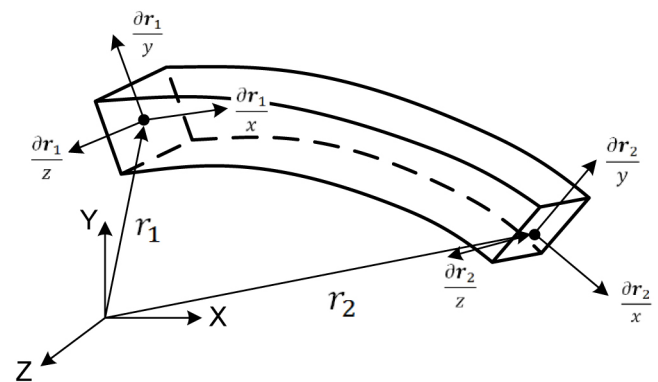


Figure 1. Fully parameterized ANCF beam element.

The position vector $\mathbf{r}$ of an arbitrary point inside the element volume is described by interpolation of the nodal degrees of freedom $\mathbf{e}$ using a set of interpolation shape functions $\mathbf{S}$:

$$\mathbf{r} = \mathbf{S} \cdot \mathbf{q} \quad (1)$$

In the ANCF, the vector of nodal degrees of freedom $\mathbf{q}$ for the element consists of the nodal positions $\mathbf{r}_i$ and a set of slope vectors. The number and types of slope vectors included in the vector of generalized coordinates can be chosen arbitrarily depending on the element application. For example, for the modeling of cables that have small cross sections and no torsional rigidity, a beam element that accommodates the nodal positions and the slope vectors in the axial direction $\partial \mathbf{r} / \partial x$ is sufficient [7]. In other cases where the shear and cross section deformation cannot be neglected, the transverse slope vectors $\partial \mathbf{r} / \partial y$, and, $\partial \mathbf{r} / \partial z$, can be included [4]. When an appropriate set of generalized coordinates has been chosen, the interpolation shape function matrix $\mathbf{S}$ can be derived.

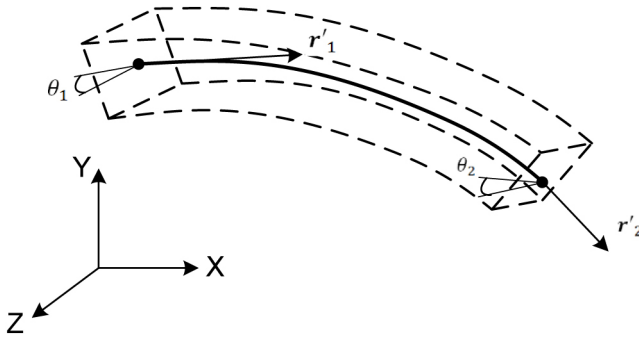


Figure 2. ANCF cable element with added rotational degree of freedom for measuring torsional deformation. The position vectors for the nodes have been omitted for clarity and the beam volume are shown with dashed lines since the beam is modeled as an elastic line.

A specific version of the cable element [8, 9] has been chosen for the modeling of the stabilizer bar (see Figure 2). This element has a simple parameterization leading to a low number of degrees of freedom, which is beneficial seen from a computational point of view. Furthermore, this element does not suffer from shear or Poisson's locking, which elements using slope vectors in the transverse direction does [4]. However, the original cable element [7] is not capable of describing torsion in its basic form. This, however, is necessary to capture the strong coupling between bending and torsion that arises in the stabilizer bar. The ability to describe torsion has been added by introducing a rotational degree of freedom θ_i at each node, see Figure 2. The rotation angle is used to define a rotation matrix that measures the orientation of the cross section with respect to a predefined reference vector, a so-called *director vector*. Using this rotation matrix, a point in the beam cross section is described by the vector $\mathbf{r}$ expressed as:

$$\mathbf{r}(x, y, z) = \mathbf{r}_c(x) + \mathbf{A}(x) \cdot \begin{bmatrix} 0 \\ y \\ z \end{bmatrix}, \quad (2)$$

where $\mathbf{r}_c$ is a vector describing the beam centerline, $\mathbf{A}$ is the rotation matrix and x, y and z are the local element coordinates. The vector $\mathbf{r}_c$ is obtained by interpolation of the nodal positions and the axial slope vectors at the nodes:

