

Analytic Engine and Transmission Models for Vehicle Fuel Consumption Estimation

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ABSTRACT

A normalized analytical vehicle fuel consumption model is developed based on an input/output description of engine fuel consumption and transmission losses. Engine properties and fuel consumption are expressed in mean effective pressure (mep) units, while vehicle road load, acceleration and grade are expressed in acceleration units.

The engine model concentrates on the low rpm operation. The fuel mep is approximately independent of speed and is a linear function of load, as long as the engine is not knock limited. A linear, two-constant engine model then covers the speed/load range of interest. The model constants are a function of well-known engine properties. Examples are discussed for naturally aspirated and turbocharged SI engines and for Diesel engines.

A similar model is developed for the transmission where the offset reflects the spin and pump losses, and the slope is the gear efficiency. Examples are given for manual, planetary automatic and continuously variable transmissions.

The engine and transmission models are combined into a vehicle model where the fuel consumption is proportional to the vehicle power requirement plus the mechanical powertrain losses, with an overall factor for the marginal energy conversion efficiency. The mechanical loss term is proportional to the product of the displacement-to-mass ratio and the rpm-to-vehicle speed ratio. The model is used on vehicles in the US EPA "Test Car List" database to break down the fuel use in the Highway test into its components.

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INTRODUCTION

The purpose of this paper is to develop a first order analytical model for the fuel consumption of conventional vehicles with no energy recuperation and storage. The intent is not to match the accuracy potential of the numerical models used in the industry to predict in detail the effects of going from a current vehicle to a future vehicle. Rather, the uses of an analytical model are to:

- identify the main parameters, and their structural relations
- identify the relative magnitude of useful work and losses
- identify the potential and limitations for reducing the losses
- show fundamental trends in a simple and general form.

Detailed numerical models are unwieldy for such purposes, and rules-of-thumb do not show interactions of parameters or the effects of a changed baseline.

An approach found to be useful in the modeling of many different energy conversion systems [1] is to look at output power versus input power. Often it is found that the relation is approximately linear, which is very convenient for modeling. The slope of the line reflects

the efficiency in converting from one energy form to another, and the offset represents the 'overhead' involved in running the machine itself. This observation was also made for steam engines [2], and it has since been known as a 'Willans line', even when applied to other engines or motors.

There appears to have been historically fairly wide recognition that internal combustion engine fuel use increases proportionally with output, with an offset related to engine losses and a slope related to indicated efficiency. Schweitzer [3] showed how the fuel could usefully be scaled to energy flow per cycle and unit displacement by expressing it as a fuel mean effective pressure (MEP). The short paper enumerated the advantageous properties of this approach, but the single example did not leverage it. Pachernegg [4] revisited the topic, and discussed the implications of running engines at very light loads versus motoring them to estimate friction.

Ross & An [5] studied fuel consumption data for a large set of engines. They found that the proportionality of fuel use per cycle and output per cycle is independent of engine speed in the commonly used speed range. This means that the engine behaves the same way in each cycle across this speed range. The linear model with two

constants covers not only a range of engine loads, but also a range of engine speeds, which makes the model much more useful. They also established that the loss term was proportional to engine displacement and speed, but they did not see the advantage of working in mep normalized units. Finally, they showed that the slope and offset for all the engines tested fell into a narrow range. Therefore, this work shows that a two-coefficient model can provide a reasonable first order estimate of fuel consumption as a function of engine displacement, load and speed.

Ross and collaborators [6,7,8] then used the simple relation between fuel use and engine output to develop models of vehicle fuel consumption. This method of evaluating fuel consumption was later used as an intermediate step in the modeling of vehicle emissions [9,10,11,12,13]. The ability to generate a reasonable estimate of fuel consumption for most vehicles with just a single set of coefficients was useful.

The fuel mep approach has also been used to compare different combustion systems [14], and to clarify the effect of weight reduction on fuel consumption [15].

