

Improved Lane-Keeping with Rear Axle Steer

Daniel E. Williams
TRW Commercial Steering Systems

ABSTRACT

The classic two-degree-of-freedom yaw-plane or “bicycle” vehicle model is augmented with two additional states to describe lane-keeping behavior, and further augmented with an additional control input to steer the rear axle. A simple driver model is hypothesized where the driver closes a loop on a projected lateral lane position. A rear axle steer control law is found to be a function of front axle steering input and vehicle speed that exhibits high speed stability and improved low speed maneuverability. The theoretically derived control law bears similarity to practical embodiments allowing a deeper understanding of the functional value of steering a rear axle.

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INTRODUCTION

This work combines relatively simple concepts from vehicle dynamics, control theory and human modelling to derive a new result that is recognized to have practical utility. The driver modelling and control theory aspects have been previously developed [1]. The present work provides a simpler vehicle dynamic based presentation of a rear axle steer control law that is designed to improve high speed stability, but is found to also improve low speed maneuverability, and resembles many rear axle steer control strategies used in practice.

Characterization of vehicle handling was first performed on two-axle passenger cars [2]. It has been long appreciated that for vehicles travelling at a relatively constant forward speed with lateral accelerations so limited to correspond to the linear range of the tires, a simple model including yaw and lateral velocity degrees of freedom can be used to predict vehicle performance. Because such a model neglects sprung mass roll and transient suspension effects the action of tires on a common axle can be combined, with the result referred to in the literature for obvious reasons as the “bicycle” model, and its parsimonious combination of simplicity and ability to characterize well known vehicle properties has resulted in its wide use, e.g. [2, 3, 4, 5, 6]. Complex multi-axle vehicles can easily be analyzed using this yaw-plane model format to evaluate the effect of steering the rearmost axle [7,8].

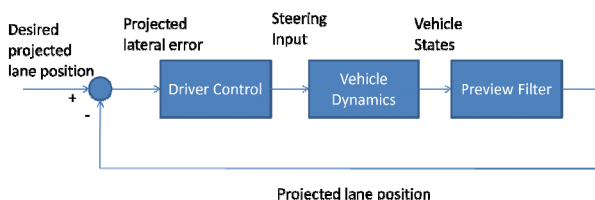


Figure 1. Driver-Vehicle Closed-loop

Steering an auxiliary axle in addition to the primary steered front axle adds an additional control input that has been appreciated by practical vehicle designers for some time. Early efforts at steering the rear axle included a mechanical gearbox connecting the front and rear wheels such that for small inputs the rear wheels steered with angular displacement in the same direction as the driver controlled front wheels (crab-steer), and at larger steering inputs the rear axle steered in the opposite direction (counter-steer). Indirectly this system allowed for crab-steer at high vehicle speeds as the magnitude of steering inputs is practically limited, and counter-steer at lower speeds where larger steering inputs can be achieved. A similar control strategy was used by the electronically controlled Delphi QuadraSteer system, where high-speed crab-steer was featured to improve stability and towing characteristics and counter-steer at low speeds to improve maneuverability [9]. Currently electronically controlled rear axle steering systems are offered on European production commercial vehicles that fully counter-steer at low speeds, and remain at their nominal center positions at higher vehicle speeds [10]. Thus a number of practically achievable rear axle steering concepts make use of low-speed counter-steer and transition to either crab-steer or a fixed axle at high speeds. This work will develop a theoretical understanding of the functionality of these systems that are available in practice.

The ability of a vehicle to maintain a lane is dependent upon the driver. MacAdam [11] gives an excellent review of prior work in driver modelling, in addition to providing a state-of-the-art literature review. MacAdam cites prior work [12] that measured the “driver ‘sight-point’ ahead of the vehicle and developed a steering model based on deviations between the sight point and the desired course... Preview allows the driver to directly observe the time-advanced path input requirement and to then project into the future a predicted vehicle response based upon the estimated control response needed to achieve a certain goal (path error minimization...etc.)”