$$\mathbf{r}_c(x) = \mathbf{S}_r \cdot \mathbf{q}_r \quad (3)$$

where

$$\mathbf{S}_r = [\mathbf{S}_1 \mathbf{I} \quad \mathbf{S}_2 \mathbf{I} \quad \mathbf{S}_3 \mathbf{I} \quad \mathbf{S}_4 \mathbf{I}] \quad (4)$$

and

$$\mathbf{q}_r = [(\mathbf{r}_1)^T \quad (\mathbf{r}'_1)^T \quad (\mathbf{r}_2)^T \quad (\mathbf{r}'_2)^T]^T \quad (5)$$

In Equation (3) and (4), $\mathbf{S}_r$ is the shape function matrix associated with interpolation of the nodal position and slope vectors consisting of four individual shape functions S_i given by:

$$\begin{aligned} S_1 &= 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3} & S_2 &= x - \frac{2x^2}{L} + \frac{x^3}{L^2} \\ S_3 &= \frac{3x^2}{L^2} - \frac{2x^3}{L^3} & S_4 &= -\frac{x^2}{L} + \frac{x^3}{L^2} \end{aligned} \quad (6)$$

L is the un-deformed length of the beam element and x is the axial coordinate defined in the range $0 \leq x \leq L$. In Equation 5, $\mathbf{q}_r$ is the vector of nodal generalized coordinates and $\mathbf{I}$ is a 3×3 identity matrix. Note that the partial derivatives of the position with respect the axial coordinate x is denoted by $\mathbf{r}'_i = \partial \mathbf{r}_i / \partial x$.

The rotation matrix $\mathbf{A}$ is a function of the cross section orientation angle along the beam centerline $\theta(x)$. This angle is described by linear interpolation of the angles at the nodes:

$$\theta(x) = \mathbf{S}_\theta \cdot \mathbf{q}_\theta = \left[1 - \frac{x}{L} \quad \frac{x}{L} \right] \cdot [\theta_1 \quad \theta_2]^T \quad (7)$$

The orientation angle is used to describe three vectors $\mathbf{e}_i$ that form an orthogonal basis in which the cross section is described.

$$\mathbf{A}(x) = [\mathbf{e}_1(x) \quad \mathbf{e}_2(x) \quad \mathbf{e}_3(x)] \quad (8)$$

The three basis vectors are given by:

$$\begin{aligned} \mathbf{e}_1 &= \frac{\mathbf{r}'}{|\mathbf{r}'|} \\ \mathbf{e}_2 &= \mathbf{e}_{20} \cdot \cos(\theta) + \mathbf{e}_{30} \cdot \sin(\theta) \\ \mathbf{e}_3 &= \mathbf{e}_{30} \cdot \cos(\theta) - \mathbf{e}_{20} \cdot \sin(\theta) \end{aligned} \quad (9)$$

The two intermediate vectors $\mathbf{e}_{20}$ and $\mathbf{e}_{30}$ are given by:

$$\mathbf{e}_{30} = \frac{\check{\mathbf{e}}_{30}}{|\check{\mathbf{e}}_{30}|} \quad \check{\mathbf{e}}_{30} = \mathbf{d} - (\mathbf{d}^T \cdot \mathbf{e}_1) \mathbf{e}_1 \quad (10)$$

and

$$\mathbf{e}_{20} = \mathbf{e}_{30} \times \mathbf{e}_1 \quad (11)$$

The vector $\mathbf{d}$ in Equation (10) is the director vector, which can be chosen arbitrarily. However, by inspection of Equation (10) it is obvious that the director vector must be chosen such that it does not become parallel with the axial slope vector to avoid a singular

rotation matrix. In this study, it is chosen as $\mathbf{d} = [0 \ 0 \ 1]^T$ with success. In extreme cases, e.g. if the vehicle rolls over, the director may need to be updated during simulation using the algorithm described in [8].

Another characteristic regarding the kinematics of this particular ANCF beam element, is that the two components of the rotation matrix $\mathbf{e}_2$ and $\mathbf{e}_3$ are calculated such that they span the normal plane of the axial slope vector (see Equations (10) and (11)). This means that the beam cross section is constrained to remain orthogonal to the beam centerline. This property is the basis of the Euler-Bernoulli beam theory and by that, transverse shear deformation cannot be described.