The above studies treated the linear relation between power and fuel flow as a useful and convenient experimental observation. The observed model constants were not expressed as a function of engine properties, which may be one reason the models weren't used much for engine or fuel economy development. It was shown in [16] that, by working in mep units, the model constants follow directly from a fundamental description of the engine process in terms of established properties. Here it will be shown that it also applies to boosted Spark Ignition (SI or 'gas') and Compression Ignition (CI or Diesel) engines, and that it is sensitive enough to quantify the fuel economy benefits of different engine technologies.

Earlier studies of transmission efficiency [17,18] have shown that the losses in a gearbox often also increase approximately linearly with speed and load. The same methods of scaling and using separate conversion efficiency and 'overhead' losses are therefore applied to model transmission efficiency in a compact manner. The method is shown to work well for manual, planetary automatic and continuously variable transmissions.

The models for engine and transmission will be combined into an overall model of powertrain fuel flow as a function powertrain output power. During power-on driving in conventional vehicles without energy recovery or storage, the powertrain output power must match the demanded vehicle driving power. Examples of uses of the model will be given at the end of the paper.

The system being considered is shown in figure 1. Fuel energy is provided to the engine at a finite rate, mechanical power leaves the engine, to go through a torque converter and then a gearbox. The losses incurred at each step of the process, which end up as heat, are estimated. Properties at different stages of the process, such as energy flow, power, torque or rotation speed, are distinguished by the

subscripts shown in figure 1. The powertrain output is the power required by the vehicle. Similarly, to develop the powertrain model, a description of each step in the process is found, and then the steps are combined into an overall model which determines how much fuel is needed to achieve the desired power under given conditions.

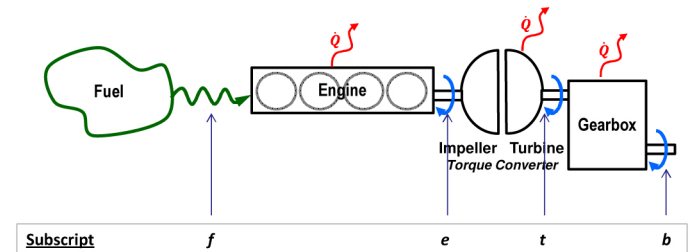


Figure 1. Fuel is provided to a system consisting of an engine, a torque converter and a gearbox. The output of the gearbox is the power required by the vehicle. Subscripts are used to differentiate different stages of the process.

NATURALLY ASPIRATED ENGINES

In this section useful metrics of engine fuel consumption and the linear first order model previously developed [16] are reviewed. The method is then used to quantify the benefit of a few technologies.

Engine Fuel Consumption Metrics

The fuel consumption of a conventional automotive naturally aspirated spark ignition engine is shown in figure 2. The traditional metric of brake specific fuel consumption (bsfc) is shown as a function of engine load in the most used speed range from 1,500 to 2,500 rpm.

The engine is a 2 liter four cylinder inline, with 4 valves per cylinder, dual overhead cams, port fuel injection, and a central spark plug, with a rated output of 107kW at 6,000 rpm. It was typical of the state of the art when it was introduced [19] in the year 2000.

Recognizing that engines large and small behave similarly, but that absolute numbers are difficult to compare for engines of different sizes and operating at different speeds, it is customary [20] to scale torques by engine displacement, and rate variables like power by engine displacement and per engine cycle instead of per unit of time,

$$bmep = P / (Dn/2) = 4\pi T / D \quad (1)$$

thus defining the brake mean effective pressure, *bmep*, where *P* is engine power, *T* is engine torque, *D* is the displacement and *n* is engine speed. The factor ½ is introduced because on 4 stroke engines one cycle takes two engine revolutions.

