

New Slip Control System Considering Actuator Dynamics

Amir Soltani
Cranfield Univ.

Francis Assadian
Univ. of California

ABSTRACT

A new control strategy for wheel slip control, considering the complete dynamics of the electro-hydraulic brake (EHB) system, is developed and experimentally validated in Cranfield University's HiL system. The control system is based on closed loop shaping Youla-parameterization method. The plant model is linearized about the nominal operating point, a Youla parameter is defined for all stabilizing feedback controller and control performance is achieved by employing closed loop shaping technique. The stability and performance of the controller are investigated in frequency and time domain, and verified by experiments using real EHB smart actuator fitted into the HiL system with driver in the loop.

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INTRODUCTION

Wheel slip dynamics is characterized by highly nonlinear and uncertain behaviour of tire forces and fast changing dynamics of the wheel during braking. Because of the complex and variable dynamics of the system, design of a slip control system is a challenging task. Several linear and nonlinear control design approaches have been proposed in the literatures, ranging from linear and nonlinear PID control system [1], fuzzy logic [2], gain scheduling [3] to nonlinear methodologies such as sliding mode [4] and Lyapunov-based [5] control design approaches. However, it is worth noting that most of the proposed brake controllers, which can be found in the published literatures, consider wheel slip (and/or angular velocity) differential equations as the plant model, without taking the complete dynamics of the brake system (such as calliper and valve dynamics) and their constraints into consideration. Interestingly, it is shown in [5] that it is impossible to employ a single linear PID controller for complete brake dynamics to provide stability and performance at all operational conditions.

In this paper, a new wheel slip control system based on Youla parameterization approach is proposed [6]. The controller provides stability and good control performance over the whole range of operating conditions of the system, considers all the existing dynamics and constraints of the brake system. In the next sections, the mathematical model of the system is introduced, then the proposed closed loop control system based on Youla parameterization is presented, and finally the stability and performance of the control system is validated by simulation and HiL testing at different driving conditions.

SYSTEM MODELLING

A new control law for continuous control of wheel slip is presented in this paper. It is assumed that the smart brake actuation (i.e. brake pressure control) is already provided by an off-the-shelf electro hydraulic brake (EHB) system, also called Sensotronic Brake Control (SBC), which has been developed by Daimler and Bosch [7]. Schematic diagram of Bosch EHB SBC system is shown in Fig 1 [8].

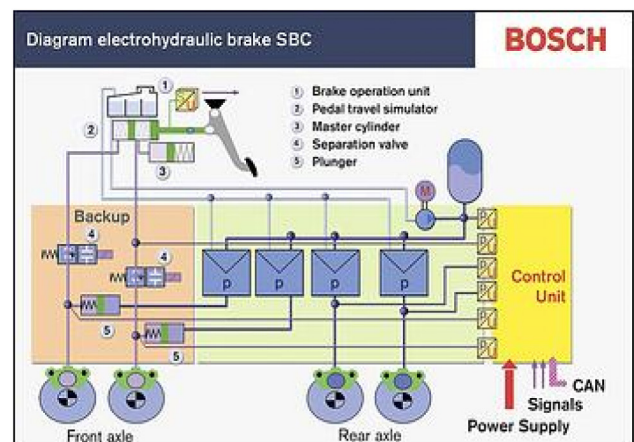


Figure 1. Schematic diagram of Bosch SBC system [8]

Employing electro hydraulic brake system as a smart brake actuator provides the possibility to control individual wheel brake line pressure to follow the desired target pressure continuously [9]. The relevant reference brake pressure for each wheel (to be generated by EHB system) is provided by the proposed slip control system.

continuous control over brake pressure provides several advantages such as driver comfort as well as the possibility of fast and precise control over the tires longitudinal force (and slip) [10].

As the brake pressure is regulated for each wheel individually (through EHB valve modulation unit, as shown in Figure 1), there are 4 independent inputs and 4 similar plants, and four similar SISO closed loop brake control systems exist in this architecture. Design of the closed loop brake control system based on wheel slip feedback for one wheel (i -th wheel), as shown in Figure 2, is presented in this section.

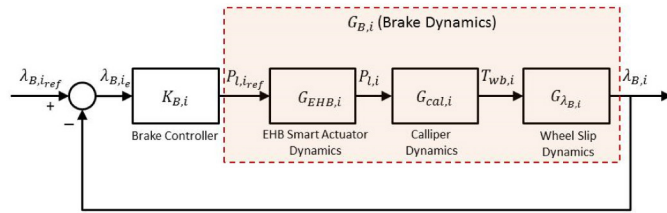


Figure 2. Closed Loop Brake Control system

The starting point in designing the control system is to derive the plant dynamics model. Here the plant, so called single corner wheel model, consists of the quarter of car mass moving in x direction with a single independent rotating wheel subject to braking torque $T_{b,i}$ at the wheel hub (centre) and longitudinal force $F_{x,i}$ at the centre of tire-ground contact patch, as shown in Figure 3. It should be noted that the quarter car model relies on some simplifications; however, this is a simple and effective model which considers the major braking dynamics and is widely applied in active braking control system designs [4, 5, 11].

