

Automatic Transmission Gear Ratio Optimization and Monte Carlo Simulation of Fuel Consumption with Parasitic Loss Uncertainty

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ABSTRACT

This investigation utilizes energy analysis and statistical methods to optimize step gear automatic transmissions gear selection for fuel consumption. A full factorial matrix of simulations using energy analysis was performed to determine the optimal number of gears and gear ratios that provide the best fuel consumption performance for a particular vehicle - engine application. The full factorial matrix setup as a design of experiment (DOE) was applied to five vehicle applications, each with two engines to examine the potential differences that variations in road load and engine characteristics might have on optimal transmission gearing selection. The transmission gearing options considered in the DOE were number of gears, launch gear ratio and top gear ratio. Final drive ratio was also included due to its global influence on vehicle performance and powertrain operating speeds and torque. An enterprise approach for vehicle fleet balancing of fuel consumption objectives based upon selection of automatic transmission gearing strategy is presented. Monte Carlo simulations for the influence of parasitic loss uncertainty for the automatic transmissions selected from the generalized DOE were conducted to determine the range of fuel consumption that might be realized in production. Specifically, the focus of uncertainty analysis was on automatic transmission parasitic loss variation that might be encountered due to manufacturing and assembly. The overall findings of the investigation show that the optimal number of transmission gears begins to plateau above 8 speeds and reaches a minimum at 10 speeds, but gearing selection is highly dependent on vehicle and engine application. Automatic transmission gearing parameter selection was unaffected by the variation introduced by the Monte Carlo simulations, but did show fuel consumption performance to vary upwards of 9 g CO₂/km depending on applications and to have a direct impact on powertrain operating parameters.

CITATION: Robinette, D. and Wehrwein, D., "Automatic Transmission Gear Ratio Optimization and Monte Carlo Simulation of Fuel Consumption with Parasitic Loss Uncertainty," *SAE Int. J. Commer. Veh.* 8(1):2015, doi:10.4271/2015-01-1145.

INTRODUCTION

As the timeline shortens to achieve the regulated emissions levels of the 101.3 g CO₂/km by 2025 as mandated by EPA and NHTSA as part of CAFE, [1], solutions for conventional powertrain must be sought that synergize design and operation for optimal efficiency. The step gear automatic transmission is one of the critical powertrain components that can contribute to a reduction in fuel consumption while also enhancing acceleration performance. With recent trends in engine engineering towards downsizing displacement and boosting to increase torque output, the selection of automatic transmission gear ratios that harmonize with the engine characteristics is more critical. With the number of technologies available, the number of powertrain permutations will require a large number of analytical iterations to adequately balance transmission, engine and vehicle design choices, see [2] for example. To date there has been a number of studies published in literature on selecting the optimal number of automatic transmission gear ratios to minimize fuel consumption, see [2, 3, 4 and 5], with nearly all pointing to increasing the number of gears and overall ratio spread while reducing parasitic losses. The approach reported by [5] incorporated design for six sigma (DFSS) with energy analysis as a means of reducing the number of simulations required to determine the most important design parameters while looking at

multiple engine and vehicle combinations. This approach potentially enables a shorter route to getting at the essence of design selection, but can potentially confound parameter interactions due to the high degree of interactions and multifactorial response the automatic transmission can produce.

The objective of this investigation is to execute a full factorial matrix of simulations using energy analysis to determine the optimal automatic transmission gearing selection for fuel consumption across a fleet of vehicle and engine applications. The test schedule to assess fuel consumption performance will be the Worldwide Harmonized Light Duty Test Procedure, WLTP, for class 3 vehicles with a power to weight ratio greater than 34 and maximum velocity greater than 120 kph. The findings from [5] will serve as a starting point for the DOE parameters included in the analysis. One of the major conclusions reported by [3, 4 and 5] is that parasitic losses dominate the fuel consumption paradigm for automatic transmission optimization. Number of forward gears were found to be a relatively minor contributor in comparison to parasitic losses. Adding forward gears then serves mainly as a drive quality and acceleration performance enabler, while also slightly improving fuel consumption. For this study, the focus will be exclusively on gearing selection.

