

Road Profile Estimation for Active Suspension Applications

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ABSTRACT

The road profile has been shown to have significant effects on various vehicle conditions including ride, handling, fatigue or even energy efficiency; as a result it has become a variable of interest in the design and control of numerous vehicle parts. In this study, an integrated state estimation algorithm is proposed that can provide continuous information on road elevation and profile variations, primarily to be used in active suspension controls. A novel tire instrumentation technology (smart tire) is adopted together with a sensor couple of wheel attached accelerometer and suspension deflection sensor as observer inputs. The algorithm utilizes an adaptive Kalman filter (AKF) structure that provides the sprung and unsprung mass displacements to a sliding-mode differentiator, which then yields to the estimation of road elevations and the corresponding road profile along with the quarter car states. The proposed method is tested through numerical analysis under MATLAB/Simulink environment using a multi-body vehicle model from CarSim software and road profile samples from the Federal Highway Administration's Long Term Pavement Performance (LTPP) database.

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INTRODUCTION

The design of an effective suspension system requires satisfying a number of conflicting requirements. The suspension setup has to ensure a comfortable ride while proving good road holding and cornering characteristics. Also, optimal contact between tires and the road surface is needed in various driving conditions in order to maximize braking capacity and thereby safety. The conventional suspension designs in use today come with preset characteristics and aim at achieving vibration isolation through passive means, which might not be able to resolve the trade-off between such conflicting performance requirements. Therefore, active systems with actuation ability are being proposed to allow adjustment of such characteristics on the go.

Active suspension systems are becoming available and feasible in a considerably increasing rate due to the increasing computational capacity of modern vehicles. They are designed not only to improve the ride quality and comfort but also the safety of the vehicle. In this context, active suspensions can improve the roll stability of the vehicle during extreme steering and cornering maneuvers that may be required to remain on the road or to avoid an obstacle. Moreover, as the active part of the suspension improves the cornering behavior, the suspension springs can be tuned for optimizing comfort and road holding.

The design of active suspensions can be approached from many avenues. The early introduction of the technology was by means of classical techniques, i.e. root locus, Bode, and Nichols plots [1]; which is followed by the implementation of conventional passive design analysis i.e. modal analysis ([2, 3, 4]); eigenvalue assignment [5]; sensitivity analysis [6]; regional pole constraints [7]; or parameterization and linear fractional transformation [8]. The majority of the research on the other hand has been focused on the use of modern control theories, i.e. velocity feedback [9]; model order reduction [10]; bond graph modeling methodologies [11]; optimization -e.g. nonlinear programming and multi-criteria optimization-[12, 13, 14, 15, 16] optimal control [17, 18, 19, 20], as well as on more practical oriented methods i.e. simulation using multi-body dynamics computer models [21, 22, 23, 24] or design of experiments [25]. More recent studies extend to the use of advanced algorithms in controls i.e. neural networks, fuzzy-logic, evolution strategy and learning automata [26, 27, 28, 30]. While each of these approaches can bring useful perspectives, one significant aspect in accurate control of vehicle suspension is adaptation to fast alterations in dynamics and disturbances which can be provided by an accurate knowledge of the road excitations or the road profile. Numerous systems have been developed and studied for such measurements ([31, 32, 33, 34, 35]); nevertheless they are mainly designed for the purpose of serviceability and maintenance, hence

not feasible to go on-board of production line vehicles. Yet a common solution opted by many so far is to adopt a simplified form of an accelerometer based system on the vehicle's suspension. A significant challenge in this approach though is that the method is heavily limited and depending on sensor location and noise. This study details the development and application of a road profile estimation methodology taking advantage of the robust nature of a higher order sliding mode observer (SMO) scheme and integrated with an adaptive Kalman filter algorithm, which uses inputs from conventional sensors in addition to a prototype tire state measurement system (TSMS) developed at Virginia Tech Center for Tire Research (Figure 1) that proposes instrumenting the tire with three-axial accelerometers for dynamically monitoring tire road contact [36, 37, 38, 39]. In what follows the observer is combined and evaluated together with an adaptive control algorithm [40, 41] for active suspension systems to improve occupant comfort while maintaining sufficient road holding. Simulations using a multibody vehicle model shows the proposed method is capable of reducing the sprung mass vibrations below the comfort limit as indicated in ISO-2631 [42] as well as guaranteeing stability.

