

Analytic Solution for the Flow Distribution and Pressure Drop of Ceramic Partially-Plugged Wall Flow Diesel Particulate Filters

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ABSTRACT

A 1-dimensional analytic solution has been developed to evaluate the pressure drop and filtration performance of ceramic wall-flow partial diesel particulate filters (PFs). An axially resolved mathematical model for the static pressure and velocity profiles prevailing inside wall-flow filters, with such unique plugging configurations, is being proposed for the first time. So far, the *PF* models that have been developed are either iterative/numerical in nature [1], or based on commercial CFD packages [7]. In comparison, an analytic solution approach is a transparent and *computationally inexpensive* tool that is capable of accurately predicting trends as well as, offering explanations to fundamental performance behavior. The simple mathematical expressions that have been obtained facilitate rational decision-making when designing partial filters, and could also reduce the complexity of OBD logic necessary to control onboard filter performance. The boundary conditions applicable to these partially-plugged filter configurations have been established before by our research group [1]. Removing few of the rear or frontal plugs from a standard wall-flow DPF reduces the (i) overall pressure drop (ΔP) across the filter, (ii) filtration efficiency (γ) of the DPF, and (iii) manufacturing cost. Partial filters have been deployed in diesel exhaust after-treatment systems for the emerging markets that follow Euro 4 emission regulations, especially in light-duty vehicle applications.

In the present work, the filtration efficiency and ΔP across PFs have been computed at various soot loads and flow rates for filter sizes typical of light-duty diesel applications. The effects of channel asymmetry, multi-staging of the PFs, and varying soot loads have been successfully captured in the proposed mathematical solution. The model predictions are in good agreement with experimental data obtained from both, single as well as multi-staged PF systems. The model results have also been demonstrated to be similar to simulations obtained from a numerical/iterative PF model reported in literature. [2].

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1. INTRODUCTION

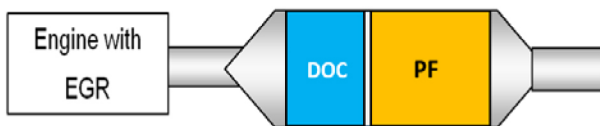


Fig.1. After-treatment architecture (DOC+PF) for Euro4 applications.

The popular choice for partial particulate filters are the *Sintered Metal* fiber filters. These filters are a promising candidate for Tier 4 and Euro 4 applications and are capable of reducing both particle mass as well as particle number. Their filtration efficiency lies in the range of 30-70% [3]. However, the metallic partial filters are expensive. Ceramic partial filters are a cost-effective alternate choice with some sacrifice in filtration performance. Aluminum Titanate is a promising ceramic substrate material for such partial filter applications [4, 5] and in general, for next-generation filters. Typically, a symmetric-channel geometry ceramic PF can achieve a filtration efficiency of ~40-45% which is less than the ~60-70% filtration achieved by the metal fiber filters. However, by adopting an asymmetric channel geometry [4, 6] in the ceramic PFs (enlarging the plugged inlet

channels), the filter's filtration capability can be enhanced to levels comparable to its metallic counterpart. The asymmetric channel concept has been matured over the years and is now available in various channel shapes such as square, octo-square and hexagonal [4]. Also, the dominant mode of burning out soot inside the partial filters is through NO_2 -aided continuous passive regeneration [3].

The emerging markets follow the (Euro 4, Euro 5, BS IV, NS 4/5, China 3-5) emission regulations for On-highway and (Tier 4, Euro Stage I/II, BS II/III) for Off-highway applications. The particulate matter (PM) emission requirements by these regulation standards are less stringent than those followed in North America or Europe. Wall-flow ceramic particulate filters, capable of offering >95% filtration efficiency, are the prime path to mitigate PM emissions and attain acceptable particle number (PN) levels from diesel exhaust for both, light-duty and heavy-duty applications. The PM emissions typically consist of dry carbon, ash, and SOFs (soluble organic fraction) that typically comprise volatile unburnt hydrocarbons (HCs) and sulfur compounds. The SOFs are contributed by the unreacted fuel and lube oil vapors. However, fully-plugged DPFs may not be

needed for meeting the Euro 4 PM standards. A passive regeneration-based after-treatment system comprising a diesel oxidation catalyst (DOC), along with a partially-plugged DPF downstream, is capable of reducing the PM mass by 40-65%. The DOC alone can reduce the PM mass by 20-30%, but the PN is mostly handled in the filter. The DOC ensures oxidation of the CO, HCs, SOFs, and some of the NO to NO₂, while the PF oxidizes the carbonaceous soot appreciably (>50%) with the aid of NO₂.