Parasitic losses for each unique transmission design were generated using a methodology for open clutch spinloss, transmission pumploss, gear mesh efficiency and were scaled as the number of gear states increase to account for additional content required to achieve more gear states typical for a planetary transmission architecture. All automatic transmissions considered in this investigation are hypothetical, existing only on paper, but are representative for meaningful conclusions to be drawn from the analysis. Furthermore, all the analysis and results presented are for automatic transmissions of the planetary gear type with a torque converter as the launch device. Once the optimal transmission design(s) has been selected based on the full factorial DOE, the next phase of the analysis was to determine the impact that variation in parasitic losses introduced by manufacturing and assembly will have on fuel consumption. Monte Carlo simulations assuming different statistical distributions are used to show the range of predicted fuel consumption to anticipate with the assumed variation and the influence it might have on transmission gearing selection as well as overall powertrain operation on test schedule.

FULL FACTORIAL MATRIX

The number of design and operating point parameters that can be adjusted for an automatic transmission are numerous and are highly interactive with other powertrain components and technologies that create a multifactorial response. This can make it difficult to isolate individual parameters for main effects. The DFSS study by [5] accomplished this to a point, however, a review of multi-collinearity of transmission design parameters showed a degree of correlation between number of gears and parasitic loss mechanisms of spinloss and gear mesh efficiency. This effect cannot be eliminated unless the same parasitic losses are assumed irrespective of transmission gearing. The goal of this study is to isolate the gearing and ratio selection of the automatic transmission. The transmission design parameters of number of gears, launch gear ratio and top gear ratio were considered as input to the full factorial matrix. Since parasitic losses are foundational to the fuel consumption performance of any automatic transmission, a consistent methodology of developing the parasitic losses for each unique combination of gearing was applied. The purpose of which was to minimize any major correlated effects to more precisely determine the main effects of number of gears and ratio selection. For the main effects, six levels of transmission number of gears, three levels of launch gear ratio and three levels of top gear ratio were considered. This setup created a $3^2 \times 6$ full factorial matrix with 54 unique automatic transmission designs, the details of which will be described next.

Transmission Parameters

The level values for automatic transmission design parameters used in the $3^2 \times 6$ full factorial matrix are contained in Table 1. The number of transmission gears varied from 6 forward gear states, which constitutes the majority of current automatic transmission production and up to 11 forward gears that goes beyond the number of gears announced in upcoming automatic transmission variants. Launch and top gear ratios were selected based upon the range of automatic transmission products available on the market or those recently announced. The combination of launch and top gear ratios leads to

nine unique overall ratio (OAR) spreads, ranging from 5.67 to 9.55 and overlap between the three launch gear ratios considered as shown in Figure 1. The range of gear ratios and resulting OAR's selected for the study encompass current and emerging planetary automatic transmission designs utilized in passenger and light duty vehicle applications. These levels of parameters should provide adequate limits on step gear automatic transmissions to match with current and future engine and vehicle applications. It is worth mentioning again that all the discussion, analysis and results for this investigation are for automatic transmissions of the planetary gear type with a torque converter as the launch device.

Table 1. Automatic transmission number of forward gears, launch and top gear ratio included in $3^2 \times 6$ full factorial experiment.

Level	1	2	3	4	5	6
Number of Gears	6	7	8	9	10	11
Launch Gear Ratio	4.25	4.75	5.25			
Top Gear Ratio	0.55	0.65	0.75			

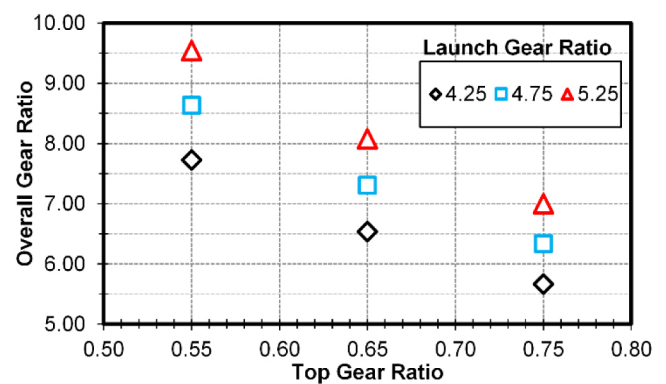


Figure 1. Transmission overall ratio spread, OAR, as a function of launch and top gear ratio for range of gear parameters chosen for $3^2 \times 6$ full factorial study.

