

Study on Vehicle Stability Control by Using Model Predictive Controller and Tire-Road Force Robust Optimal Allocation

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ABSTRACT

The vehicle chassis integrated control system can improve the stability of vehicles under extreme conditions using tire force allocation algorithm, in which, the nonlinearity and uncertainty of tire-road contact condition need to be taken into consideration. Thus, An MPC (Model Predictive Control) controller is designed to obtain the additional steering angle and the additional yaw moment. By using a robust optimal allocation algorithm, the additional yaw moment is allocated to the slip ratios of four wheels. An SMC (Sliding-Mode Control) controller is designed to maintain the desired slip ratio of each wheel. Finally, the control performance is verified in MATLAB-CarSim co-simulation environment with open-loop manoeuvres.

CITATION: Dengbo, H., Hui, L., and Fan, Y., "Study on Vehicle Stability Control by Using Model Predictive Controller and Tire-road Force Robust Optimal Allocation," *SAE Int. J. Commer. Veh.* 8(1):2015, doi:10.4271/2015-01-1580.

INTRODUCTION

The past 30 years have witnessed the development of vehicle chassis control systems [1]: from single wheel control system (Anti-lock Braking Control-ABC, Traction Control System-TCS) to vehicle system dynamics control (Electronic Stability Control-ESC [2]) and chassis integrated control systems [3, 4]. Nowadays, the vehicle chassis controllers can be classified into three categories according to the tire force direction: lateral tire force control (Active Front Steering - AFS, Four Wheel Steer Control - 4WS), longitudinal tire force control (ABS, TCS, Vehicle Stability Control - VSC, Vector Torque Control-VTC) and vertical tire force control (Continuous Damping Control - CDC, Anti Roll Control - ARC, Active Rear Spoiler Control).

Braking/driving and steering are two important control systems on longitudinal and lateral directions that have attracted many researchers. The coordination between ESC and EPS (Electric Power Steering) are studied by using the limitation information of tires calculated from EPS as the reference of ESC controller to avoid saturation of tire forces [5]. This kind of integrated control is based on the information interaction of different systems. Also, another kind of controller was proposed by many researchers, of which the 4WS control is of higher priority. When the control objective is beyond the ability of 4WS, the ESC controller or ESC& 4WS integrated controller which is combined with the active front steering and braking force allocation algorithm [6, 7] works. Finally, the integrated controllers considering the integration of different control inputs are also discussed in many papers, which are based on multi-input and multi-output vehicle model with the non-linear tire model taken into consideration [8, 9]. This paper will focus on an integrated chassis controller aiming at improving the vehicle handling stability.

By using model predictive control, the vehicle will be able to follow the driver's steering intent better so that the handling stability can be improved [10, 11]. For the model based control algorithm, an accurate model is highly valuable because the model error will bring down the performance of the vehicle especially when the tires enter non-linear region. Considering the computation burden, a compromise between the accuracy and the model complexity is necessary [12]. In this paper, the LTV-MPC (Linearized Time Varying MPC) controller adopts a single track linear model for the computational consideration. The outputs of the LTV-MPC controller are additional steering angle and additional yaw moment.

In order to allocate the additional yaw moment to the slip-ratios of each wheel, a robust optimal allocation algorithm is designed to ensure the robustness of the vehicle stability controller proposed in this paper. Selected single wheel braking logic and 2-4 braking channel with quadratic programming (QP) or nonlinear programming are implemented by many researchers [13, 14, 15]. However, the effect of the optimal braking force allocation largely relies on the force capacity of each wheel and vehicle parameters, which can hardly be obtained accurately. Hence, a robust optimal algorithm is proposed to deal with the uncertainty of the vehicle parameters and the mismatch of the tire model in this paper. Considering the saturation phenomena of them and the complicated relationship between the tire longitudinal force and lateral force, the Burkhardt tire model [16] is adopted and the slip-ratios of four wheels are allocated.