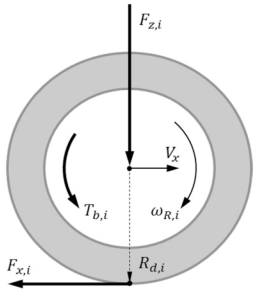


Figure 3. Wheel Dynamics Model

The equation of wheel dynamics is:

$$J_{y,i} \dot{\omega}_{R,i} = T_{b,i} - R_{d,i} F_{x,i} \quad (1)$$

where $J_{y,i}$ is the (i -th) wheel moment of inertia, $R_{d,i}$ is the wheel radius and $\omega_{R,i}$ is the wheel angular velocity. The quarter car dynamics in the longitudinal direction can be written as:

$$-\dot{V}_x m_s = -F_{x,i} \quad (2)$$

where m_s is the quarter car mass (as described above) and $\dot{V}_x$ is the vehicle longitudinal acceleration. The tire longitudinal force $F_{x,i}$ is a nonlinear function of slip $\lambda_{B,i}$, i.e.

$$F_{x,i}(\lambda_{B,i}) = \mu_{x,i}(\lambda_{B,i}) F_{z,i} \quad (3)$$

The tire slip ratio during braking $\lambda_{B,i}$ was defined as [7]:

$$\lambda_{B,i} = \frac{R_{d,i} \omega_{R,i}}{V_{x,i}} - 1 \quad (-1 \leq \lambda_{B,i} \leq 0) \quad (4)$$

where $V_{x,i}$ is the forward speed of the tire centre which is here assumed that is equal to vehicle longitudinal speed V_x .

Combining Eq. (1) with Eq. (4), the relationship between braking torque input and tire slip output can be derived as:

$$\dot{\lambda}_{B,i} = -\frac{1}{V_x} \left(\dot{V}_x (\lambda_{B,i} + 1) + \frac{R_{d,i}^2}{J_{y,i}} F_{x,i} \right) + \frac{R_{d,i}}{J_{y,i} V_x} T_{wb,i} \quad (5)$$

Substituting $\dot{V}_x$ from Eq. (2) into Eq. (5), the dynamics of the system can be formulated as a first order model of the wheel slip dynamics as:

$$\dot{\lambda}_{B,i} = -\frac{1}{V_x} \left(\frac{(\lambda_{B,i} + 1)}{m_s} + \frac{R_{d,i}^2}{J_{y,i}} \right) F_{x,i} + \frac{R_{d,i}}{J_{y,i} V_x} T_{wb,i} \quad (6)$$

Eq. (6) is a nonlinear function of two state variables V_x and $\lambda_{B,i}$.

To be able to design a linear control system, one should linearize Eq. (6) around nominal operating points $V_{x,n}$ and $\lambda_{B,i,n}$ (we will discuss this nominal operating point in the following paragraphs).

Considering $\Delta \lambda_{B,i} = \lambda_{B,i} - \lambda_{B,i,n}$ and $\Delta V_x = V_x - V_{x,n}$ and $\Delta T_{wb,i} = T_{wb,i} - T_{wb,i,n}$, the linear transfer function for plant dynamics (from brake torque input to tire slip output) can be derived by application of first order Taylor series expansion as [5]

$$G_{\lambda_{B,i}}(s) = \frac{\lambda_{B,i,n}}{T_{wb,i,n}} = \frac{R_{d,i}}{J_{y,i} V_{x,n}} \left[\frac{1}{s + \frac{\dot{\mu}_{x,i,n} F_{z,i}}{m_s V_{x,n}} \left((1 + \lambda_{B,i,n}) + \frac{m_s R_{d,i}^2}{J_{y,i}} \right)} \right] \quad (7)$$

where $\dot{\mu}_{x,i,n}(\lambda_{B,i,n}) = \frac{\partial \mu_{x,i}}{\partial \lambda_{B,i}}|_{\lambda_{B,i,n}}$, represents the slope of the $\mu_{x,i}$ curve around a nominal operating point. It should be noted that calculation of $\dot{\mu}_{x,i,n}$ in Eq. (7) is not dependent on any specific tire model. It could be obtained either by differentiation of a tire model, such as Burckhardt tire model as cited in the literatures (see [12] for example), or by implementation of an online algorithm to detect the sign of the friction curve slope, which is suitable for practical applications [5, 13].