The first attempt on modeling partial filters was reported in Ref. [7] where a 3D rear-plugged PF was simulated using a commercial CFD solver. However, the governing equations used and boundary conditions applied at the PF outlet needed to be clearly stated. The pressure boundary condition used in the simulation requires a revisit.

Ensuing, a numerical iterative approach was illustrated in Ref. [1] to model rear-plugged as well as front-plugged PFs. The boundary conditions of these filter configurations were defined and discussed in this study. The model was supported by experimental data measured on a real-world partial filter. Also, a general methodology was presented, using which, one could model a particulate filter plugged to any extent (zero to unity).

A recent study reports ‘approximate’ analytic expressions for the filtration efficiency and ΔP of partially-plugged filters [2]. This study adopts the correct boundary conditions, for the filter configurations, as have been reported in the past by our research group in Ref. [1]. The axial velocity and pressure profiles in this study were still obtained using an iterative numerical approach. The analytic expression of the filtration efficiency involves approximations in the soot cake thickness estimation, and varies by ~6% from the results obtained through the numerical approach. Two important approximations made when deriving the expressions. (i) The wall velocity was assumed linear i.e. the dashed green curve in Fig. 9a in the present study was assumed to simplify the derivation. (ii) The cake layer pressure drop term was assumed to resemble the substrate wall ΔP term ($\frac{w_{sc}}{k_{sc}}$) instead of a logarithmic term $\left[\frac{a_1}{2k_{sc}} \ln \left(\frac{a_1}{a_1 - 2w_{sc}} \right) \right]$ in order to simplify the derivation. In other words, the soot cake layer was assumed to be a slab instead of trapezoidal. See Eq. (5) to get a better perspective on this.

In brief, the system of differential equations and boundary conditions describing the partial filters is much more complex than their fully-plugged counterparts. A straight-forward analytic solution to this problem is difficult and does not exist. The present work is an attempt to bridge this gap. Two decades ago, Konstandopoulos et al [8] developed an analytical model for a fully-plugged, symmetric channel wall-flow ‘clean’ DPF- trap with no soot load. The transport equations of mass and momentum conservation were solved for the wall-flow trap with appropriate boundary conditions. Their reported solution is discussed in section 2.1.3. In the present work, we have used the same conservation equations of mass and momentum as used in Ref. [8], and derive the analytic solution to not only represent partially-plugged filters but also, describe asymmetric channel geometry as well as soot load.

Summarizing, in the present work, we have extended the 1D mathematical solution for regular wall-flow DPFs to represent the axial pressure and velocity fields prevailing inside ceramic partially-plugged filters. Analytic expressions for 1-dimensional axially resolved channel pressure and velocity profiles of rear-plugged and frontal-plugged PF configurations have been developed. The specific objectives of the work are to:

- (1). Develop 1D analytic expressions for accurately predicting the steady-state pressure-drop and filtration efficiency of partial DPFs- including asymmetric channel effects.
- (2). Compare the differences in fundamental performance behavior between a front and rear-plugged ceramic PF.
- (3). Investigate the effect on ΔP and FE across a PF of the following parameters: (a) cell density, (b) asymmetric channel hydraulic diameter, (c) multi-staging of PFs, and (f) exhaust mass flow.
- (4). Benchmark model data with experimental data obtained from a single as well as multi-staged PF system.
- (5). Predict representative numbers for the ΔP and FE performance of a PF, across a range of soot loads, for a filter size typical of light-duty diesel applications.