In its simplest embodiment the driver senses a lane position - perhaps previewed as just discussed - and determines the appropriate handwheel input, thus closing a loop on lane position. Embedded within this closed-loop are possibly driver dynamics in series with vehicle dynamics.

This work will combine well known concepts from the literature to form the man-machine closed-loop shown in [Figure 1](#). The driver control is assumed to convert a projected lane position error to a steering input. Preview will be applied so that the lane position error will be projected in front of the moving vehicle, and steering input occurs if the vehicle is not projected to achieve its desired lane position. Starting with a conventional yaw-plane vehicle model, inertially referenced states and a rear axle steer control input are appended. A rear axle steer control strategy is suggested that insures generally complex zeros of the open-loop transfer function between steering input and projected lane position are real and repeated. The driver and vehicle models are combined and analyzed using concepts from linear system theory, so the effect of various nonlinearities that might be present in the vehicle are beyond the scope of this work.

The proposed control strategy is shown to be generally familiar to practical vehicle designers: counter-steering at low vehicle speed and crab-steering at high speed. The main contribution of this work is therefore a classical control theoretic analysis of the man-machine system providing insight into the theoretical functional performance enhancement achieved by a practically applied speed dependent rear-axle steering strategy.

VEHICLE MODEL WITH REAR AXLE STEERING

Neglecting aerodynamics, external forces that determine vehicle response are transmitted by its tires. These forces can propel, brake or turn the vehicle with a lateral force. The simplest expression for side force generated by a rolling tire is a linear dependence upon slip angle, defined as the angular difference between the tire's center-line and its instantaneous velocity, as shown in [Figure 2](#). The side force F_i of a displaced tire i can be expressed as:

$$F_i = C_i \left(\delta_i - \frac{v + x_i r}{u} \right) \quad (1)$$

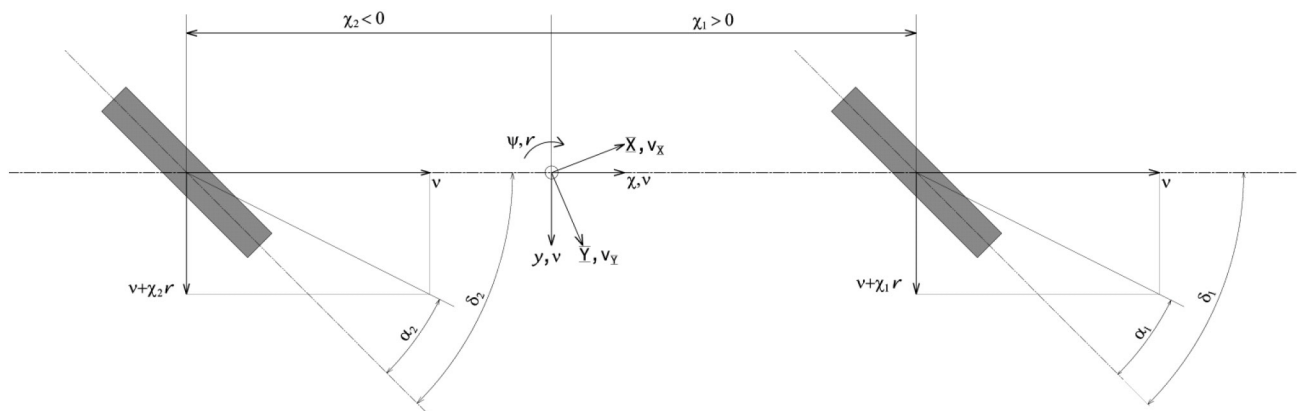


Figure 2. Yaw-Plane Model

where C_i is the positive linear cornering coefficient of the i th tire, δ_i is its angular displacement input, x_i is the longitudinal location of the tire forward of the vehicle center of gravity, v is the lateral velocity, r the yaw rate, and u the constant longitudinal velocity. Using this generalized tire side force model, it is important to keep in mind that an axle located behind the vehicle center of gravity is located by a negatively signed distance x_i , as seen in [Figure 2](#). The steer angle δ_i can be a variable control input, or in the case of a non-steered or rigid axle it becomes zero.