Determining Intermediate Gear Ratios

The intermediate gear ratios between the launch and top gear ratios listed in Table 1 for all transmissions were determined by developing a ratio step, RS , profile that was progressively decaying. Equation 1 is an adaptation from [6], where the ratio step size of each intermediate gear step is solved for as a function of the top ratio step, RS_{top} , number of ratio steps, z , the gear, n , being shifted to and two coefficients, ϕ_1 and ϕ_2 . Coefficient ϕ_1 represents the base ratio step size and ϕ_2 the ratio progression factor. Equation 2 gives the formulation for ϕ_1 coefficient, which is a function of ϕ_2 , number of ratio steps, z , and the overall ratio spread of the transmission, OAR . A numerical search for a suitable value of ϕ_2 was determined using a generalized reduced gradient algorithm with the variables z , n , RS_{top} , and OAR specified for the transmission. Once ϕ_2 was found given the specifications of the transmission all intermediate ratio steps and gear ratios were attained.

$$RS_i = RS_{top} \phi_1^{(z-n)} \phi_2^{0.5(z-n)(z-n-1)}$$

$$\phi_1 = \left(\frac{1}{\phi_2^{0.5(z-1)(z-2)}} \cdot OAR \right)^{\frac{1}{z-1}} \quad (2)$$

Table 2 summarizes the value of top ratio step size, RS_{top} , specified for each grouping of transmissions by number of forward gears. A fixed value of RS_{top} was assumed for a given number of forward gears, ranging from 1.3 for a 6 speed down to 1.125 for an 11 speed transmission. The values of RS_1 , the first ratio step for the 1-2 or 2-1 shift was adjusted for each grouping of transmissions by number of forward gears. This was done so that a converged search solution on ϕ_2 was found that met the specifications of the transmission design. The maximum and minimum values for RS_1 are listed in Table 2 by number of transmission forward gears and decrease in value as the number of gears increases.

Table 2. Summary of maximum and minimum first ratio step size, RS_1 , and top ratio step, RS_{top} , sizes assumed for 6 to 11 speed transmissions.

Number of Gears	max RS_1	min RS_1	RS_{top}
6	1.750	1.550	1.300
7	1.600	1.480	1.250
8	1.550	1.450	1.200
9	1.550	1.425	1.150
10	1.500	1.400	1.150
11	1.450	1.360	1.125

Example gear ratios and ratio steps are shown in Figures 2 and 3, respectively for 6 to 11 speed transmissions with a launch gear ratio of 4.75:1 and a top gear ratio of 0.65:1. As the number of forward gears increase, so too does the gear ratio for all the intermediate gears as shown in Figure 2. This is a direct result of the progressive ratio step profiles getting shallower as the number of gears increases. It is worthwhile to mention that the transmissions in this study are all hypothetical designs, following the idealized progressive ratio step profiles determined by Eq.'s (1) and (2). Actual automatic transmissions will deviate from the idealized profiles due to constraints of planetary gearsets and their various kinematic arrangements to achieve the desired number of gear states.

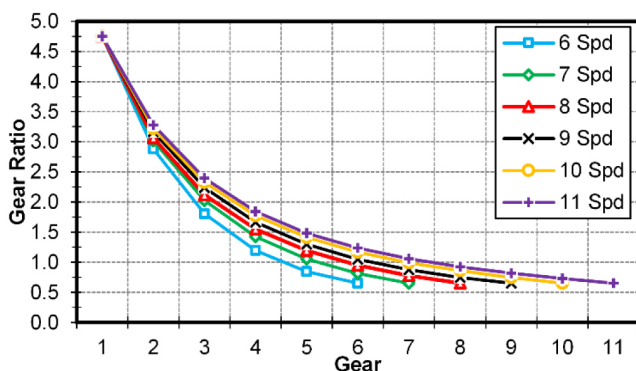


Figure 2. Example gear ratios for 6 to 11 speed transmissions with a 4.75:1 launch gear ratio and 0.65:1 top gear ratio.