The curves for bsfc show the typical 'fish hook' shape as a function of brake mean effective pressure (*bmep*), and the fuel consumption behavior does not change appreciably with engine speed at the lighter loads. At the highest loads the bsfc is degraded at 1,500 rpm due to knock sensitivity, but this is nearly eliminated by 2,500 rpm.

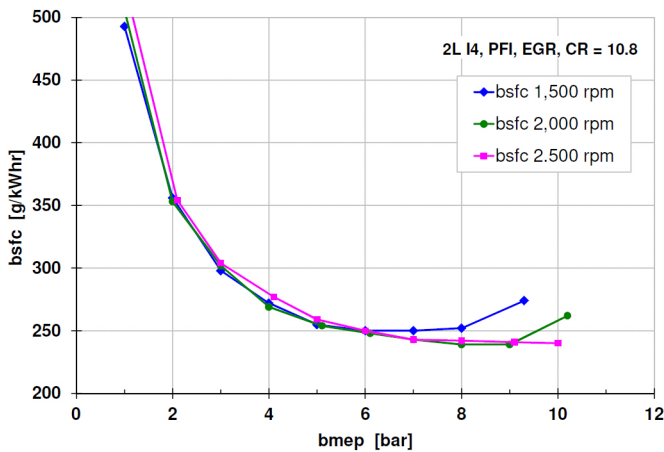


Figure 2. Brake specific fuel consumption (bsfc) of an automotive 2L PFI NA SI engine as a function of load (bmep) over a range of typical engine speeds.

As a metric of the engine's fuel consumption, bsfc has some deficiencies. The first is that it is unsuitable for interpolation or extrapolation at light loads, because of the singularity at zero load. Another concern is that bsfc data is worthless without knowing the heating value of the fuel. The heating value of US gasoline typically varies $\pm 2\%$, and ethanol/gasoline blends can vary much more. Finally, the 'fish hook' curves do not provide any clue that an engine works in a nearly linear fashion.

One way to make the linear operation of the engine visible is to plot absolute metrics, for example fuel mass flow rate versus engine power, as in [15]. This has the disadvantage that no impression of the efficiency can be gained, because of the mixed units. It is then better to convert the fuel mass flow rate to a fuel energy flow rate by using the heating value, as shown in figure 3.

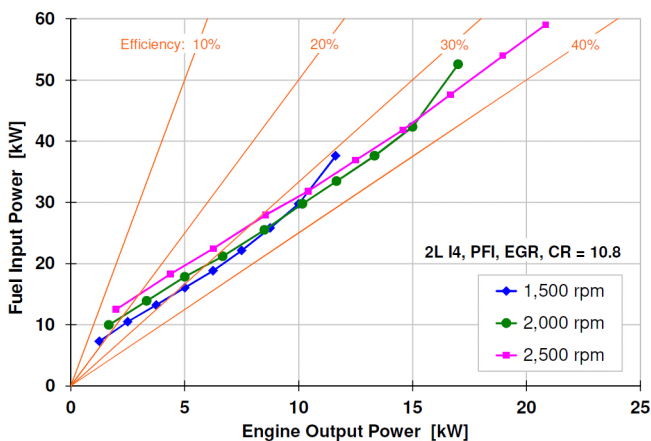


Figure 3. Absolute fuel input power of an automotive 2L PFI NA SI engine as a function of output power over a range of typical engine speeds. Same engine as in figure 2.

In this representation, the linear behavior at part load is obvious, and one sees that the marginal efficiency of the engine, i.e. the slope of the fuel consumption lines, is constant in this speed range. This representation also makes clear that at a constant low power level, the fuel usage increases markedly with engine speed. Finally, as input

and output are expressed in the same units, lines of constant efficiency are diagonals, and an impression can be gained of how efficiency varies.