Given the assumptions of the yaw-plane model, all tires on a given axle experience approximately the same slip angles. When using the tire model of (1) all tires on the same axle can therefore be combined. Side force coefficients of the right and left tires can be added to form a side force coefficient for the axle, and then the side force produced by that axle is calculated from the linear tire model of (1). A vehicle model can be constructed by summing the forces and moments produced by the combined tires on each axle.

$$m(\dot{v} + ur) = F_1 + F_2 \quad (2)$$

$$I\dot{r} = x_1 F_1 + x_2 F_2 \quad (3)$$

where m is the vehicle mass and I its moment of yaw inertia.

Together, (2) and (3) describe the dynamic behavior of a vehicle with a constant longitudinal velocity u . A vehicle sideslip angle β can be formed by scaling the time-varying lateral velocity v by the constant longitudinal velocity u [13], and β can be physically interpreted as the "angle of attack" of the vehicle with respect to a body-fixed coordinate frame.

Relative to the inertial frame, the body fixed coordinate system is rotated an angle ψ which is the integral of the yaw rate as shown in [Figure 2](#). Using ψ the body fixed vehicle dynamic velocities can be transformed to velocities relative to the inertial frame where lane position is referenced.

Using the definition of the vehicle side-slip angle β , and if β and ψ are both small, a condition consistent with lane-keeping, velocities in the inertial frame can be written as linear functions of the state variables.

$$V_x = u \quad (4)$$

$$V_y = u(\psi + \beta) \quad (5)$$

In the lane maintenance mode the longitudinal velocity is the same constant value in both the body fixed and inertially fixed reference frames as seen in (4). Lateral velocity in the inertially fixed frame is a function of the constant forward velocity and state variables, and is the derivative of lateral displacement Y with respect to time.

$$\dot{Y} = u\beta + u\psi \quad (6)$$

Another dynamic state is apparent by noting that the time derivative of yaw angle is simply yaw rate.

$$\dot{\psi} = r \quad (7)$$

Equations (2), (3), (6) and (7) can be written in state-space form, recalling that C_1 and C_2 are the combined side force coefficients for the front and rear axles respectively.

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \\ \dot{\psi} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} \frac{-C_1 - C_2}{mu} & \begin{pmatrix} -x_1 C_1 - x_2 C_2 \\ mu^2 \end{pmatrix} & 0 & 0 \\ -x_1 C_1 - x_2 C_2 & \frac{-x_1^2 C_1 - x_2^2 C_2}{lu} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ u & 0 & u & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ \psi \\ Y \end{bmatrix} + \begin{bmatrix} \frac{C_1}{mu} & \frac{C_2}{mu} \\ x_1 C_1 & x_2 C_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \quad (8)$$

Some nomenclature is introduced for compactness, and allows the findings of this work to be generalized to vehicles of any arbitrary axle configuration [7, 8].

$$C_a = C_1 + C_2 \quad (9)$$

$$C_b = x_1 C_1 + x_2 C_2 \quad (10)$$

$$C_c = x_1^2 C_1 + x_2^2 C_2 \quad (11)$$

A single-input system is formed if it is assumed that the rear steering input δ_2 is proportional to the front steering input δ_1 .

$$\delta_2 = \eta \delta_1 \quad (12)$$

When $\eta = 1$ the front and rear axles are steered with the same steering angle in crab-steer, and when $\eta = -1$ they oppose each other in counter-steer. The dynamic response of the augmented two-axle system can be expressed using methods generally described in [14].