Finally, the allocated slip-ratios need to be tracked by each wheel quickly and accurately. The most commonly used slip-ratio tracking algorithm is logic threshold control [17], which can guarantee control performance without depending on dynamics models. However, it

lacks theoretical base and requires lots of tuning work. The PID logic [18] is also widely used in vehicle dynamics controller. However, the parameters of PID controller highly rely on the tire-road contact condition. Thus it cannot guarantee consistent control performance without knowing the road conditions. In this paper, considering the non-linearity of the tire and uncertainty of tire-road contact situation, an SMC controller is used, which is insensitive to model parameters and responses quickly if finely tuned.

The entire structure of the proposed hierarchical controller is shown in Figure 1:

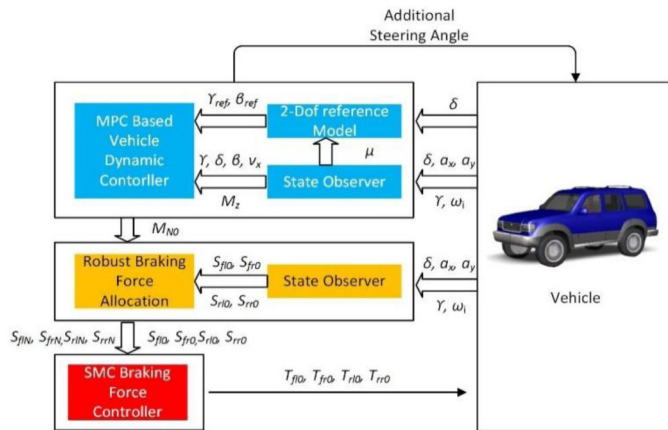


Figure 1. Configuration of the control system

The following paper is structured as follows: First, the design of the MPC controller will be introduced. Then, the optimal robust allocation algorithm will be described. Next, the sliding-mode algorithm to track the slip-ratio of the wheel will be proposed. Finally, we will present and analyze the simulation results and a conclusion will be made at the end of this paper.

THE MPC CONTROLLER

Vehicle Model and Tire Characteristics

In this section, the vehicle models are depicted. A linear bicycle model is used to generate the reference yaw velocity and the vehicle side slip angle. And a time-varying linearized model is used as the predictive model of the MPC controller. Both of them are based on the single track assumption, as shown in Figure 2:

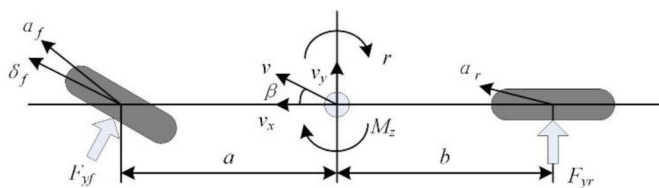


Figure 2. Single track bicycle model

where, v represents the velocity at the center of gravity; v_x and v_y are the longitudinal and lateral velocity, respectively; γ is the vehicle yaw rate and M_z is the additional yaw moment applied on the vehicle; β is the vehicle side slip angle; a , b represent the distances of the front axle and the rear axle from the center of gravity; F_{yf} and F_{yr} are the

lateral forces of the front and the rear axles; a_f and a_r are the tire side slip angles of the front and the rear axles; δ_f is the steering angle of the front wheel.

Linear Bicycle Model and Reference Yaw Rate & Sideslip Angle

As shown in Figure 2, supposing the vehicle speed v_x is constant, then the vehicle dynamics equations are as follows:

$$\dot{\beta} = \frac{F_{yf} + F_{yr}}{mv_x} - \gamma \quad (1)$$

$$\dot{\gamma} = \frac{aF_{yf} - bF_{yr}}{I_{zz}} \quad (2)$$

While, the lateral force of the front and the rear axles are presented as follows:

$$F_{yf} = -C_{af0}\alpha_f, F_{yr} = -C_{ar0}\alpha_r \quad (3)$$

where, C_{af0} and C_{ar0} are the cornering stiffness of the tires obtained by linearizing the nonlinear curve of tire force from the origin. The tire slip angles of the front and the rear axles are:

$$\alpha_f = \beta + \frac{a\gamma}{v_x} - \delta_f, \alpha_r = \beta - \frac{b\gamma}{v_x} \quad (4)$$