To normalize the fuel flow, a fuel energy flow rate, P_f , is first defined as the fuel mass flow rate multiplied by the lower heating value of the fuel, H_p ,

$$P_f = H_p \dot{m}_f \quad (2)$$

This fuel 'power' can be normalized for displacement D and engine speed n , just as power is normalized to bmep in (1), to define a 'fuel mean effective pressure', $fumep$,

$$P_f = \frac{1}{2} D n fumep \quad (3)$$

where the factor $\frac{1}{2}$ is for the four stroke cycle that takes two revolutions. One can convert between the different ways of expressing engine efficiency, bsfc, efficiency and fumep using

$$fumep = H_p bmep bsfc / 3600 = bmep / \eta(bmep) \quad (4)$$

The same engine data is re-plotted in figure 4 as inputs and outputs normalized per unit displacement, and normalized per engine cycle instead of per unit time, i.e. as $fumep(bmep)$.

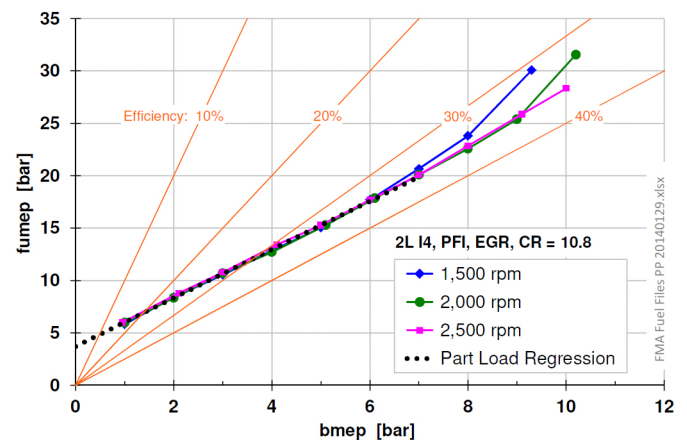


Figure 4. Normalized fuel input per cycle (fumep) of an automotive 2L PFI NA SI engine as a function of normalized load (bmep) over a range of typical engine speeds. Same engine as in figures 2 and 3.

Now the measurements at the three engine speeds collapse onto a single straight line at part load, there is no singularity or zero at zero load, and the results are independent of the heating value of the fuel. Numerical interpolation or extrapolation to lighter loads is straightforward, and a visual check will identify suspicious data points. The visual check of experimental data points is not possible at light loads in the traditional bsfc representation such as figure 2, because of the singularity at zero load. Fuel mep increases nearly linearly over most of the load (bmep) range, except at the highest loads, and a finite amount of fuel is needed at zero output. This offset

is the energetic cost of running the engine itself. As a result, the higher the engine load, the higher the efficiency, as the penalty for running the engine becomes a smaller fraction of the fuel used.

The linear approximation is clearly not valid near full load, where the engine becomes knock limited. Ignition and combustion then need to be retarded from the most efficient location, so that bsfc reaches a minimum. This makes this approach less suitable for the modeling of Hybrid Electric Vehicles that operate their engine mainly near the minimum fuel consumption condition [21]. For conventional vehicles without energy storage, where the powertrain output needs to match the demanded power instantaneously, the situation is different.

The distribution of speed/load points in an engine map over the EPA drive cycles for a vehicle with an advanced automatic transmission are shown in figure 5. The operating points are seen to be concentrated in the low speed and low to mid load ranges. This is what the advanced automatic transmission does to reduce fuel consumption. The high loads are only rarely used on the EPA City, Highway or NEDC cycles, which is why the linear part load model is accurate and useful. In this example with very aggressive upshifting for fuel economy, fewer than 1% of the operating points are above 8 bar, where the deviations from linearity become significant. With a less advanced transmission strategy and naturally aspirated engine, no operating points on the EPA City and Highway tests would be above 8 bar.

