

Allocation-Based Control with Actuator Dynamics for Four-Wheel Independently Actuated Electric Vehicles

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ABSTRACT

This paper proposes a novel allocation-based control method for four-wheel independently actuated electric vehicles. In the proposed method, both actuator dynamics and input/output constraints are fully taken into consideration in the control design. First, the actuators are modeled as first-order dynamic systems with delay. Then, the control allocation is formulated as an optimization problem, with the primary objective of minimizing errors between the actual and desired control outputs. Other objectives include minimizing the power consumption and the slew rate of the actuator outputs. As a result, this leads to frequency-dependent allocation that reflects the bandwidth of each actuator. To solve the optimization problem, an efficient numerical algorithm is employed. Finally the proposed control allocation method is implemented to control a four-wheel independently actuated electric vehicle. Simulation results show that the proposed method is valid and effective in improving the actuator response and subsequently the handling performance.

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INTRODUCTION

Four-wheel independently actuated electric vehicles (FWIAEV) have been widely studied for over a decade [1-2]. Their individually and independently controlled driving and braking torques for each wheel offer great advantage through higher controllable degrees of freedom, faster and more precise control responses for improved vehicle performance in drivability, handling and stability [3]. Since the number of actuators exceeds the number of control variables, control allocation has been regarded as an effective approach in handling system over-actuation problems, and has been widely investigated in prior research [4]. The objective of control allocation for over-actuated vehicles is to find a set of control commands that are appropriately allocated on each actuator to fulfill the higher level control commands, in this case, desired vehicle body forces.

Although many control allocation methods have been proposed in literature, some of them such as redistributed pseudo-inverse, direct allocation, daisy-chain allocation, and optimization-based allocation [5] tend to treat control commands as scalars with magnitude and do not consider the phase effect. The dynamics of the actuators has been largely ignored on control allocation with the underlying assumption that actuators can and will respond instantaneously. However, the dynamic characteristics of each actuator can be far from ideal and different from one another, which could greatly impact the control performance both in magnitude and in phase. There was mounting research effort devoted in control allocation with actuator dynamics, which in general falls into two approaches. One is compensation-

based approach, which attempts to compensate actuator dynamics by overriding the output of magnitude-based control allocation; the other is frequency domain based approach, which deals with control allocation in frequency domain based on different bandwidths of actuators. Compensation-based approach is relatively simple and easy to use, and can noticeably increase the bandwidth of the overall system. For example, Oppenheimer [6] proposed a method that post-processed the output of control allocation to amplify the actuators' commands such that at the end of each sample interval the actual value is equivalent to the desired one. Härkegård modified the natural actuator dynamics by a feedforward compensator to increase the actuator's bandwidth [7]. However, compensation-based approach deals with control allocation process independent of actuators' bandwidths. Therefore, even assuming high bandwidth actuators, the fast response characteristics of these actuators cannot be effectively utilized. Furthermore, compensation-based approach is realized by amplifying the actuators' commands, thereby potentially subject to higher cost or physical limitations such as bus voltage. On the other hand, frequency domain-based approach can overcome the weakness of the compensation based approach, given it is independent of input limitation of actuators, while increasing the bandwidth of the overall systems. For example, Davison [8] proposed a frequency apportioned control allocation method, which partitioned the control commands into high and low frequency components using a low-pass filter, and respectively distributed them to either a faster or a slower group of actuators. However, frequency-domain based approach is not as straight forward as it need to choose, for example, the cutoff

frequency of the low-pass filter [8] or the weighting matrices [7], mainly as a trade-off between utilization and wear out of high-bandwidth actuators. Some other studies [9] proposed to use model predictive control (MPC) method since it is straightforward in handling actuator dynamics, however, it is often computationally prohibitive for practical applications.

A novel allocation-based control method for four-wheel independently actuated electric vehicles is proposed. The main contribution of this paper is in proposing an allocation-based method which takes actuators' dynamics along with input and output constraints into an optimization problem. An additional objective which minimizes the slew rate of the actuator outputs leads to frequency-dependent allocation that reflects the bandwidth of each actuator. To solve the optimization problem, an efficient numerical algorithm is employed. Finally the proposed control allocation method is implemented to control a four-wheel independently actuated electric vehicle.

The remainder of the paper is organized as follows: firstly, the proposed allocation method with actuator dynamics is derived and the effect is verified by an example. Secondly, the proposed method is established and applied to a four-wheel independently actuated electric vehicle. Finally, simulation results are presented followed by discussion and conclusive remarks.