Defining $x = [\beta, \gamma]^T$ and substituting Eq. (3) and Eq. (4) into Eq. (1) and Eq. (2), the linear bicycle model can be described in a linear state-space form:

$$\dot{x} = Ax + Bu \quad (5)$$

where,

$$A = \begin{bmatrix} \frac{-(C_{af} + C_{ar})}{mv_x} & \frac{-(aC_{af} - bC_{ar})}{mv_x} - 1 \\ \frac{-(aC_{af} - bC_{ar})}{I_{zz}} & \frac{-(a^2C_{af} + b^2C_{ar})}{I_{zz}v_x} \end{bmatrix}, B = \begin{bmatrix} \frac{C_{af}}{mv_x} \\ \frac{aC_{af}}{I_{zz}} \end{bmatrix} \quad (6)$$

Hence, the reference output can be obtained as:

$$\gamma_{N0} = v_x \delta_f / \{(a + b) [1 + (v_x/v_{ch})^2]\} \quad (7)$$

where, $v_{ch} = \frac{C_{af}C_{ar}(a+b)^2}{m(C_{ar}b - C_{af}a)}$. The maximal tire-road friction force will limit the reference value,

$$\gamma_{refmax} = |\mu g / v_x| \text{sign}(\delta)$$

where, μ is the coefficient of friction, g is the acceleration of gravity. Hence, the reference vehicle yaw rate is given by

$$\gamma_{ref} = \min\{|\gamma_{N0}|, |\mu g/v_x|\} \text{sign}(\delta) \quad (9)$$

The vehicle slip angle is also limited by a function of vehicle speed, which, if exceeded, the vehicle will lose control [19]:

$$\beta_{max} = 10^\circ - 7^\circ \cdot \frac{v_{c0}^2}{v_{ch}^2} \quad (10)$$

So, the reference vehicle side slip angle is given by:

$$\beta_{ref} = \begin{cases} \beta, & |\beta| < |\beta_{max}| \\ \pm \beta_{max}, & |\beta| \geq |\beta_{max}| \end{cases} \quad (11)$$

Then the reference yaw rate and sideslip angle are as follows:

$$\gamma_{t+i,t}^{ref} = [\beta_{ref}, \gamma_{ref}] \quad (12)$$

Linearized Time Varying Bicycle Model

As the cornering stiffness is constant in the above bicycle model, it can only predict the dynamic response in linear area. In order to keep the predictive precision in both linear and non-linear regions, a linearized time varying bicycle model is applied.

Aiming at reflecting the accurate tire characteristics, the tire cornering stiffness is defined as the slope at the working point, which is called the instantaneous cornering stiffness C_{ins} , as shown in Figure 3. For the front and rear axles:

$$F_{yi} = \hat{F}_{yi,0} - \hat{C}_{ai,0}(\alpha_i - \alpha_{i,0}), i = f, r \quad (13)$$

where, $\hat{F}_{yi,0}$ and $\alpha_{i,0}$ represent the lateral force and the tire slip angle at the working point, respectively. $\hat{C}_{af,0}$ and $\hat{C}_{ar,0}$ are the instantaneous cornering stiffness of the front and the rear axles, which can be calculated online from the tire model. In this paper, the Burckhardt tire model is used, which will be discussed in following sections.

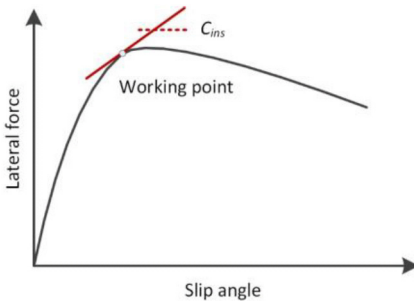


Figure 3. The relationship between tire lateral force and tire slip angle

Then we substitute Eq. (13) into Eq. (1) and Eq. (2) so that a linearized bicycle model can be obtained with the tire model linearized at the working point:

$$\dot{\beta} = \frac{\hat{F}_{yf,0} - \hat{C}_{af,0}(\alpha_f - \alpha_{f,0}) + \hat{F}_{yr,0} - \hat{C}_{ar,0}(\alpha_r - \alpha_{r,0})}{mv_x} - \gamma \quad (14)$$

$$\dot{\gamma} = \frac{a(\hat{F}_{yf,0} - \hat{C}_{af,0}(\alpha_f - \alpha_{f,0})) - b(\hat{F}_{yr,0} - \hat{C}_{ar,0}(\alpha_r - \alpha_{r,0})) + M_z}{I_{zz}} \quad (15)$$