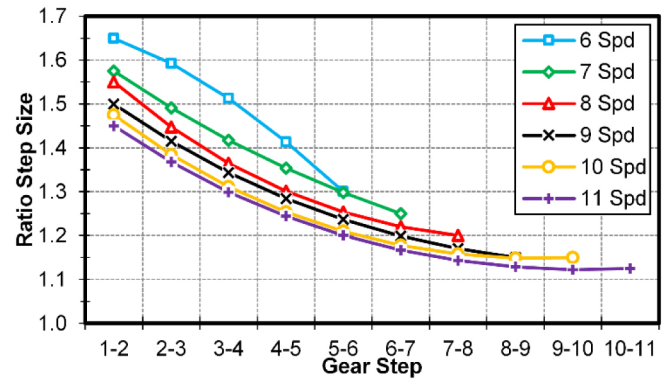


Figure 3. Example gear ratio steps for 6 to 11 speed transmissions with a 4.75:1 launch gear ratio and 0.65:1 top gear ratio.

Transmission and Drivetrain Losses

Transmission parasitic losses are known to be the major contributor in determining the optimal set of design parameters as shown by [2, 4 and 5]. For this study, the focus is on the selection of number of gears and the intermediate ratios between launch and top gears. To facilitate the most relevant set of comparisons across a wide range of geared automatic transmissions a single methodology for estimating parasitic losses was utilized and applied throughout the investigation. Four parasitic loss mechanisms were assumed for this investigation and were summed together as an overall loss of the transmission and driveline.

The first loss mechanisms included was the torque required to energize the transmission pump as a function of input torque and speed. The torque loss is based on leading edge binary, off axis, chain drive pump. Figure 4 contains a subset of the assumed pumploss dataset for transmission input torques of 75 and 150 Nm across input speeds from 500 to 6000 rpm.

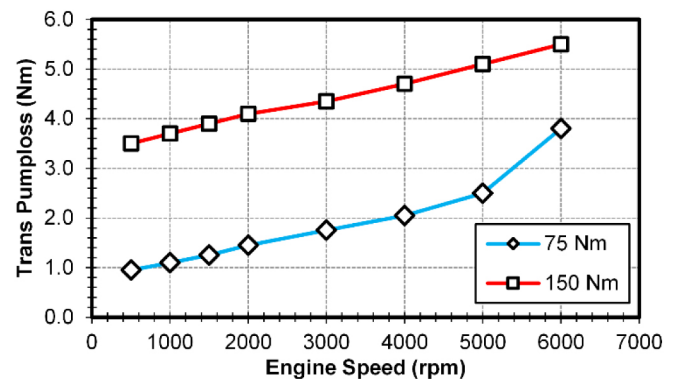


Figure 4. Transmission binary pump parasitic torque loss as a function of input torque and speed.

The second parasitic loss mechanism considered was the torque lost from hydrodynamic drag of fluid between unapplied clutch packs within the powerflow of the transmission as a function of speed and input torque. This mechanism is highly dependent on the layout of gearsets and clutches and for a given transmission, so a generalized assumption was made and scaled by the number of transmission gears and increased from a 0.8 scale factor for launch gear to a 1.0 scale factor in top gear. Figure 5 contains the data for the generalized

spinloss assumption as a function of input speed and torque along with the scale factors used to modify spinloss as a function of the details of the transmission.