ALLOCATION WITH ACTUATOR DYNAMICS

Basic Problem and Constraints

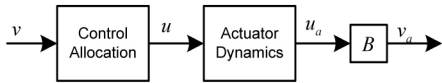


Figure 1. Schematic of control allocation

Assume the relationship between the virtual control inputs and the output of actuators is linear, Fig.1 illustrates the structure of control allocation problem, where v is the higher level control command vector, u is the control command vector of actuator, u_a is the output of actuator and v_a is the virtual control input vector. The basic problem of control allocation is to find a set of control commands of actuator to make the virtual control input equal to the higher level control command, i.e. $v_a = v$, it is often expressed by:

$$v = Bu_a \quad (1)$$

To simplify the derivation, actuator dynamics is treated as a first-order system:

$$\frac{U_{a,i}(s)}{U_i(s)} = \frac{1}{\tau_i s + 1} \quad (2)$$

where τ_i is the time constant of i th actuator. The transfer function of the actuator dynamics (2) can be discretized as follows:

$$u_{a,i}(k+1) = u_{a,i}(k) + \frac{\Delta t}{\tau_i} (u_i(k) - u_{a,i}(k)) \quad (3)$$

where Δt is the sampling interval. Substituting (3) into (1) with u as variable:

$$v(k) - Bu_a(k) + \Delta t \cdot BW_\tau^{-1} u_a(k) = \Delta t \cdot BW_\tau^{-1} u(k) \quad (4)$$

where $W_\tau = \text{diag}(\tau_1, \tau_2, \dots, \tau_n)$.

Other actuators' constraints, such as position and slew rate, can be expressed by:

$$u_{a,\min} \leq u_a \leq u_{a,\max} \quad (5)$$

$$|\dot{u}_a| \leq u_{a,\text{rate}} \quad (6)$$

where $u_{a,\min}$ and $u_{a,\max}$ are the lower and upper bounds of actuators' positions and $u_{a,\text{rate}}$ are the slew rate limits. Assume small step size in numerical computation, it is reasonable to re-write the constraints (5) and (6) into an equivalent position constraint as:

$$\underline{u}_a(k+1) \leq u_a(k+1) \leq \bar{u}_a(k+1) \quad (7)$$

Where,

$$\begin{aligned} \underline{u}_a(k+1) &= \max(u_{a,\min}(k+1), u_a(k) - u_{a,\text{rate}} \cdot \Delta t) \\ \bar{u}_a(k+1) &= \min(u_{a,\max}(k+1), u_a(k) + u_{a,\text{rate}} \cdot \Delta t) \end{aligned} \quad (8)$$

In practice, the actuators' control commands are also restricted by constraints, which usually include only the actuator position constraint:

$$u_{\min} \leq u \leq u_{\max} \quad (9)$$

Substituting (3) into (7) and merging the result with (9), the constraints with u as variables can be obtained:

$$\underline{u}(k) \leq u(k) \leq \bar{u}(k) \quad (10)$$

where

$$\begin{aligned} \underline{u}(k) &= \max\left(\frac{W_\tau}{\Delta t} (\underline{u}_a(k+1) - u_a(k)) + u_a(k), u_{\min}(k)\right) \\ \bar{u}(k) &= \min\left(\frac{W_\tau}{\Delta t} (\bar{u}_a(k+1) - u_a(k)) + u_a(k), u_{\max}(k)\right) \end{aligned} \quad (11)$$

Bandwidth-Based Allocation

Allocation with actuator dynamics is realized by solving a constrained optimization problem. The primary objective is to minimize the error between the virtual control input and the higher level control command, which guarantees the existence of an optimal solution. To obtain a unique solution, a secondary objective is introduced to minimize power consumption. Besides, a third objective is added with the aim of minimizing the change of the actuator output in two consecutive sampling instants. These three objectives are expressed as below, respectively:

$$\begin{aligned} J_1 &= \|W_v(v(k) - Bu_a(k+1))\|_2 \\ J_2 &= \|W_u u_a(k+1)\|_2 \\ J_3 &= \|W_{du}(u_a(k+1) - u_a(k))\|_2 \end{aligned} \quad (12)$$

where W_v , W_u and W_{du} are diagonal matrices to adjust the weights, $\|\cdot\|_2$ denotes the Euclidean 2-norm defined by $\|u\|_2 = \sqrt{u^T u}$.