$$\begin{bmatrix} \beta(s) \\ \delta_1(s) \\ r(s) \\ \delta_1(s) \\ \psi(s) \\ \delta_1(s) \\ Y(s) \\ \delta_1(s) \end{bmatrix} = \frac{\begin{bmatrix} \frac{s^2(lu + C_c)}{lu} & \frac{-s^2(C_b + mu^2)}{mu^2} & 0 & 0 \\ \frac{-s^2 C_b}{s^2(mu + C_a)} & \frac{mu}{s(mu + C_a)} & 0 & 0 \\ \frac{-s C_b}{I} & \frac{s(mu + C_a)}{mu} & \frac{\Delta}{s} & 0 \\ \frac{I}{(s^2 lu + s C_c - u C_b)} & \frac{mu}{(-s C_b + u C_a)} & \frac{u \Delta}{s} & \Delta \\ \frac{I}{mu} & \frac{mu}{mu} & \frac{s^2}{s} & \frac{s}{s} \end{bmatrix} \begin{bmatrix} \frac{C_1 + C_2}{mu} \eta \\ \frac{x_1 C_1 + x_2 C_2}{I} \eta \\ 0 \\ 0 \end{bmatrix}}{\frac{s^2(s^2 mu^2 I + su(mu C_c + I C_a) + C_a C_c - C_b^2 - C_b mu^2)}{mu^2 I}} \quad (13)$$

Thus the transfer functions from the single control input to the various state variables can be written when the rear axle is steered using the simple proportional relationship of (12).

$$\frac{\beta(s)}{\delta_1(s)} = \frac{(s^2 lu + C_c)(C_1 + C_2 \eta) - (C_b + mu^2)(x_1 C_1 + x_2 C_2 \eta)}{s^2 mu^2 I + su(mu C_c + I C_a) + C_a C_c - C_b^2 - C_b mu^2} \quad (14)$$

$$\frac{r(s)}{\delta_1(s)} = \frac{u(smu + C_a)(x_1 C_1 + x_2 C_2 \eta) - u C_b(C_1 + C_2 \eta)}{s^2 mu^2 I + su(mu C_c + I C_a) + C_a C_c - C_b^2 - C_b mu^2} \quad (15)$$

$$\frac{\psi(s)}{\delta_1(s)} = \frac{u(smu + C_a)(x_1 C_1 + x_2 C_2 \eta) - u C_b(C_1 + C_2 \eta)}{s(s^2 mu^2 I + su(mu C_c + I C_a) + C_a C_c - C_b^2 - C_b mu^2)} \quad (16)$$

$$\frac{Y(s)}{\delta_1(s)} = \frac{u(u C_a - s C_b)(x_1 C_1 + x_2 C_2 \eta) + u(s^2 lu + s C_c - u C_b)(C_1 + C_2 \eta)}{s^2(s^2 mu^2 I + su(mu C_c + I C_a) + C_a C_c - C_b^2 - C_b mu^2)} \quad (17)$$

When there is no rear steering and $\eta = 0$, (14), (15), (16), (17) describe the behavior of a conventionally steered vehicle.

The lateral position Y_p of any point located on the projected velocity vector of the vehicle a distance x_p from the center of mass can be expressed in terms of the state variables. This can be interpreted as the vehicle's lateral position error that would occur at some time in future if the vehicle continues along its current velocity vector.

$$Y_p = Y + (\beta + \psi)x_p \quad (18)$$

Alternatively the lateral position Y_p could be projected along the longitudinal axis of the vehicle, creating slightly different subsequent derived expressions. However anecdotal evidence seems to suggest projection along the velocity vector is a more appropriate assumption when constructing the "time advanced" present trajectory of the vehicle that will ultimately be used to form a lane position error by the driver. The lateral position Y_p can be expressed relative to the steering input δ_1 using (14), (16) and (17).

$$\frac{Y_p(s)}{\delta_1(s)} = \left(\frac{sx_p + u}{s^2} \right) \left(\frac{C_1(s^2 lu + s(C_c - x_1 C_b) + u(x_1 C_a - C_b)) + \eta C_2(s^2 lu + s(C_c - x_2 C_b) + u(x_2 C_a - C_b))}{s^2 mu^2 I + su(mu C_c + I C_a) + C_a C_c - C_b^2 - C_b mu^2} \right) \quad (19)$$