The linearized plant (7) (herein after called plant) is of first order in form of:

$$G_{\lambda_{B,i}}(s) = \frac{k_i}{s + p_i} \quad (8)$$

with a gain of:

$$k_i = \frac{R_{d,i}}{J_{y,i} V_{x,n}} \quad (9)$$

and a single pole of:

$$p_i = \frac{\dot{\mu}_{x,i,n} F_{z,i}}{m_s V_{x,n}} \left((1 + \lambda_{B,i,n}) + \frac{m_s R_{d,i}^2}{J_{y,i}} \right) \quad (10)$$

To derive a control oriented linear plant model, it is essential to select an appropriate operating point, where the plant is being linearized around it. By studying the slip dynamics transfer function, Eq. (7), one can conclude that the tire normal load, $F_{z,i}$, the vehicle velocity, $V_{x,n}$, and the road condition (which is reflected on $\dot{\mu}_{x,i,n}$) are the main parameters that are considerably changing during braking actuation time period¹. Therefore, to investigate the variation of plant dynamics, it is required to derive the maximum and minimum values of longitudinal speed V_x , $F_{z,i}$ and $\dot{\mu}_{x,i}$ in the plant dynamics transfer function, Eq. (7).

As V_x is always positive (forward driving assumption), its variation cannot make the plant unstable, but, it will impact the value of gain (k_i). On the other hand, the change of $F_{z,i}$ and $\dot{\mu}_{x,i,n}$ will move the location of pole on s plan, which will have an effect on the stability of plant. More specifically, the effect of $\dot{\mu}_{x,i,n}$ variation on the plant dynamics is significant as it can make the plant unstable, as discussed below.

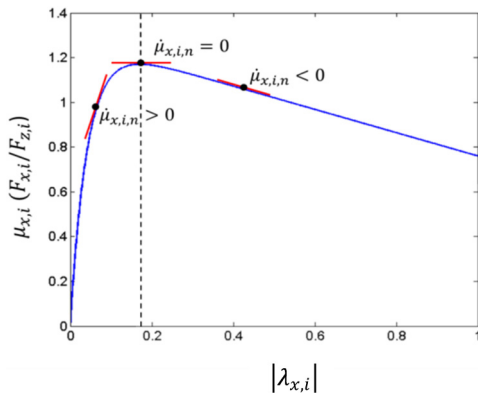


Figure 4. Slope of $\mu(x, i)$ at different operating points

Considering a typical longitudinal tire force/slip curve, as shown in Figure 4, one can conclude from Eq. (10) that the open loop transfer function $G_{\lambda_{B,i}}(s)$ is stable before slip peak (as $\dot{\mu}_{x,i,n} > 0$, therefore $p_i > 0$), and become unstable when the slip goes beyond the peak point (as $\dot{\mu}_{x,i,n} < 0$, therefore $p_i < 0$). Therefore, the worst case operating points are corresponding to high longitudinal speed V_x , low normal load $F_{z,i}$ and smallest positive slope of $\mu_{x,i}$ which is equivalent to the smallest distance of stable pole from imaginary axis in s plan. Moreover, when the linearization point is close to the peak of the curve, i.e. $\dot{\mu}_{x,i,n} \cong 0$, the plant becomes a pure integrator:

$$G_{\lambda_{B,i}}(s) = \frac{k_i}{s} \quad (11)$$

To obtain the value of tire friction slope, $\dot{\mu}_{x,i}(\lambda_{B,i})$, one can employ the Burckhardt tire model [12]. Based on Burckhardt tire model, the relationship between tire slip and normalised longitudinal tire force is defined as:

$$\mu_{x,i}(\lambda_{B,i}) = \frac{F_{x,i}(\lambda_{B,i})}{F_{z,i}} = c_1(1 - e^{-c_2\lambda_{B,i}}) - c_3\lambda_{B,i} \quad (12)$$

where c_1 , c_2 and c_3 are constants, as defined for different road surfaces in Table 1. Therefore

$$\dot{\mu}_{x,i}(\lambda_{B,i}) = \frac{\partial \mu_{x,i}(\lambda_{B,i})}{\partial \lambda_{B,i}} = c_1 c_2 e^{-c_2 \lambda_{B,i}} - c_3 \quad (13)$$

Table 1. Values of the Burckhardt tire model coefficients [12]

Road Surface	c_1	c_2	c_3
Asphalt, dry	1.2801	23.99	0.52
Asphalt, wet	0.857	33.822	0.347
Concrete, dry	1.1973	25.168	0.5373
Cobblestone, dry	1.3713	6.4565	0.6691
Snow	0.1946	94.129	0.0646
Ice	0.05	306.39	0

By investigating different road surfaces at different slips, one can conclude that the maximum and minimum values of $\dot{\mu}_{x,i}$ happen on dry asphalt at $\lambda_{B,i} = 0$ with the value of 30.2 and on dry cobblestone at slip $|\lambda_{B,i}| = 1$ with the value of -0.658 , respectively. Therefore the slip slope bound is:

$$-0.658 \leq \dot{\mu}_{x,i} \leq 30.2 \quad (14)$$

The tire normal load on each wheel $F_{z,i}$ is a function of the vehicle weight distribution (static load) and load transfer due to longitudinal and lateral accelerations (dynamic loads), which can be estimated from the following equations [14]:

1. By other word, it is assumed that the vehicle mass, wheel inertia, and wheel radius are constant.