2. THEORY

The PF is modeled using a 1-dimensional grid with the cell centers being the computational node points. Figure 2 shows the cross-section of a typical *cake* (filtering) channel in a filter. The computational domain comprised a representative volume element (RVE) of a pair of adjacent filter channels (see Figures 3, 4, 5) assuming that all remaining channel pairs in the filter are identical. The key model assumptions are:

- a). The exhaust flow is laminar
- b). The filter is composed of identical adjacent filter channel pairs. The computational domain is a RVE
- c). The filter is isothermal axially and radially. The gas temperature in the filter is equal to the measured inlet temperature
- d). The transverse pressure drop across the soot layer and substrate wall follows the Darcy law [4, 5]
- e). The formation and deposition of ash have not been considered. The effect of plug length has also been neglected
- f). The pressure losses at the filter inlet due to contraction, and at the filter outlet due to expansion, have been neglected
- g). Secondary flow effects have been neglected
- h). The model assumes 100% capture of particulates from the exhaust that pass through the filter wall
- i). Filtration efficiency (FE) describes the flow distribution between the plugged and flow-through channels. The FE of a regular DPF is 100%

2.1. Governing Equations and Principles of Operation

The resistances offered to the exhaust flow are due to: (i) friction along the channel length, (ii) contraction losses at the filter inlet, (iii) expansion losses at filter outlet, and (iv) transverse pressure losses due to the substrate wall thickness, soot cake and soot depth layers.

The resistances (ii) and (iii) have been neglected in this analysis considering their relative magnitudes to be significantly smaller than the other ones. This primarily helps to reduce the complexity of the analytic solution. The soot cake layer thickness and the wall velocity in the transverse direction, according to Darcy Law, are described by Eqs. (1) and (2) respectively [9].

$$w_{sc} = \frac{1}{2} \left[a_1 - \sqrt{a_1^2 - \frac{m_{sc}}{\rho_{sc} V_{DPF} \times c_{psi} \times 1550}} \right] \quad (1)$$

$$u_w = \frac{P_1 - P_2}{\left(\frac{a_1 - 2w_{sc}}{a_1} \right) \left[\frac{\mu w_s}{k_w} + \frac{a_1 \mu}{2k_{sc}} \ln \left(\frac{a_1}{a_1 - 2w_{sc}} \right) + \frac{\mu w_{sd}}{k_{sd}} \right]} \quad (2)$$

Note that u_w is the transverse wall velocity at the surface of the inlet (cake) channel. It is the superficial velocity through the wall based on the soot loaded width of the cake channel. Also, note that the factor '1550' in Eq. (1) converts square inches to square meters. The preliminary non-dimensional parameters are defined as [8]:

$$\hat{x} = \frac{x}{L}; \hat{u}_1 = \frac{u_i}{U}; \hat{u}_w = \frac{4L}{(a_1 - 2w_{sc})U} u_w \quad (3)$$

$$\hat{P}_1 = \frac{P_i - P_{atm}}{P^*} u_w; P^* = \frac{\mu U a_1 w_{eff}}{4L k_{eff}} \quad (4)$$

where,

$$\frac{w_{eff}}{k_{eff}} = \left(\frac{a_1 - 2w_{sc}}{a_1} \right) \left[\frac{w_s}{k_w} + \frac{w_{sd}}{k_{sd}} + \frac{a_1}{2k_{sc}} \log \left(\frac{a_1}{a_1 - 2w_{sc}} \right) \right] \quad (5)$$

$$A_1' = \frac{k}{a_1 w_s} \frac{4L}{a_1} Re; A_2' = \frac{4Fk}{a_1 w_s} \left(\frac{L}{a_1} \right)^2; A_2'^* = \frac{4Fk}{a_2 w_s} \left(\frac{L}{a_2} \right)^2 \quad (6)$$

The non-dimensional mass and momentum balance equations are listed in Eqs. (7), (8), (9), (10), (11), (12).

$$\frac{d\hat{u}_1}{d\hat{x}} = -\hat{u}_w \quad (7)$$

$$\frac{d\hat{u}_2}{d\hat{x}} = +\varphi \hat{u}_w \quad (8)$$

where,

$$\varphi = \frac{a_1 - 2w_{sc}}{a_2} \left(1 - \frac{2w_{sc}}{a_1} \right) \quad (9)$$

$$\hat{P}_1 - \hat{P}_2 = \Delta \hat{P} = \hat{u}_w \quad (10)$$

$$\frac{d\hat{P}_1}{d\hat{x}} + A_1' \frac{d\hat{u}_1^2}{d\hat{x}} + A_2' \hat{u}_1 = 0 \quad (11)$$

$$\frac{d\hat{P}_2}{d\hat{x}} + A_1' \frac{d\hat{u}_2^2}{d\hat{x}} + A_2'^* \hat{u}_2 = 0 \quad (12)$$

