

Transmission Dynamic Modeling and Parametric NVH Analysis

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ABSTRACT

A new approach for modeling and analysis of a transmission and driveline system is proposed. By considering the stiffness, damping and inertias, model equations based on lumped parameters can be created through standard Lagrangian Mechanics techniques. A sensitivity analysis method has then been proposed on the eigenspace of the system characteristic equation to reveal the dynamic nature of a transmission and driveline system. The relative sensitivity calculated can clearly show the vibration modes of the system and the key contributing components. The usefulness of the method is demonstrated through the GM 6-speed RWD transmission by analyzing the dynamic nature of the driveline system. The results can provide a fundamental explanation of the vibration issue experienced and the solution adopted for the transmission. The proposed method can greatly simplify the NVH related analysis, which is useful for architecture selection, shift dynamic analysis, TCC damper design, driveline analysis, hardware and control design. It is also useful for other applications beyond transmission and driveline systems.

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INTRODUCTION

Driveline NVH characteristics are very important to vehicle drive quality and possible issues may be perceived by a customer through audible sound, drive torque and chassis etc. The strong need to further improve fuel efficiency demands powertrain systems of increased complexity that may result in new NVH issues. Many NVH phenomena are determined by powertrain architecture and components, such as engine, transmission as well as electrical motors. Each driveline architecture, such as front wheel drive (FWD), rear wheel drive (RWD) and all-wheel drive (AWD), has its unique NVH characteristics, and so does the transmission architecture. As the driveline components are highly coupled, identification and isolation of the components that contribute to specific driveline vibration can be very challenging. In addition, transmission damper design has become increasingly important as engine downsizing with boost and active cylinder deactivation continue to be the trend in future automotive powertrains. Driveline NVH analysis and parameter tuning for the damper design has become more difficult under all possible operation conditions. In this context, development of a refined driveline system with desired performance requires a deep understanding of such a system, and a useful tool for quick concept assessment, selection, analysis and troubleshooting is highly desired.

When it comes to NVH related analysis and troubleshooting, simulation of dynamic driveline model is often used. Simulation can reveal the detailed dynamic processes of the system in the selected scenarios. However, it is inefficient to be used in the study of the

impact from the subsystems, components and their design parameters on the overall system behavior, especially for the identification and isolation of the components that contribute to certain vibration modes. Simulation intensity will grow with the system complexity, and it can quickly become a formidable task if multiple operating conditions and component parameters need to be considered with their combinations. Some commercial software tools offer functionalities that can help study the parameter dependent system dynamic characteristics for the analysis and control purposes. However, they are still simulation based and the results are usually tied to specific system parameters and simulation scenarios. When a study involves multiple components and a broad frequency range as well as their impact on overall system's behavior, overall system simulation in an exhaustive fashion is inevitable, which is challenging and inefficient for any simulation based tool. Therefore, a quick and efficient method or tool is highly desired for the study of the parametric influence on the system dynamics. To simplify this kind of NVH related analysis, a systematic and analytic method is proposed in this paper. Based on the system equation, the dynamic nature of a transmission and driveline system can be studied without simulation. The results can reveal fundamental properties of the system that can help engineers better understand NVH tendencies resulting from interactions among driveline components.

In the subsequent sections, the system structure, model equations based on Lagrangian Mechanics techniques, and analysis methods will be presented. An analysis example will then be described to demonstrate the effectiveness of the proposed method. Summary and concluding remarks will be drawn in the end.

TRANSMISSION & DRIVELINE SYSTEM EQUATIONS

Transmission & Driveline Model Structure

An automotive driveline is usually modelled as a series of inertia-spring systems with lumped parameters. Figure 1 shows an example of a general driveline model, where the transmission is simplified as a lumped inertia. When it comes to transmission related NVH analysis, such an approach is over simplified, where possible dynamic responses from the transmission components are neglected.

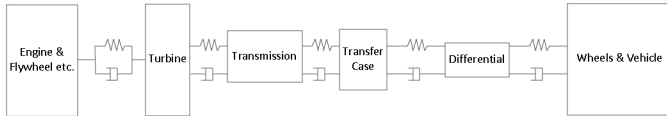


Figure 1. Lumped Parameter Driveline Model Structure

For analysis and control characterization, an inertia-based transmission model has often been used. The effect of transmission internal inertias have been well-studied and documented for shift dynamic analysis and control design as well as clutch design and sizing [1]-[2]. Using the basic free body diagram of an automatic transmission presented in [2] as in Figure 2, the transmission can be broken into two lumped inertias before and after the oncoming clutch. The equivalent inertias can be calculated with respect to the input acceleration, which can further be used to calculate engine torque, clutch torque, as well as shift time for a desired shift performance. However, when it comes to detailed dynamics and the NVH nature of a transmission, this simplified inertia-based model is no longer sufficient.