It should be pointed out that J_3 is a key to the bandwidth-based allocation. The high bandwidth actuators should take on more effort when the higher level control command changes rapidly and the output increment of actuators should be proportional to their bandwidth. Accordingly, the weight matrix W_{du} is chosen as:

$$W_{du} = \left(\prod_{i=1}^n \omega_{\text{bandwidth},i} \right) \text{diag}(\omega_{\text{bandwidth},1}, \omega_{\text{bandwidth},2}, \dots, \omega_{\text{bandwidth},n})^{-1} \quad (13)$$

where $\omega_{\text{bandwidth},i}$ is the bandwidth of i th actuator.

Substituting (3) into (12), the three objectives which take u as variables can be alternatively described as:

$$\begin{aligned} J_1 &= \|W_v(v(k) - Bu_a(k) + \Delta t \cdot BW_\tau^{-1} u_a(k) - \Delta t \cdot BW_\tau^{-1} u(k))\|_2 \\ J_2 &= \|W_u(u_a(k) - \Delta t \cdot W_\tau^{-1} u_a(k) + \Delta t \cdot W_\tau^{-1} u(k))\|_2 \\ J_3 &= \|W_{du} \cdot \Delta t \cdot W_\tau^{-1} (u(k) - u_a(k))\|_2 \end{aligned} \quad (14)$$

Then, the overall optimization problem can be formulated as:

$$\begin{aligned} \min \quad & J = p_1 J_1 + p_2 J_2 + p_3 J_3 \\ \text{s.t.} \quad & \underline{u}(k) \leq u(k) \leq \bar{u}(k) \end{aligned} \quad (15)$$

where p_1 , p_2 and p_3 are the coefficients to adjust the weight of each objective, with p_1 is much larger than p_2 or p_3 . Now, it is apparent that the optimization problem becomes a quadratic programming (QP) problem which is strictly convex. Therefore a feasible solution always exists and the problem always has a unique optimal solution. QP problems are usually solved by numerical methods such as active set method. A quadratic-programming (QP) solver suitable for real-time implementation [10] is used to find the solution.

Design Example

The proposed allocation method is applied to a design example which is also used in [7]. Via this design example, the proposed control allocation method is compared with the existing allocation methods mentioned in the introduction section.

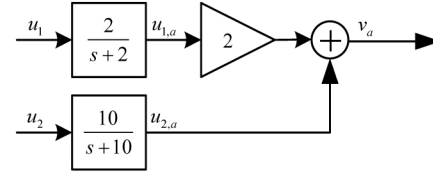


Figure 2. System with one slow and one fast actuator

Consider the system depicted in Fig. 2, with two actuators whose outputs $u_{1,a}$ and $u_{2,a}$ produce a total control effort of

$$v_a = 2u_{1,a} + u_{2,a} = Bu_a \quad B = \begin{bmatrix} 2 & 1 \end{bmatrix} \quad (16)$$

The actuators have first-order dynamics and their bandwidths are 2 and 10 rad/s, respectively. Thus, the first actuator is slow yet effective, whereas the second one is fast but less effective. The actuator position limits are given by $|u_{1,a}| \leq 1$, $|u_{2,a}| \leq 2$, $|u_1| \leq 1$, $|u_2| \leq 2$.

The allocation methods being compared in this paper for performance evaluation of the proposed method include constrained optimization (CO), compensation by time domain response (CT) [6], compensation in frequency domain (CF) [7], frequency-based allocation (AF) [8], and dynamic control allocation (DCA) [7]. The higher level control command is set to be a smoothing step. Based on extensive simulation, the control commands and actuator outputs are shown in Fig. 3, 4, 5, 6, 7, 8; The higher level control command and the virtual control input obtained based on each of the methods are shown in Fig. 9. For ease of notation, the proposed method in this paper is marked as AAD, referring to allocation with actuator dynamics.

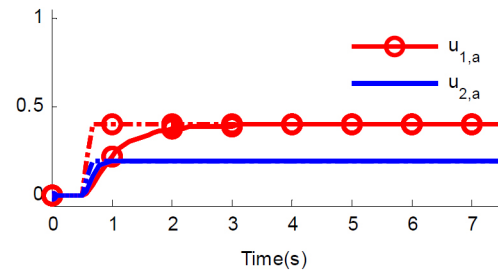


Figure 3. Results of constrained optimization

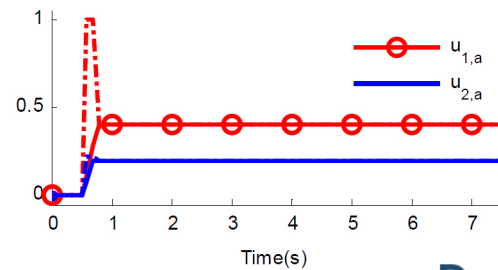


Figure 4. Results of compensation by time domain response