Figure 2 shows the typical cross-sectional view of a square channel in a ceramic honeycomb filter. The fraction of exhaust gas that gets filtered, flows in the transverse direction through the wall. The correct sequence of soot cake layer, soot depth layer and the substrate wall are shown in Fig. 2. Note that 'soot depth layer' is the fraction of soot in the filter that deposits in the substrate pores, and does not contribute to the soot cake volume. Approximately, the first 0.5 to 1 g/L of soot that enters a filter deposits in the substrate pores and does not occupy a separate volume in the hydraulic channel. The soot depth layer and the substrate wall are represented as slab-like resistances in the Darcy equation. The soot cake layer is trapezoidal in shape. The hydraulic diameter of the inlet (cake) channel is a_1 while, the outlet (clean) channel's width is a_2 . Both these channel widths will be equal to each other in case of a symmetric channel geometry DPF.

In essence, the governing equations of mass and momentum are the same as those used for modeling wall-plugged regular DPFs. Handling the asymmetry in channel hydraulic diameters introduces a few additional multipliers to the coefficients of the conservation equations. The boundary conditions (BCs) however, are different. The BCs for a rear-plugged PF, front-plugged PF, and a regular DPF are listed in the following sections. The subscript '1' denotes the inlet or the filtering 'cake' channel while, the subscript '2' denotes the outlet or the 'clean' non-filtering channel.

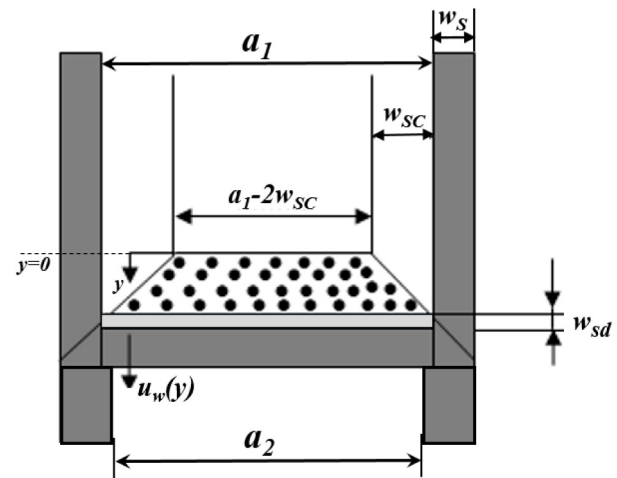


Fig. 2. Cross-sectional area of a computational cell.

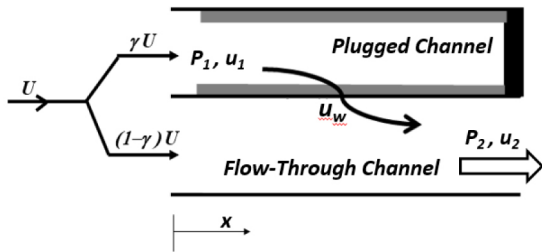
2.1.1. Rear-plugged PF

The biggest challenge associated with this plugging pattern is to accurately predict the exhaust split (γ) between the two open channels at the filter inlet. The boundary conditions to be applied on this particular PF are listed below [1] in Eqs. (13), (14), (15). The corresponding non-dimensional interpretation of each boundary condition is also shown.