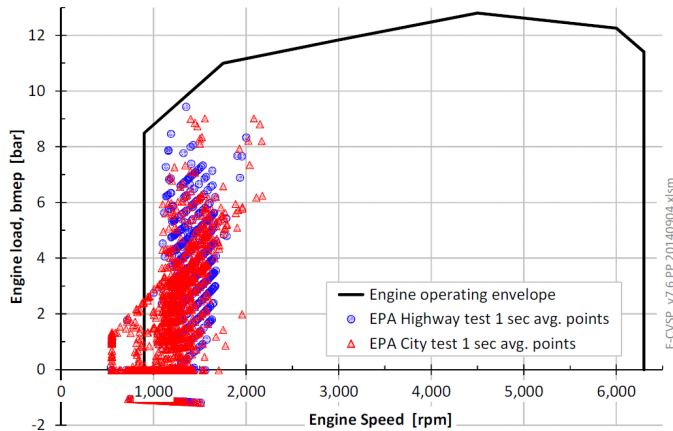


Figure 5. Second by second operating points for a naturally aspirated engine with an eight speed planetary automatic transmission on the EPA City and Highway tests.

Naturally Aspirated Engine Model

To illustrate the fundamental nature of the linear relation between fuel input and work, the steps of the engine process are followed in an example. Measurements are used from a production 1.6L naturally aspirated PFI engine [22] with cylinder pressure indication installed for experimental purposes. Data from a load sweep at 2,000 rpm are shown in figure 6.

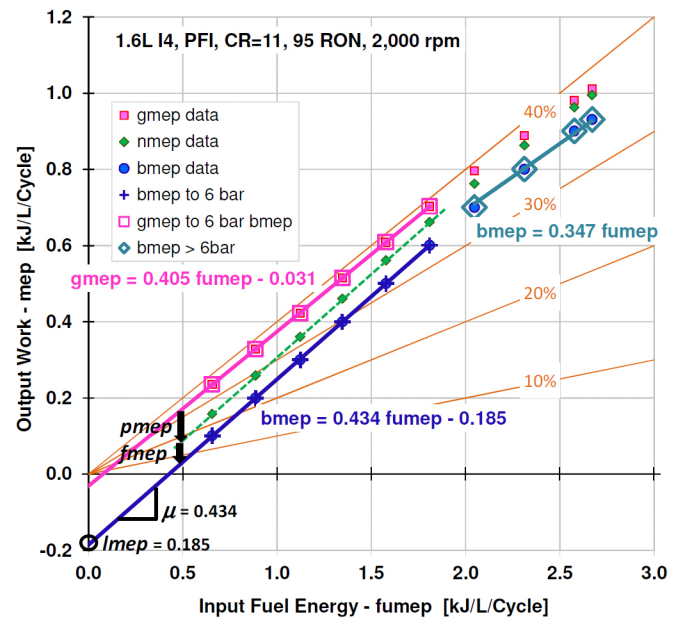


Figure 6. Gross, net and brake mean effective pressure as a function of fuel input (fumeq) for a 1998 naturally aspirated SI engine.

The efficiency of fuel energy conversion to gas work on the piston over the compression and expansion strokes (gross imep = gmeq) can also be examined as input and output, as illustrated by the square symbols. At low to mid loads, the relation can be described as an efficiency and offset,

$$gmeq = \eta_c fumeq - cmeq \quad (5)$$

where the marginal fuel conversion efficiency $\eta_c = 40\%$, and the combustion loss mep, $cmeq = 0.23$ bar. The formulation (5) provides a good description of the data, and it fits with our understanding of combustion. According to Heywood [20], about 2% of the fuel does not burn, and exits the engine as HC and CO emissions. Some additional fuel burns slowly or late in the cycle, where it contributes little to the work output.

The work developed during the high pressure revolution must then overcome pumping in the low pressure revolution. The pumping losses (pmeq) are seen to decrease linearly with increasing engine load. In a stoichiometric SI engine, the amounts of air and fuel are proportional and are controlled by throttling, i.e. reducing the pressure and density of the intake air, so that a simple model of the throttling is

$$pmeq = PMEP(1 - fumeq/FUMEP) \\ \text{where } FUMEP = H_i \eta_v \rho_a / AFR \quad (6)$$

and $PMEP$ is the pumping loss extrapolated back to $gmeq = 0$, with a typical value of about 1 bar for throttled engines.