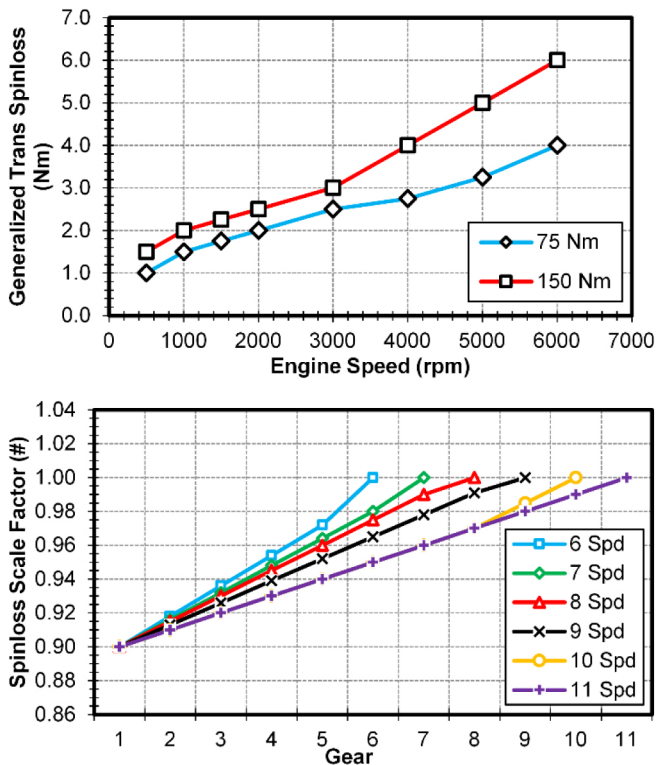


Figure 5. Generalized transmissions spinloss for unapplied clutches as function of input torque and input speed and spinloss scale factor as a function of transmission number of gears and gear state.

Gear mesh efficiency of the automatic transmissions planetary gear train was the third parasitic loss mechanism considered. A single efficiency parabola was assumed to cover entire range of transmission designs formed from the full factorial matrix the details of which are contained in Figure 6. The gear mesh efficiency assumed is taken from data and analysis contained in [7 and 8]. A peak efficiency of 100% is noted at a gear ratio of 1:1 and decreases as transmission gear ratio departs from unity. Minimum efficiency occurs at the underdrive ratios, particularly as the launch gear becomes numerically larger. Similarly as the overdrive gears decrease numerically, the mesh efficiency decreases. For gear ratios falling between the discrete points in Figure 6, separate 4th order polynomial curve fits above and below unity gear ratio were used to estimate a value of mesh efficiency. The range of efficiencies assumed falls within a typical range of gear mesh efficiencies of modern planetary automatic transmissions indicated by the shaded region in Figure 6; see [6, 9, 10, 11, 12, 13].

The final parasitic loss mechanism included is that of the final drive. A fixed value of efficiency as a function of transmission input torque was assumed to simplify the analysis; however, it is known that losses are typically highly dependent on selected ratio, speed and operating torque. For this study front wheel drive, FWD, applications an efficiency of 98% was assumed while for rear wheel drive, RWD, applications an axle efficiency of 92% was assumed.

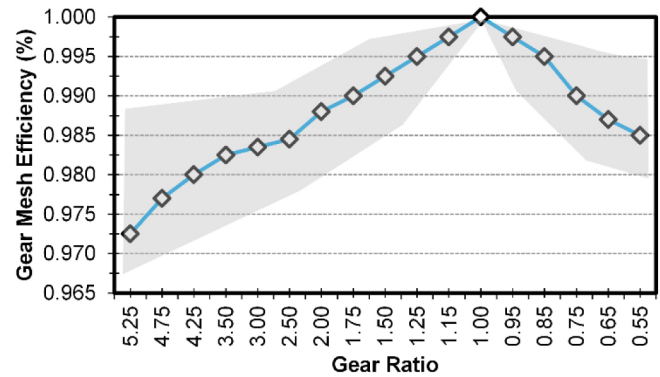


Figure 6. Assumed gear mesh efficiency as function of gear ratio, showing typical range of mesh efficiencies reported for production transmissions within shaded region.

For perspective, the overall parasitic torque loss for FWD 6 to 11 speed transmissions with a launch gear ratio of 4.75:1 and top gear ratio of 0.65:1 operating at 150 Nm of input torque and 1500 rpm input speed is shown in Figure 7. The minimum torque loss corresponds to the gear ratio closest to unity. The highest overall losses are found at launch or top gears.

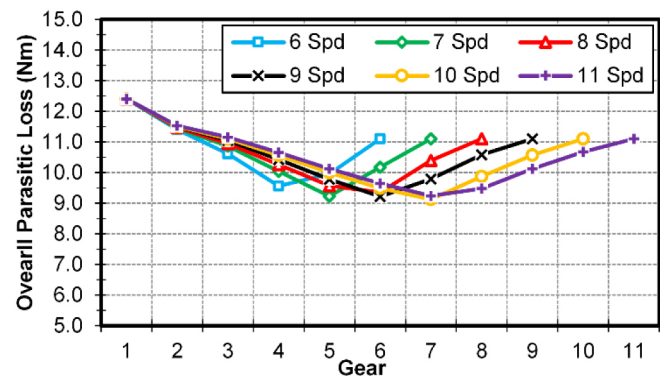


Figure 7. Overall parasitic torque loss at 150 Nm input torque for FWD 6 to 11 speed transmissions with 4.75:1 launch gear ratio and 0.65:1 top gear ratio.