$$u_l=0 \text{ @ } x=L: \hat{u}_2(1) = 1 \quad (13)$$

$$P_2=P_{atm} \text{ @ } x=L: \hat{P}_2(1) = 0 \quad (14)$$

$$P_l=P_2 \text{ @ } x=0: \hat{u}_w(0) = 0 \quad (15)$$



1: Inlet/Plugged channel, 2: Outlet/Flow-through channel

Fig. 3. Rear-plugged PF

Equations (11) and (12) reduce to:

$$\frac{d^2 \hat{u}_2}{d\hat{x}^2} - 2A_1 \frac{d\hat{u}_2}{d\hat{x}} - (A_2 + A_2^*) \hat{u}_2 = -A_2 \quad (16)$$

$$A_1 = \phi A_1'; A_2 = \phi A_2'; A_2^* = \phi A_2'^* \quad (17)$$

The boundary conditions are:

$$\hat{u}_2(1) = 1; \hat{u}_2(0) = 1 - \gamma \quad (18)$$

The solution to the above set of equations is:

$$\hat{u}_2(\hat{x}) = c_1 \exp(g_1 \hat{x}) + c_2 \exp(g_2 \hat{x}) + \frac{A_2}{A_2 + A_2^*} \quad (19)$$

where,

$$g_1 = A_1 - \sqrt{A_1^2 + (A_2 + A_2^*)} \quad (20)$$

$$g_2 = A_1 + \sqrt{A_1^2 + (A_2 + A_2^*)} \quad (21)$$

$$c_2 = \frac{\exp(g_1)(1 - \gamma) - 1 + \frac{A_2}{A_2 + A_2^*} [1 - \exp(g_1)]}{\exp(g_1) - \exp(g_2)} \quad (22)$$

$$c_1 = 1 - \gamma - c_2 - \frac{A_2}{A_2 + A_2^*} \quad (23)$$

Details of how the above-mentioned mathematical solution was obtained is presented in Eqs. (68), (69), (70), (71), (72), (73) of the Appendix. The 'filtration efficiency' or the fraction of incoming exhaust flow (γ) that enters the plugged (filtering) channel turns out to be:

$$\gamma = 1 + \left(\frac{g_1 - g_2 + g_1 \beta (\exp(g_2) - 1) + g_2 \beta (1 - \exp(g_2))}{g_2 \exp(g_1) - g_1 \exp(g_2)} \right) \quad (24)$$

$$\beta = \left(\frac{A_2}{A_2 + A_2^*} \right) \quad (25)$$

The plugged channel axial velocity and wall velocity profiles are:

$$\hat{u}_1(\hat{x}) = 1 - u_2(\hat{x}) \quad (26)$$

$$\hat{u}_w(\hat{x}) = \frac{1}{\phi} \frac{d\hat{u}_2}{d\hat{x}} = \frac{1}{\phi} [c_1 g_1 \exp(g_1 \hat{x}) + c_2 g_2 \exp(g_2 \hat{x})] \quad (27)$$

The flow-through channel pressure profile is:

$$\hat{P}_2(\hat{x}) = A_1' [1 - \hat{u}_2^2(\hat{x})] + A_2'^* \left[\frac{c_1}{g_1} (\exp(g_1) - \exp(g_1 \hat{x})) + \frac{c_2}{g_2} (\exp(g_2) - \exp(g_2 \hat{x})) + \frac{A_2}{A_2 + A_2^*} (1 - \hat{x}) \right] \quad (28)$$

The plugged channel static pressure profile is:

$$\hat{P}_1(\hat{x}) = \hat{u}_w(\hat{x}) + \hat{P}_2(\hat{x}) \quad (29)$$

Please note that the above expressions are valid for both clean and soot-loaded filters. They also hold for a rear-plugged trap with symmetric or asymmetric channel geometry. The details of the above derivation are illustrated in the Appendix. In case of a clean trap, w_{sc} is set to zero. In case of a symmetric channel geometry filter, the channel width ratio is set to 1 (i.e., $w_1 = w_2$).