FUMEP is an estimate of the full load fuel mep, based on the amount of air the engine is taking in, which is proportional to the air density ρ_a and the volumetric efficiency η_v , and **AFR** is the stoichiometric Air Fuel Ratio. Note that the pumping is modeled in terms of an input to the combustion process, and not an output from it, which will be a substantial simplification in the use of the model.

The difference between the net indicated mep, **nmep**, which accounts for pumping losses, and the brake mep is the friction mep, **fmep**. This engine has a friction level of about 0.6 bar, with a slight tendency to increase with load.

Taken together, these results show why a linear variation of work output with fuel input may be expected, as each step in the process exhibits approximately linear behavior. The net output, bmeP, is simply the gmep minus the losses,

$$bmeP = gmep - pmep - fmep = nmep - fmep \quad (7)$$

and from (5), (6), and (7) the corresponding mathematical model formulation is

$$\begin{aligned} fumeP &= (bmeP + lmeP) / \mu \\ \text{where } \mu &\approx \eta_c + \frac{PMEP}{FUMEP} \\ lmeP &\approx cmeP + PMEP + fmep \end{aligned} \quad (8)$$

to first order in the loss coefficients. Neglecting higher order terms is appropriate, as this is only a first order description of the engine phenomena. The marginal efficiency μ is the slope of the line, and is primarily a function of the fuel conversion efficiency, but also of pumping losses reducing with increasing load. The loss mep **lmeP** is the sum of the engine losses, combustion, friction and pumping extrapolated to zero load. Compared to Ross & An [3], the loss offset term in (8) is a constant because all variables have been normalized for displacement and engine speed. In addition, while they had a physical argument to relate μ to indicated efficiency and **lmeP** to mechanical losses, the relations here are more detailed and derived from first principles.

The structure of (8) is similar to (4), but the undefined function $\eta(bmeP)$ has been replaced by two constants, resulting in a major simplification. To put the outcome of the engine modeling in words, evaluating fuel mep as brake mep divided by a variable absolute efficiency in (4) is approximated well and more simply in (8) by brake mep plus mechanical loss mep, divided by the marginal thermodynamic efficiency, which is nearly constant at part load.

For the engine shown in figure 6 ($PMEP = 0.95$ bar, $\eta_c = 40\%$, $cmeP = 0.23$ bar, $fmep = 0.6$ bar), the slope and offset are estimated with (7) to be 0.439 and 1.85 bar, very close to the 0.435 and 1.85 bar observed. The simple model describes the observed behavior to within a few percent, and shows how the major physical parameters are connected. It enables a physically reasonable estimate of the effect of changes in one or several of them.

The eventual purpose of the model is to predict fuel consumption, so if (8) is used to find the fuel energy flow (3),

$$P_f = (P_e + \frac{1}{2} lmeP D n_e) / \mu \quad (9)$$

The fuel energy flow rate is approximated by the output power plus the mechanical loss power, divided by the marginal efficiency.

Fuel Consumption Reduction Technologies

As the Willans line performs an averaging of the trend over the part load, one may ask whether it is still sensitive enough to distinguish between the typical changes seen as engines develop from one generation to the next, through the refinement of technology and the adoption of new technology.

Examples will be shown of technical evolution aimed at the two descriptive parameters, the marginal efficiency μ and the zero load loss mep, **lmeP**.