Equations of motion

The equation of motion for the considered ANCF beam element can be derived using the Lagrange Equation and expressed in matrix form:

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{Q}_f - \mathbf{Q}_e \quad (12)$$

where $\mathbf{M}$ is the mass matrix and $\ddot{\mathbf{q}}$ is the vector of generalized accelerations consisting of both the nodal positions, nodal slopes and orientation angles: $\mathbf{q} = [(\mathbf{q}_r)^T (\mathbf{q}_\theta)^T]^T$. On the right hand side, $\mathbf{Q}_e$ is the vector of internal elastic forces arising due to deformation of the beam element and $\mathbf{Q}_f$ is the vector of external applied forces.

In general, the mass matrix is time invariant when considering ANCF elements [4], but in this case where the cross section is described using a rotation matrix, additional time dependent inertia terms must be added [8]. However, a good approximation of the mass matrix that is constant in time can be obtained as [9]:

$$\begin{aligned} \mathbf{M} &= \begin{bmatrix} \mathbf{M}_r & 0 \\ 0 & \mathbf{M}_\theta \end{bmatrix} \\ \mathbf{M}_r &= A\rho \int_0^L (\mathbf{S}_r)^T \mathbf{S}_r \, dx \\ \mathbf{M}_\theta &= I_x \rho \int_0^L (\mathbf{S}_\theta)^T \mathbf{S}_\theta \, dx \end{aligned} \quad (13)$$

Here, A is the cross section area of the beam, I_x is the polar moment of inertia and ρ is the material density. This approximation holds if the products $\dot{\mathbf{e}}_3^T \dot{\mathbf{e}}_3$ and $\dot{\mathbf{e}}_2^T \dot{\mathbf{e}}_2$ are not significantly different [9].

The vector of elastic forces is derived by calculating the partial derivative of the elastic energy with respect to the generalized coordinates:

$$\mathbf{Q}_e = \frac{\partial U_e}{\partial \mathbf{q}} \quad (14)$$

The internal elastic energy U_e is calculated as the sum of the energy stored in the beam due to axial elongation, bending and twisting of the beam centerline. The internal elastic energy is given by:

$$U_e = \frac{1}{2} \int_0^L \left(\underbrace{EA\varepsilon^2}_{\text{Axial}} + \underbrace{GI_x\kappa_1^2}_{\text{Torsion}} + \underbrace{EI_y\kappa_2^2 + EI_z\kappa_3^2}_{\text{Bending}} \right) dx \quad (15)$$

where E is the Young's modulus, G is the shear modulus, and I_y and I_z are the moments of inertia with respect to the local y and z axis, respectively. The axial strain ε is calculated using the Green strain:

$$\varepsilon = \frac{1}{2} (\mathbf{r}'^T \mathbf{r}' - 1) \quad (16)$$

The torsion κ_1 and the bending about the two principle axes of the cross section κ_2 and κ_3 , respectively, are calculated based on the basis vectors $\mathbf{e}_i$ (see Equation (8) and (9)) that defines the rotation matrix:

$$\begin{Bmatrix} \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix} = \boldsymbol{\kappa} = \sum_{i=1}^3 k_i \cdot \mathbf{e}_i \quad (17)$$

In which

$$\begin{Bmatrix} k_1 \\ k_2 \\ k_3 \end{Bmatrix} = \mathbf{k} = \frac{1}{2} \sum_{i=1}^3 \mathbf{e}_i \times \mathbf{e}'_i \quad (18)$$

Again, the prime symbol denotes the derivative with respect to the axial coordinate x .

The expressions listed in Equation (16), (17), (18) are only valid for initially straight beam elements, and these apply in this case since a very simple model of a stabilizer bar is considered. In the case where initially curved elements must be utilized in the modeling, the expressions must be expanded to take the initial strain in to account [8].

Implementation of ANCF in the Multibody Dynamics Framework

The motion of a general multibody system is governed by the Newton-Euler equations that describe the relation between forces acting on the system and its accelerations. Furthermore, a set of algebraic constraint equations are added to model idealized joints. The equation of motion for the ANCF elements (see Equation (12)) can be directly implemented in these equations, such that combined equations of motion for the rigid bodies, ANCF elements and the