An additional assumption regarding the thermal state of the engine and transmission was made for all simulations to eliminate the fuel consumption benefits afforded by various powertrain combinations, transmission architectures and thermal warm-up strategies. The bulk transmission oil temperature was set at 90 C, the engine oil and coolant temperatures were set at 100 and 90 C, respectively, throughout the entirety of the WLTP test schedule simulations.

Vehicles

Five vehicle platforms were included in this investigation to examine the trend in the ideal number of automatic transmission gears and gear ratios to achieve optimal fuel consumption as function of vehicle and road load power. Figure 8 contains the vehicle mass versus the road load power requirement on a 0% grade at 88.5 kph.

Each combination of vehicle and engine were simulated with three final drive ratios to cover the range of ratios used to balance fuel consumption and acceleration performance. The range of final drive ratios used for the five vehicle platforms are listed in Table 3. The

final drive ratios span the range that are typically utilized for the mass and road load of the vehicles modeled and can modify the perception of the vehicle from that of fuel economy to performance orientated.

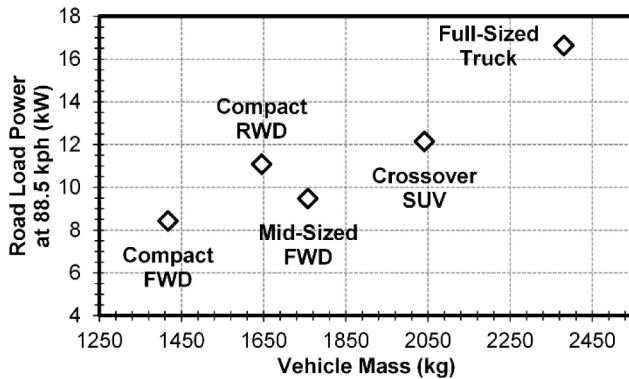


Figure 8. Road load power vs. mass for the five vehicle applications investigated in full factorial DOE.

Table 3. Final drive ratios used in simulations by vehicle platform.

	FDR 1	FDR 2	FDR 3
Compact FWD	3.17	3.32	3.47
Mid-Sized FWD	2.89	3.23	3.55
Crossover SUV FWD	2.89	3.23	3.55
Compact RWD	2.85	3.08	3.23
Full-Sized Truck	3.08	3.23	3.42

Engines

Each vehicle platform considered was modeled with two gasoline engines. A total of eight different engines were included in the investigation, listed as Eng 1 to Eng 8 in Figure 9 showing the ratio of peak engine power divided by displacement along with the engines minimum brake specific fuel consumption, BSFC, at 2000 rpm. The smallest displacement engine is Eng 1 and the largest displacement engine is Eng 8. All engines are direct injected, with engines 1, 2, 3, 5 and 7 turbocharged and engines 4, 6 and 8 naturally aspirated. Cylinder deactivation was a technology featured on Eng 8 as a further means of reducing pumping losses and fuel consumption.

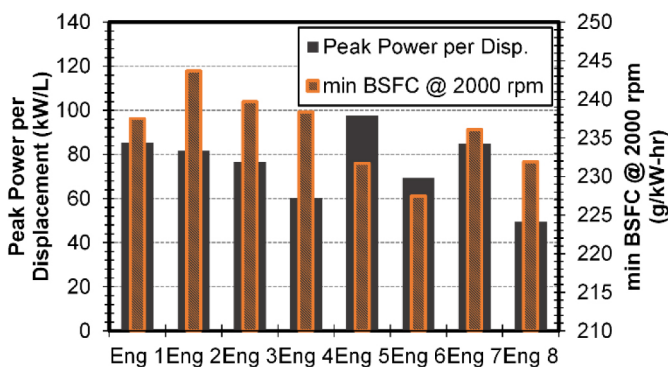


Figure 9. Engine peak power to displacement ratio and minimum BSFC at 2000 rpm for all engines considered in the full factorial matrix.

ANALYSIS SETUP

Energy Analysis Method

The technique used to simulate the fuel consumption performance of the vehicle - powertrain system is a first principles, energy based modeling methodology. All the key characteristics of the vehicle and powertrain are included as inputs to the model along with simplified control algorithms for key powertrain operating features.