When the active steering angle δ_{f_add} is taken into consideration, the front wheel steering angle can be written as:

$$\delta_f = \delta_{f_driver} + \delta_{f_add} \quad (16)$$

where, δ_{f_driver} is the steering angle of front wheel exerted by the driver. Denoting $x = [\beta, \gamma]^T$ and $u = [\delta_{f_add}, M_z]^T$, an LTV state space model can be obtained from the equations above:

$$\dot{x} = A_{c,t}x + B_{c,t}u + d_{c,t} \quad (17)$$

where,

$$A_{c,t} = \begin{bmatrix} \frac{-\hat{C}_{af,0} + \hat{C}_{ar,0}}{mv_x} & \frac{-a\hat{C}_{af,0} + b\hat{C}_{ar,0}}{mv_x^2} - 1 \\ \frac{-a\hat{C}_{af,0} + b\hat{C}_{ar,0}}{I_{zz}} & \frac{(a^2\hat{C}_{af,0} + b^2\hat{C}_{ar,0})}{v_x I_{zz}} \end{bmatrix}, B_{c,t} = \begin{bmatrix} \hat{C}_{af,0} & 0 \\ a\hat{C}_{af,0} & \frac{1}{I_{zz}} \end{bmatrix}$$

$$d_{c,t} = \begin{bmatrix} \frac{\hat{F}_{yf,0} + \hat{C}_{af,0}(\alpha_{f,0} + \delta_{f_driver}) + \hat{F}_{yr,0} + \hat{C}_{ar,0}\alpha_{r,0}}{mv_x} \\ \frac{a(\hat{F}_{yf,0} + \hat{C}_{af,0}(\alpha_{f,0} + \delta_{f_driver})) - b(\hat{F}_{yr,0} + \hat{C}_{ar,0}\alpha_{r,0}) + M_z}{I_{zz}} \end{bmatrix} \quad (18)$$

$d_{c,t}$ is considered to be constant in every sampling step. The tire slip angles of each axle are obtained through Eq. (4). In this paper, we assume that the steering angle δ_{f_driver} , the slip angle of the vehicle β and the yaw rate of the vehicle γ can be measured directly.

Design of the MPC Algorithm

The nominal vehicle slip angle and the yaw rate are generated from Eq. (5). The linearized vehicle model described in Eq. (17) is used as the predictive model with sampling time T_s . In the predictive model, the cornering stiffness of the tire is updated each time at working points and the steering angle of the front wheel exerted by the driver is assumed to be constant in each step. The outputs of the MPC controller are additional front wheel steering angle δ_{f_add} and additional yaw moment M_z , of which, the former is applied to the vehicle directly and the latter is used as the input of the robust optimal allocation algorithm. The MPC control algorithm is described as follows:

$$\min J(x_t, u_t, d_t) = \sum_{i=1}^{H_p} \|y_{t+i,t} - y_{t+i,t}^{ref}\|_Q^2 + \dots + \sum_{i=1}^{H_c} \|\Delta u_{t+i,t}\|_R^2 + \rho \varepsilon^2 \quad (19)$$

$$\text{s. t. } x_{t+i+1,t} = A_{d,t}x_{t+i,t} + B_{d,t}u_{t+i,t} + \dots + d_{d,t}, \quad i = 0, 1, \dots, H_p - 1 \quad (20)$$

$$y_t = x_t \quad (21)$$

$$u_{min} \leq u_{t+i,t} \leq u_{max}, \quad i = 0, 1, \dots, H_c \quad (22)$$

$$\Delta u_{min} \leq \Delta u_{t+i,t} \leq \Delta u_{max}, \quad i = 0, 1, \dots, H_c \quad (23)$$