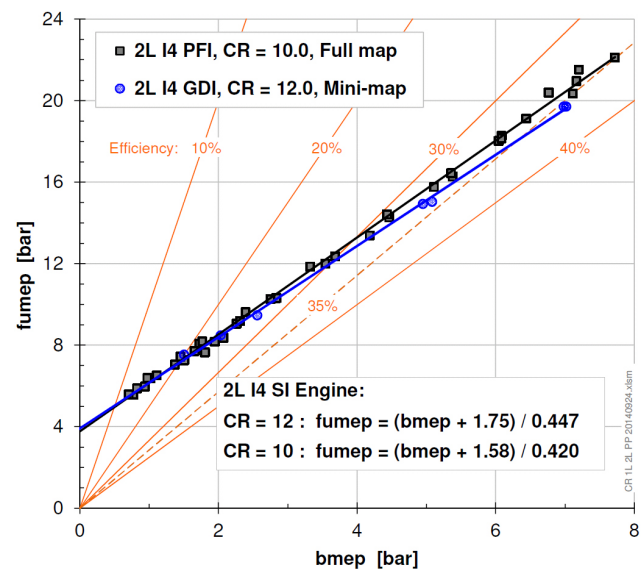


Figure 7. fuel mep as a function of brake mep for two versions of a 2L I4 SI engine, with compression ratios 10:1 and 12:1. These are different versions of the engine shown in figures 2, 3, 4.

The first example concerns the same engine family as in figures 2, 3, 4. It was initially introduced in the US market with Port Fuel Injection and a compression ratio of 10:1. In 2011 a new version with Direct Injection and compression ratio 12:1 replaced it. The low speed, part load fumeP(bmeP) data for both versions shown in figure 7 are of different types: for the CR=10 version a full engine map was available, whereas for the CR=12 version a standardized subset of points known as a mini-map was used. At very light loads the fuel consumption is very similar, but as the load increases, the high compression engine becomes increasingly more efficient. This is reflected in the marginal efficiency, which is 6% higher. The standard error on the marginal efficiency estimate for each version is about 0.7%, so that the difference between them is statistically significant. The effect of raising compression ratio by one unit is an improvement of marginal efficiency by 3% in this application.

The second example concerns a 3L 6 cylinder engine that went through several valvetrain upgrades. The oldest version shown as base in figure 8 had Intake Variable Cam Timing (IVCT), but this operated only near full load. It was first upgraded to wider range, higher speed Twin Independent VCT that improves part load efficiency, and then later Continuous Variable Valve Lift (CVVL) capability was added.

These technologies primarily reduce the pumping work but also improve the thermodynamic efficiency. The improvements in $imep$ are substantial, but are accompanied by deteriorations in μ , as expected from (7). This means there is a substantial improvement at light loads, that drops to zero by about seven bar load. The newer engines had improvements of $imep$ of 10% and 40% versus the base, which is statistically significant in view of standard errors of 2% and 3.5%.

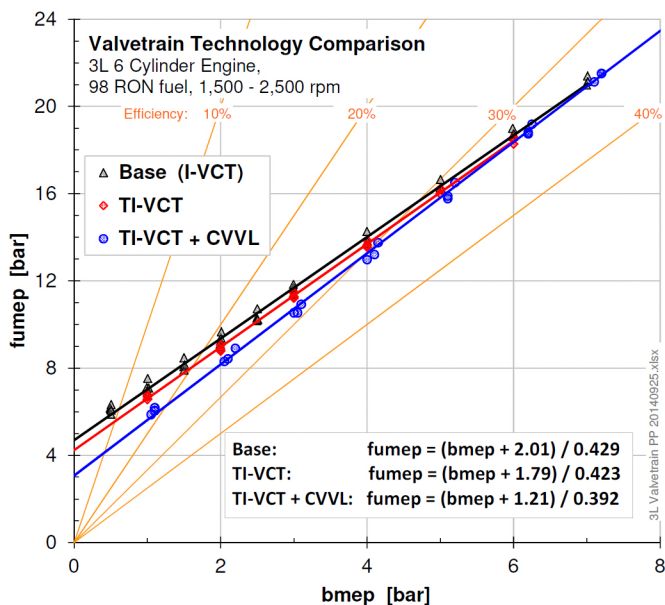


Figure 8. Fuel mep as a function of brake mep for three successive generations of an engine, with increasing levels of valvetrain technology: Base 4 Valve, Twin Independent VCT, and twin VCT with Continuously Variable Valve Lift.