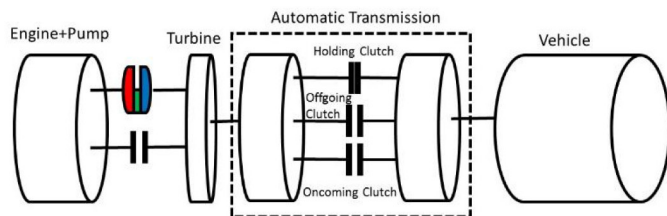


Figure 2. Free Body Diagram of an Automatic Transmission [2]

To understand the transmission and driveline dynamics during a shift from the NVH perspective, a more detailed model needs to be considered. Built upon the inertia-based model in Figure 2, a new transmission and driveline model can be created with internal inertias, stiffness, and damping taken into account. With the extended transmission and driveline model structure illustrated in Figure 3, the driveline NVH nature in response to all the possible inputs such as the engine, clutches or brakes can be better comprehended through a theoretical analysis.

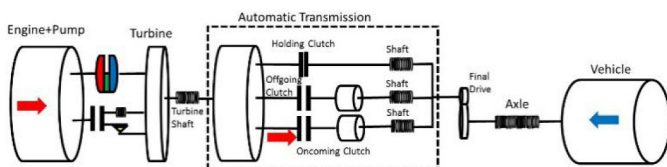


Figure 3. Detailed Transmission Model Structure in a Driveline Environment

State Space Model Equation

From the model structures presented in the previous section, a system equation can be formulated in the standard procedure used in Lagrangian Mechanics [3]. To formulate the system equation, state variables need to be defined based on the nodes selected along the driveline and transmission system. From the free body diagrams illustrated in Figures 1, 2, 3, the nodes with lumped parameters, such as inertia, stiffness, damping, can be defined according to the rotational speeds. The components that rotate at a same speed can be lumped together into a node, and in a transmission system, nodes are connected through either a clutch, gear, shaft or friction surfaces. In general, let us denote by an $n \times 1$ vector φ the state variables, where entry φ_i represents the rotational angle of the i^{th} node. The speed and acceleration vector can be denoted by $\dot{\varphi}$ and $\ddot{\varphi}$ respectively. With the nodes selected, the system equations can be formulated based on Newton's second law of motion with the constraints on the motion relevant to each node. Such constraints may come from the transmission components like planetary gear sets and clutches etc. In a driveline system, the constraints can be written as

$$\Phi \varphi = 0 \text{ (or } \Phi \ddot{\varphi} = 0 \text{)} \quad (1)$$

Here, Φ is an $m \times n$ matrix with m representing the number of the constraints considered in the system. Use clutch as an example to illustrate how the constraint equation can be formulated. Suppose that a clutch is used to connect to node i and j . When locked, the two nodes become coupled and rotate together. The motion of the two nodes satisfy the following equation.

$$\varphi_i - \varphi_j = 0$$

This is equivalent to $\bar{\Phi} \varphi = 0$ with $\bar{\Phi}$ defined as

$$\bar{\Phi} = [0 \quad \dots \quad 0 \quad 1 \quad 0 \quad \dots \quad 0 \quad -1 \quad 0 \quad \dots \quad 0]$$

Here, the entries of vector $\bar{\Phi}$ are zero except for the i^{th} and j^{th} entries, which are 1 and -1 respectively.

Constraint matrix Φ can also reveal the interaction forces among the nodes coupled through constraints. Such forces acting on the nodes can be written as $\Phi^T \Lambda$, with Λ being an $m \times 1$ force vector to be solved. Here, Λ is essentially the Lagrangian defined in Lagrangian Mechanics [3]. In the clutch constraint example, the torque acting on node i and j on either side are equal but in opposite direction, which can be written as $\bar{\Phi} \lambda$ with scalar λ representing the clutch torque.

Differential equations can be written using the second law of motion, and they can be put into a matrix form as

$$\bar{I} \ddot{\varphi} + C \dot{\varphi} + K \varphi = \Sigma T + T \quad (2)$$

In eq. (2), φ is an $n \times 1$ vector where each entry represents the angular position of the corresponding node defined; n is the number of nodes modeled; $\bar{I}$ is an $n \times n$ diagonal matrix, where each entry on the diagonal represents the rotational moment of inertia associated with the corresponding node; C is an $n \times n$ matrix with diagonal entries

presenting damping elements; ΣT is the $n \times 1$ matrix with diagonal entries presenting the effective stiffness associated with the relative subcomponents; T is an $n \times 1$ vector where each entry represents the sum of the reaction forces acting on corresponding node; $\bar{T}$ is an $n \times 1$ vector where each entry represents the external force directly applying on relevant nodes. The reaction forces represented by ΣT are the forces from the constraints to be resolved. The external input T to the driveline system may come from the engine, clutch and brakes etc. Through Lagrangian Mechanics, the internal forces ΣT can be written as

$$\Sigma T = -\Phi^T \Lambda$$

Here, the minus sign is chosen arbitrarily for simplicity.