$$\Delta u_{t+i,t} = 0, H_c < i \leq H_p \quad (24)$$

$$y_{min} \leq y_{t+i,t} \leq y_{max}, \quad k = 0, 1, \dots, H_p \quad (25)$$

where, Eq. (20) and (21) are from linearized model described by Eq. (17). $\Delta u_{t+i,t}$ is the optimal control input at time $t+i$ obtained from x_t , u_t , d_t through predictive model at time t . $y_{t+i,t}^{ref}$ is the reference output generated from Eq. (12). H_c and H_p are control horizon and predictive horizon, respectively. $\rho \varepsilon^2$ is the relaxation factor to guarantee the feasibility of the problem. Q is the weighting matrix for the target error, R is the weighting matrix for the control input. The increment of the control input signal is assumed to be zero for all $H_p > i > H_c$.

The problem can be formulated as a Quadratic Programming problem with constraints and only the first increment of the control input $\Delta u_{t+i,t}|_{i=1}$ is used as the input to the vehicle and the robust optimal allocation algorithm.

TIRE-ROAD FORCE ROBUST OPTIMAL ALLOCATION

Considering the uncertainty of the tire-road contact condition and vehicle parameters, a robust optimal allocation algorithm needs to be designed to allocate the additional yaw moment generated from the MPC controller to the slip ratios of each wheel. In this paper, we only consider the braking and steering mixed working condition of the vehicle. A 7-DOF vehicle model with the Burckhardt tire model is linearized to obtain the efficiency matrix for the robust optimal allocation algorithm. As a result, the uncertainty of the allocation algorithm consists of 3 parts: the uncertainty of vehicle parameters, the tire-road contact condition and the linearization process.

Vehicle Model with Non-linear Tire Model

First, a tire model is selected. Commonly used tire models include Dugoff Model [20], Brush Model [21], Burckhardt Model [22], Pacejka Magic Formula Model [23], etc. To reduce the complexity of derivation process, the Burckhardt Model is adapted here, which can be expressed as:

$$\mu_{Res}(S_{Res}) = \{c_1[1 - \exp(-c_2 S_{Res}) - c_3 S_{Res}]\} e^{-c_4 v} \quad (26)$$

where, μ_{Res} is the comprehensive coefficient of friction. c_1 , c_2 , c_3 , c_4 are coefficients that are decided by tire-road conditions and can be fitted based on experimental data. Among them, $e^{-c_4 v}$ reflects the influence of vehicle speed and, usually, $c_4 = 0.02 \sim 0.04 \text{ s/m}$, which will be ignored in this paper. S_{Res} is the comprehensive tire slip ratio and can be defined as follows:

$$S_{Res} = \sqrt{S_L^2 + S_S^2} \quad (27)$$

S_L and S_S are longitudinal ratio and lateral slip ratio respectively and in the braking condition, they can be expressed as follows:

$$S_L = (v_R \cos \alpha - v_w) / v_w \quad (28)$$

$$S_S = (1 + S_L) \tan \alpha \quad (29)$$

where, v_R is the linear velocity of wheel rotation, which can be measured directly. v_w is the linear velocity at the wheel center. α is the tire slip angle. Both α and v_w can be obtained easily from the knowledge of the vehicle longitudinal speed v_x , the lateral speed v_y , the steering angle of the front wheel δ_f and the vehicle yaw rate γ . For simplification, the steering angle of left and right front wheels are assumed to be the same. Thus, the longitudinal force and the lateral force of tires in the tire coordination Yw-ow-Xw can be obtained, as shown in Figure 4:

$$f_l = \frac{S_L \mu_{Res} F_z}{S_{Res}} \quad (30)$$

$$f_s = k_s \frac{S_S \mu_{Res} F_z}{S_{Res}} \quad (31)$$

k_s is a coefficient that indicates the difference between the longitudinal and lateral tire forces. Usually, $k_s = 0.9 \sim 0.95$ [18]. In this paper, we choose $k_s = 0.95$. F_z is the vertical tire force, which is assumed to be measurable in this paper. A 225/65 R17 tire is chosen and tire experiments are conducted in CarSim on both low- μ ($\mu=0.4$) and high- μ ($\mu=0.8$) roads. The data is fitted using nonlinear regression tool in MATLAB.