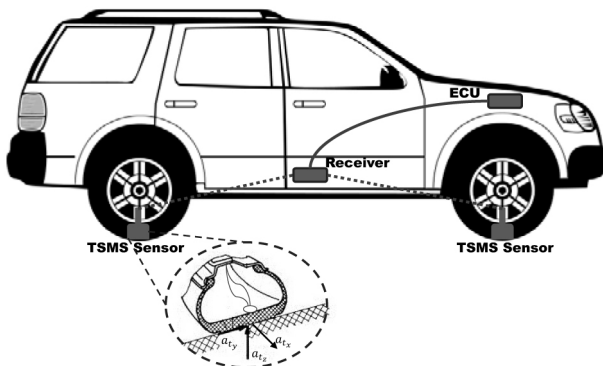


Figure 1. Concept diagram for the proposed smart tire sensor

The remaining of this paper is organized as follows: [Section 2](#) summarizes the mathematical background to model the vehicle suspension utilized in the design of the proposed observer and controller, [Section 3](#) introduces the observer structure that details the filter and SMO designs finishing with a comparative numerical study, [Section 4](#) outlines the proposed adaptive control structure and the results of a numerical analysis that puts the proposed system in test using a multi-body vehicle dynamics model, finally the paper is concluded by a discussion on the simulation results with possible pros and cons.

MATHEMATICAL MODEL

The observer dynamics are based on a quarter-car ride model as shown in [Figure 2](#). The model represents each individual corner of the vehicle body and consists of the vehicle chassis (sprung mass) attached to the respective wheel-tire assembly (unsprung mass) that yields to a two-degree-of-freedom quarter-car model. The masses (m_b and m_w as sprung and unsprung respectively) are assumed to move free only in the vertical and the passive suspension system is taken as linear viscous damper (b_s) and spring (k_s) element attached in parallel,

while the tire is modeled as a linear spring considering its vertical stiffness effect (k_t). In what follows, applying the principles of force balance analysis yields to the equations of motion as shown in (1):

$$\begin{aligned} m_b \ddot{z}_b &= -k_s(z_b - z_w) - b_s(\dot{z}_b - \dot{z}_w) + f_s \\ m_w \ddot{z}_w &= k_s(z_b - z_w) + b_s(\dot{z}_b - \dot{z}_w) - k_t(z_w - r) - f_s \end{aligned} \quad (1)$$

Based on the set of equations, the system states are described as:

$$z = [z_b \ z_w]^T$$

where $z \in \mathbb{R}^2$ and the variables z_b and z_w represent the displacement of the sprung and unsprung mass respectively, r stands for the road disturbance, and f_s is the force element presumably exerted by the additional active suspension. Using these definitions, equations (1) can be recapitulated as:

$$M \ddot{z} + B \dot{z} + Kz = U \quad (2)$$

The inertia matrix $M \in \mathbb{R}^{2 \times 2}$ is given as $\text{diag}(m_b, m_w)$. The damping $B \in \mathbb{R}^{2 \times 2}$ and stiffness $K \in \mathbb{R}^{2 \times 2}$ matrices are composed of the corresponding constants for individual damper and spring elements.

$$B = \begin{bmatrix} b_s & -b_s \\ -b_s & b_s \end{bmatrix}, \quad K = \begin{bmatrix} k_s & -k_s \\ -k_s & k_s + k_t \end{bmatrix} \quad (3)$$

The input to the system is defined in terms of the actuator forces and the disturbances that characterize the road profile:

$$U = B_f f_s(t) + B_d d(t) \quad (4)$$

where $B_f, B_d \in \mathbb{R}^{2 \times 2}$ define constant parameters and f_s and d represent the active suspension force and road disturbance respectively.

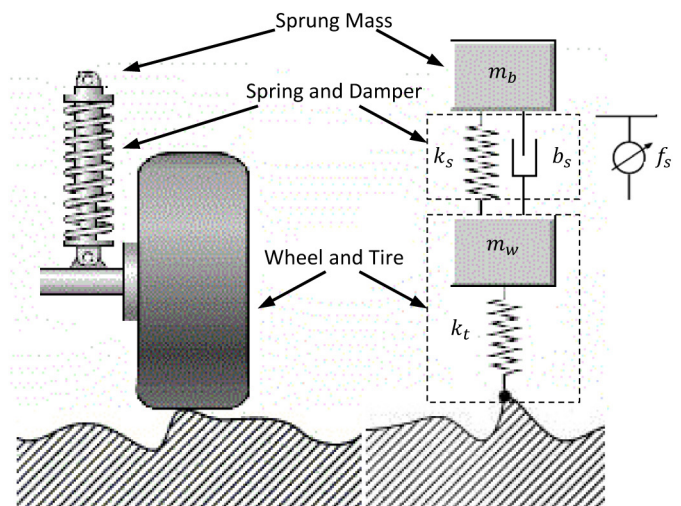


Figure 2. The quarter-car model used for observer dynamics

OBSERVER DYNAMICS

A. Second Order Sliding Mode Observer (SMO)

Equation (2) can be considered in the following state-space form as a MIMO system:

$$\begin{aligned}\dot{X} &= AX + GU \\ y &= CX\end{aligned}\quad (5)$$

where the state vector is chosen as $X = [x_1^T \ x_2^T]^T$ with $x_1 = z$ and $x_2 = \dot{z}$. More explicitly:

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -M^{-1}[(Kx_1 + Bx_2) + U]\end{aligned}\quad (6)$$

and the system output is taken as the position and speed of the states which are available through an AKF scheme and sensor fusion of conventional systems with a smart tire system as summarized in the next section.