Combine eq. (1) and (2), one can obtain the system equation.

$$Mx + \bar{C}\dot{\bar{\varphi}} + \bar{K}\bar{\varphi} = \bar{T} \quad (3)$$

The terms eq. (3) are defined as

$$M = \begin{bmatrix} \bar{I} & \Phi^T \\ \Phi & 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix}, \bar{K} = \begin{bmatrix} K & 0 \\ 0 & 0 \end{bmatrix}, \bar{T} = \begin{bmatrix} T \\ 0 \end{bmatrix}, \quad (4)$$

$$x = \begin{bmatrix} \ddot{\varphi} \\ \Lambda \end{bmatrix} \text{ and } \bar{\varphi} = \begin{bmatrix} \varphi \\ 0 \end{bmatrix}$$

where $\bar{I}$, C , K and T are defined in eq. (2); $\bar{\varphi}$ and $\bar{T}$ are defined in such a way that the dimension of the relevant terms remain consistent; x is an $(m+n) \times 1$ vector where the bottom m entries, represented by vector Λ , are the Lagrangians associated with the constraints Φ .

Eq. (3) represents a general form of the equation established for a transmission and driveline system with all possible elements (e.g., inertia, stiffness, damping) taken into account. The equation is generic enough to represent a transmission driveline system under different gears or during gear shifting with input and output at different locations. It is very useful for dynamic analysis and control design. In addition, the input and the output can be selected among the n nodes modeled depending on the problem of interest.

Eq. (3) is a linear dynamic system, so that many tools developed for the linear system theory can be readily applied. Note that this driveline system has n nodes and m constraints, so that the degree of freedom of the system is $n-m$.

System Sensitivity Analysis

In a vibrational mechanical system like the one modeled by eq. (3), component inertias $\bar{I}$ and the interacting stiffness K determine fundamental vibration modes, e.g., the associated natural frequencies and relative strength of vibrations among the relevant nodes, which is often referred to as mode shape. Damping C represents possible energy dissipation inside the system, and from a system NVH perspective, damping can reshape the mode shape, and if used properly, it can be an effective solution to some driveline vibration

problems. The effectiveness of damping C indeed depends on its locations, magnitude and the vibration frequency. From a system analysis perspective, it is important first to identify system vibration tendency. Such task can be done through the corresponding undamped system with any damping and parasitic losses ignored, as the system vibration modes are essentially driven by interacting inertias and stiffness. In fact, the frequency response of the undamped system is usually very close to the real system, unless the damping is carefully designed and in which case, the effect of the damping can be further studied through the model equation discussed here. For the identification purpose, we can first focus on an undamped system with $C=0$ in eq. (3).

For the undamped version of the system, eq. (3) can be rewritten as

$$Mx + \bar{K}\bar{\varphi} = \bar{T}. \quad (5)$$

To characterize the system dynamic nature, the corresponding system characteristic equation can be derived through the homogenous equation of free motion as written in eq. (6) [4].

$$\text{Det}(\bar{K} - M\omega^2) = 0. \quad (6)$$

In eq. (6), ω^2 is an eigenvalue of matrix $\bar{K}M^{-1}$, and ω is the natural (angular) frequency associated of each resonant mode. Note that eq. (6) is not a normal system characteristic equation described in [4] due to the Lagrangian term Λ involved. However, it can be proved they are equivalent for the analysis proposed in this paper.

As mentioned earlier, the system described by eq. (3) or (5) has the degree of freedom $n-m$, so that matrix $\bar{K}M^{-1}$ has no more than $n-m$ nonzero eigenvalues. It can further be shown that each nonzero eigenvalue of matrix $\bar{K}M^{-1}$ is real and positive. Furthermore, each nonzero eigenvalue ω_k^2 ($k=1, 2, \dots, K$) is associated with one resonant mode of the system. In other words, the system has a natural vibration tendency under periodic input at each frequency ω_k . The corresponding eigenvector v_k reveals the mode shape with each entry representing the relative vibration magnitude and phase of the vibration at the relevant node.

A particular driveline vibration mode is usually caused by the interaction among relevant components. The interacting components are not necessary close to one another and often located at different places inside the transmission or along the driveline. The fact that they can be far apart from one another makes identification or isolation of the key component to a vibration very challenging at times. During the hardware development and troubleshooting, such tasks are usually done through simulations, which can be time consuming if there are a high number of components that can potentially contribute to the vibration. Thus, the ability to quickly identify the contributing components to a NVH phenomenon is highly desired. In what follows, we will discuss a system sensitivity method inspired by perturbation analysis that can clearly reveal the system vibration mode and their contributing components. Not only does it greatly simplify the identification process, but it also provides additional insights into the dynamic nature of the driveline system.