These examples illustrate that the Willans line reduction of part load, low speed fuel consumption data is able to resolve the typical improvements made in engine upgrades.

BOOSTED ENGINES

The method described above was initially applied to naturally aspirated engines. Here it will be shown that it can also be applied to boosted engines. For boosted SI engines the knock limitation of the engine at high loads needs to be taken into account, but for Diesel engines the method works throughout the load range, as they are not knock limited.

Turbocharged SI Engines

The evolution of gross, net and brake mep as a function of fuel mep is shown in figure 9 for a turbocharged PFI SI engine of the same period as the naturally aspirated engine of figure 6. In the load range of 0 to 1

MPa (0-10 bar), the behavior of the two engines is very similar qualitatively, but for the turbocharged engine the marginal efficiency is substantially lower because of the lower compression ratio.

The drop in fuel conversion efficiency at the high loads can be clearly seen, with the consequence that the model (7) would be too optimistic at the high loads. Fortunately, an even simpler model provides a good approximation, namely that of constant, maximum brake efficiency,

$$bme = \eta_M fume \quad (10)$$

where η_M is the engine maximum brake efficiency. This model provides a very good approximation of the data in figure 9 for the load range from 1 to 1.6 MPa (10 to 16 bar), and transitions seamlessly to model (7) at lighter loads. For this older PFI turbocharged engine, the peak brake efficiency is 33% over a wide load range. In this range, the indicated efficiency is falling off at an increasing rate with increasing load; but the friction losses become a smaller fraction of the total fuel used, so that the overall efficiency is approximately constant.

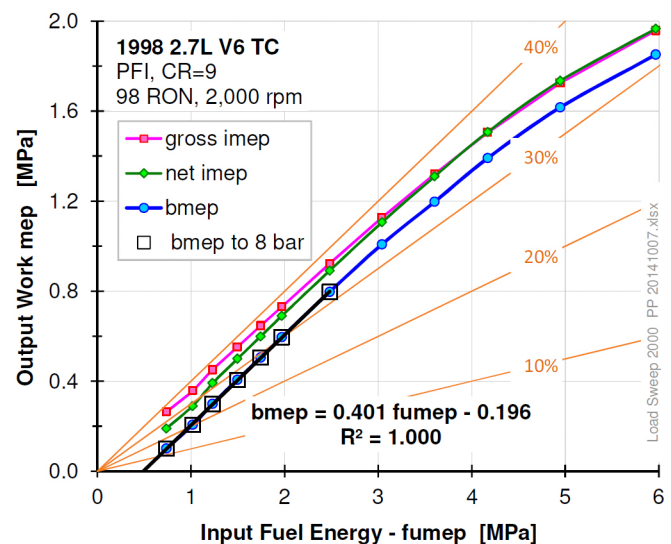


Figure 9. Gross, net and brake mean effective pressure as a function of fuel input (fume) at 2,000 rpm for a 1998 PFI turbocharged V6 engine.

Corresponding results for a modern 1L three cylinder turbocharged engine introduced in 2011 [23] are shown in figures 10, 11, 12. Through the use of direct injection and variable cam timing the compression ratio could be raised to 10:1. The engine was mapped, and the test points used for part load and full range regressions are shown in figure 10.

Figure 11 shows the full range fume(bme) of all these points. The scatter is very low, and the data is represented to a very high correlation by a third order polynomial. Only one point, at 1.7 MPa bme, deviates significantly from the trend, an indication there might be an issue with this particular measurement. The regression line could have been determined with much fewer data, saving testing time and money.