2.1.2. Front-Plugged PF

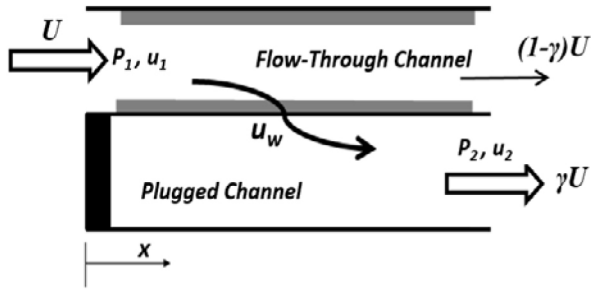


Fig. 4. Front-plugged PF

In case of an asymmetric channel front-plugged PF, the governing differential equations are similar to the rear-plugged PF case as illustrated in the previous section. The boundary conditions satisfying this configuration are as follows:

$$u_2=0 \text{ @ } x=0: \hat{u}_2(0) = 0 \quad (30)$$

$$P_2=P_{atm} \text{ @ } x=L: \hat{P}_2(1) = 0 \quad (31)$$

$$P_1=P_2 \text{ @ } x=L: \hat{u}_w(1) = 0 \quad (32)$$

The axial velocity profile expressions are identical to the rear-plugged case as shown below. Only the constants (c_1, c_2) vary in this case.

$$\hat{u}_2(\hat{x}) = c_1 \exp(g_1 \hat{x}) + c_2 \exp(g_2 \hat{x}) + \beta \quad (33)$$

$$\hat{u}_1(\hat{x}) = 1 - u_2(\hat{x}) \quad (34)$$

$$\hat{u}_w(\hat{x}) = \frac{1}{\phi} [c_1 g_1 \exp(g_1 \hat{x}) + c_2 g_2 \exp(g_2 \hat{x})] \quad (35)$$

$$c_2 = \frac{\beta g_1 \exp(g_1)}{(g_2 \exp(g_2) - g_1 \exp(g_1))} \quad (36)$$

$$c_1 = -\beta - c_2 \quad (37)$$

The pressure profile differs slightly as can be seen below.

$$\hat{P}_2(\hat{x}) = A_1' [\hat{u}_2^2(\hat{x}) - \hat{u}_2^2(1)] + A_2' \left[\frac{c_1}{g_1} (\exp(g_1 \hat{x}) - \exp(g_1)) + \frac{c_2}{g_2} (\exp(g_2 \hat{x}) - \exp(g_2)) + \beta(\hat{x} - 1) \right] \quad (38)$$

$$\hat{P}_1(\hat{x}) = \hat{u}_w(\hat{x}) + \hat{P}_2(\hat{x}) \quad (39)$$

The 'filtration efficiency' in this case turns out to be:

$$\gamma = \hat{u}_2(1) \quad (40)$$

2.1.3. Regular DPF

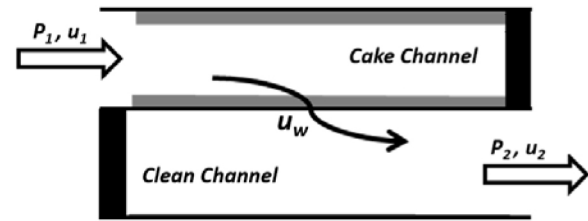


Fig. 5. Wall-plugged regular DPF

The boundary conditions in case of a regular wall-flow DPf are shown in Eqs. (41), (42), (43).

$$u_2=0 \text{ @ } z=0: \hat{u}_2(0) = 0 \quad (41)$$

$$u_1=0 \text{ @ } z=L: \hat{u}_1(1) = 0 \quad (42)$$

$$P_2=P_{atm} \text{ @ } z=L: \hat{P}_2(1) = 0 \quad (43)$$

Two decades ago, Konstandopoulos et. al. [8] reported the analytic solution for the velocity and pressure profiles of the wall-flow, symmetric channel DPf as:

$$c_2 = \frac{1}{2} \left[\frac{e^{g_1+1}}{e^{g_2}-e^{g_1}} \right] \quad (44)$$

$$c_1 = -\frac{1}{2} - c_2 \quad (45)$$

Note that the above solution corresponds to a clean trap with symmetric channels only ($a_1=a_2, w_{sc}=0$).

2.2. Effect of CPSI and Asymmetric Channels

Two methods of increasing the filtration efficiency of a PF are: (i) increasing its cpsi and (ii) incorporating asymmetry in the channel geometry. In case of filters that have asymmetric channels, the adjacent cells have different hydraulic diameters (Fig. 6). The asymmetric channel geometry helps increase soot and ash accumulation capacity when the larger channels are plugged at the DPf outlet. The typical channel width ratio between the larger and smaller hydraulic diameters can be obtained from the substrate