Configurable calibration values for engine fueled, and non-fueled operation, transmission shift schedule, shift dynamics and launch device behavior exist as user inputs. The technique is fundamentally identical to that detailed by [4, 5 and 14] so the reader is directed to those references and will not be explored at depth.

Test Schedule

To assess the fuel consumption performance of the various vehicle and engine combinations with the full factorial experiment of automatic transmission gear and ratio designs, the WLTP test schedule was selected as the best candidate. The WLTP test schedule shown in Figure 10 is an 1800 second long schedule that combines city and highway driving with a range of vehicle accelerations. Using the WLTP schedule enables one rather than multiple test schedules simulations since it essentially emulates a three-cycle average of combined FTP city and highway with the US06 in terms of fuel consumption. This reduces the number of test schedules to be run and will in general lead to the conclusions with respect to the gear and ratio selection required to achieve an optimal automatic transmission design. None of the powertrain or vehicle applications were modeled with start/stop as an operating feature as it does not serve to differentiate automatic transmission gearing selection. Note that the powertrain thermal conditions assumed for this investigation were a nominally warmed up, 100 C for engine and 90 C for transmission, state rather than starting cold at 20 C as specified by the test schedule.

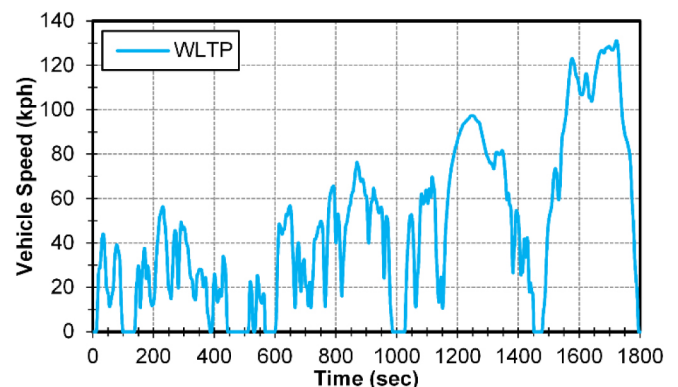


Figure 10. Vehicle speed versus time for the WLTP test schedule.

Shift Schedule Generation

The shift schedules for all the hypothetical automatic transmission designs originating from the full factorial matrix were created using an optimization procedure. Each unique transmission design and final drive pairing with a given vehicle and engine combination went through an analytical optimization procedure to develop the shift and torque converter clutch (TCC) schedules. The optimization criteria

a minimization search of best operating point based on the ratio of mass flow rate of fuel at the engine over power delivered to the vehicle axle as defined by,

$$\Psi = \min \left(\frac{\dot{m}_{fuel,engine}}{P_{axle}} \right) \quad (3)$$

The power delivered at the axle, P_{axle} , includes the torque losses associated with the torque converter, automatic transmission and final drive.

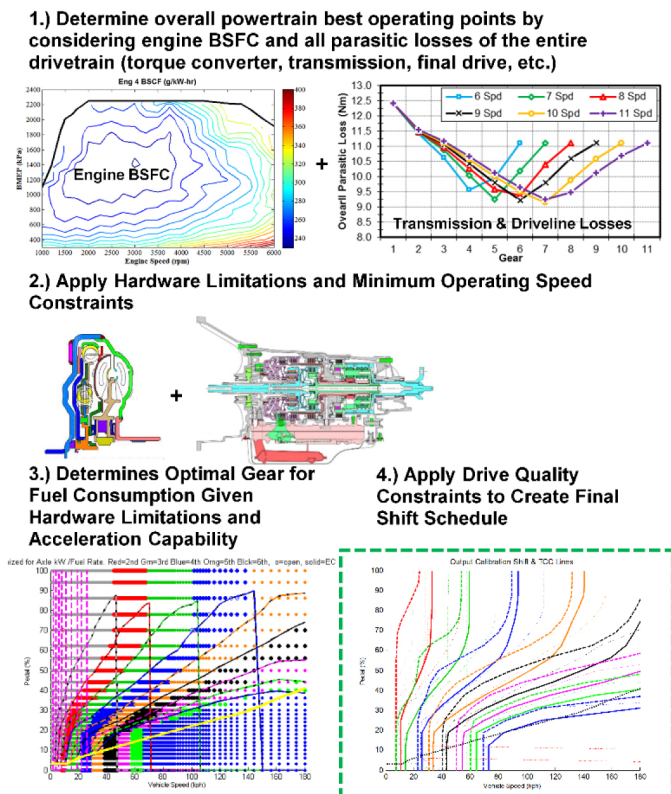


Figure 11. Flowchart of analytical optimization procedure to develop the shift and TCC schedule for each unique vehicle - powertrain combination considered in this investigation.