Based on the assumptions defined by Isidori [43], this system can be considered locally detectable as one can conclude by direct computation that:

- the system's total relative degree $\sum r_i < n$;
- the output distribution $\Gamma \triangleq \{span\{g_i\} | g_i \in G\}$ is involutive;
- and the internal dynamics of the system are asymptotically stable for the bounded inputs (e.g. $\exists F$ and D such that $|f(t)| \leq F$ and $|d(t)| \leq D$)

Fridman et al. [44] details the problem formulation for developing a higher-order SMO structure. Following the formulation to estimate the state vector X and to deduce the unknown road profile in this study, the following second-order sliding mode differentiator is proposed,

$$\begin{aligned}\xi_1 &= -H_1|x_1 - \hat{x}_1|^{1/2}sgn(x_1 - \hat{x}_1) \\ \xi_2 &= -H_2sgn(x_1 - \hat{x}_1)\end{aligned}\quad (7)$$

where H_1 and H_2 are diagonal observer gain matrices and $\hat{x}_1$ and $\hat{x}_2$ are state estimations. In what follows, the sliding mode observer dynamics are given as:

$$\begin{aligned}\dot{\hat{x}}_1 &= \hat{x}_2 + \xi_1 \\ \dot{\hat{x}}_2 &= -M^{-1}(K\hat{x}_1 + B\hat{x}_2) + U + \xi_2\end{aligned}\quad (8)$$

The convergence of the results can be analyzed using the dynamics of the estimation errors:

$$\begin{aligned}\dot{\tilde{x}}_1 &= \tilde{x}_2 - H_1|\tilde{x}_1|^{1/2}sgn(\tilde{x}_1) \\ \dot{\tilde{x}}_2 &= -M^{-1}(K\tilde{x}_1 + B\tilde{x}_2) + U - H_2sgn(\tilde{x}_2)\end{aligned}\quad (9)$$

The right hand side of equation (9) is assumed to be enlarged, in the Fillipov sense, to satisfy the upper semi-continuity property. Further assuming the existence of its solution for any initial condition as infinitely extendible in time, the derivative of $\tilde{x}_1$ satisfies:

$$\ddot{\tilde{x}}_1 \in [U^-, U^+] - \left(\frac{1}{2} H_1 \frac{\dot{\tilde{x}}_1}{|\tilde{x}_1|^{1/2}} + H_2sgn(\tilde{x}_1) \right)\quad (10)$$

where U^- and U^+ represent the minimum and maximum system inputs. In what follows, letting $\tilde{x}_1, \dot{\tilde{x}}_1 > 0$ in equation (10) leads to

$$\ddot{\tilde{x}}_1 \leq U^+ - H_2sgn(\tilde{x}_1) - \left(\frac{1}{2} H_1 \frac{\dot{\tilde{x}}_1}{|\tilde{x}_1|^{1/2}} \right) < 0\quad (11)$$

Thus we can conclude the trajectory $\tilde{x}_1$ approaches the axis $\dot{\tilde{x}}_1 = 0$. Next, considering the dynamics of $\tilde{x}_2$, one can obtain:

$$0 < H_2 - U^+ \leq |\dot{\tilde{x}}_2| \leq H_2 + U^+\quad (12)$$

which holds in small proximity of the origin, and thus

$$\dot{\tilde{x}}_1 \geq (H_2 - U^+)t\quad (13)$$

where t is the time and hence we can state the total convergence time approaches to the finite value of:

$$t \leq \sum \frac{\dot{\tilde{x}}_1}{(H_2 - U^+)}\quad (14)$$

so that the estimated states converge to the real values in finite time. A more detailed discussion of the convergence analysis is given by Davila et al in [45].

The proposed SMO scheme yields to successful estimates of the quarter-car states with first and second derivatives which are utilized to reconstruct the unknown road profile disturbance. The dynamics as given in (1) provide a basis equation for computing an estimated disturbance:

$$\hat{r}(t) = \left(\frac{m_w}{k_t} \right) a_w - ((k_s(z_b - z_w) - b_s(\dot{z}_b - \dot{z}_w) + k_t(z_w) - f_s))/k_t\quad (15)$$