An overview of the analytical procedure to determine the final shift and TCC schedule is shown in Figure 11. It begins by considering the engine brake specific fuel consumption (BSFC) and overall drivetrain efficiency defined by Eq. (3) for a specific vehicle and powertrain combination as noted in step 1 of Figure 11. Next, hardware limitations of the powertrain such as minimum operating speeds per gear state, hydraulic pump limitations, system resonances, etc. are applied in step 2 of Figure 11. At this point, an initial map of best operating point for fuel consumption given the engine, drivetrain and the system hardware constraints can be formulated as noted in step 3 of Figure 11. The various colored dots correspond to the individual gear states of the transmission and the solid lines represent tractive effort capability for a given gear. The solid yellow line is the zero grade, road load of the vehicle. The plot in step 3 closely resembles that of a shift schedule only missing consideration for drive quality and shift busyness. Finally, the drive quality constraints such as speed

and pedal hysteresis and minimum time in gear are applied in step 4 of Figure 11 to produce the final shift schedule along with the TCC apply and release schedules. All transmission variants were subject to the aforementioned procedure in addition to assuming similar types of constraints and hardware limitations. The purpose of setting up the analysis this way was not to give an advantage to any one particular transmission variant and to isolate the effect of gearing selection alone. Additional details of other shift and TCC schedule are further explained in the next sections.

Deceleration Fuel Cutoff

During a zero percent pedal position vehicle coast down with or without brake pedal input, it is common to turn off fueling to the engine, referred to as deceleration fuel cutoff (DFCO). This powertrain control feature was used in the analysis to provide a fuel consumption benefit. The shift and TCC schedules are developed around this operating feature with the torque converter required to be applied under this condition. For all the transmissions considered, a minimum operating DFCO gear ratio of approximately 1.4:1 was assumed. The DFCO condition was exited slightly before the minimum DFCO gear ratio, refueling the engine for all numerically higher gear ratios along with releasing the TCC. For all gear ratios numerically lower than the minimum DFCO gear, the torque converter was applied and DFCO enabled at a zero percent pedal position.

Minimum Operating Speed Constraints

Optimization of the shift speeds relative to the engines BSFC contours is a further enabler to minimizing fuel consumption after the gearing has been determined for the automatic transmission. For this investigation, the minimum operating speeds per gear were determined primarily by fixed offsets from the engine operating limits of idle speed and the lowest possible engine speed for DFCO. Additionally, the minimum operating speed in each gear cannot violate hardware limitations of the gearbox, launch device, hydraulic capability or prompt excessive downshifting. The hardware limitations are easily quantified, while a vehicle speed hysteresis constraint between downshift and upshifts is utilized to prevent busyness. Typically, this constraint is about 2 to 4 kph depending on the spacing of the transmission gears. The earliest upshift speeds originate from the selection of the minimum operating speed for each gear combined with hysteresis constraints. The shift schedule optimization algorithm has a supplementary feature to check that capability, acceleration of the vehicle, can still be met once in the upshifted gear. If capability cannot be met, then the upshift gets pushed out to higher vehicle speeds beyond the minimum hysteresis constraint until capability can be achieved. For all variants of automatic transmissions considered, the minimum operating speeds ranged from 850 to 1000 rpm, with minimum upshift speeds in the range of approximately 1050 to 1150 rpm.

Torque Converter Clutch Operation

For all transmission variants, the TCC schedule was assumed to be fully applied in all gears above the minimum DFCO exit gear, approximately a ratio of 1.4:1. For all other gears the TCC apply condition depended on driver's pedal position. At a zero percent pedal position the TCC was unapplied, operating as a hydrodynamic