

Stress Intensity Factor Solutions for Dissimilar Welds in Lap-Shear Specimens of Steel, Magnesium, Aluminum and Copper Sheets

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ABSTRACT

In this paper, the analytical stress intensity factor and J integral solutions for welds in lap-shear specimens of two dissimilar sheets based on the beam bending theory are first reviewed. The solutions are then presented in the normalized forms. Next, two-dimensional finite element analyses were selectively conducted to validate the analytical solutions based on the beam bending theory. The interface crack parameters, the stress intensity factor solutions, and the J integral solutions for welds in lap-shear specimens of different combinations of steel, aluminum, and magnesium, and the combination of aluminum and copper sheets of different thickness ratios are then presented for convenient fracture and fatigue analyses. The transition thickness ratios for critical crack locations for different combinations of dissimilar materials are then determined from the analytical solutions. The transition weld widths for applicable ranges of the weld widths for the analytical solutions based on the beam bending theory are also presented. Finally, fracture and fatigue behaviors of dissimilar magnesium/steel ultrasonic welds, dissimilar aluminum/copper laser welds, and similar steel laser welds in lap-shear specimens are examined and demonstrate the usefulness of the stress intensity factor solutions presented in this paper.

CITATION: Lai, W. and Pan, J., "Stress Intensity Factor Solutions for Dissimilar Welds in Lap-Shear Specimens of Steel, Magnesium, Aluminum and Copper Sheets," *SAE Int. J. Mater. Manf.* 8(2):2015, doi:10.4271/2015-01-1754.

INTRODUCTION

The automotive industry has increased the use of lightweight materials such as aluminum and magnesium alloys to reduce the vehicle weight. Aluminum and magnesium alloys are lighter than the steels commonly used in vehicles. Using aluminum and magnesium alloys extensively in a vehicle could result in a significant reduction of vehicle weight. Incorporating lightweight materials into the steel vehicle structure requires joining of these dissimilar materials. Therefore, fracture and fatigue analyses of dissimilar welds between steel, aluminum and magnesium sheets are of interest. For lithium-ion battery packs in electric and hybrid vehicles, the electrodes of these batteries are made of aluminum and copper. Joining the aluminum and copper tabs in battery packs is also needed. Therefore, fracture and fatigue analyses of dissimilar welds between aluminum and copper sheets are also of interest.

In many cases, analyses of continuous dissimilar welds can be treated as two-dimensional plane strain problems. Franklin [1] machined the lap-shear specimens of ultrasonic spot welds of dissimilar magnesium and steel sheets into a dog-bone shaped profile. Figures 1(a) and 1(b) show the lap-shear specimens with ultrasonic spot welds of magnesium and steel sheets before and after being machined into a dog-bone shape, respectively. Figure 2 shows an optical micrograph of the cross section near the ultrasonic weld in the lap-shear specimen of magnesium and steel sheets. Two-dimensional plane strain analyses can be used to model the fatigue and fracture behaviors of the dissimilar welds in this kind of dog-bone lap-shear specimens [1]. Two dimensional plane strain analyses were also conducted to analyze similar laser welds in dog-bone

shaped lap-shear specimens of high strength low alloy steel in Sripichai et al. [2]. The structural stress, stress intensity factor and J integral solutions for similar and dissimilar welds in lap-shear specimens are quite important for fracture and fatigue analyses since lap-shear specimens are commonly used to examine the fracture and fatigue of the welds. Based on the work of Zhang [3], Tran and Pan [4] recently presented analytical stress intensity factor and J integral solutions for dissimilar spot welds in lap-shear specimens of different materials and thicknesses in the normalized forms. The solutions are presented in the graphical form and they are quite conveniently used as references for fracture and fatigue analyses of spot welds in lap-shear specimens.

In this paper, the analytical stress intensity factor solutions for welds in lap-shear specimens of two dissimilar sheets based on the beam bending theory are first reviewed. The analytical solutions are then presented in the normalized forms. Next, two-dimensional finite element analyses were selectively conducted to validate the analytical solutions based on the beam bending theory. The interface crack parameters, the stress intensity factor solutions, and the J integral solutions for welds in lap-shear specimens of different combinations of steel, aluminum, and magnesium, and the combination of aluminum and copper sheets of different thickness ratios are then presented for convenient fracture and fatigue analyses. The transition thickness ratios for critical crack locations for different combinations of dissimilar materials are then determined from the analytical solutions. The transition weld widths for applicable ranges of the weld widths for the analytical solutions based on the beam bending theory are also presented. Finally, fracture and fatigue behaviors of

dissimilar magnesium/steel ultrasonic welds, dissimilar aluminum/copper laser welds, and similar steel laser welds in lap-shear specimens are examined and demonstrate the usefulness of the stress intensity factor solutions presented in this paper.

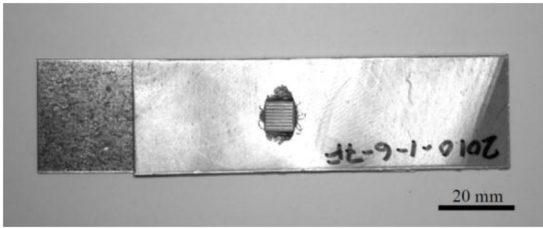


Figure 1. An ultrasonic spot welded lap-shear specimen of magnesium and steel sheets (a) before being machined and (b) after being machined into a dog-bone shape.



Figure 2. An optical micrograph of the cross section of an ultrasonic welded lap-shear specimen of magnesium and steel sheets.

ANALYTICAL STRESS INTENSITY FACTOR SOLUTIONS BASED ON THE BEAM BENDING THEORY

Figure 3 shows a schematic of a lap-shear specimen. The applied force per unit width, F/b , is shown as the bold arrows. Here, b represents the specimen width. For the lap-shear specimen considered here, the weld width w is much larger than the upper and lower sheet thicknesses so that the beam bending theory is applicable. The upper and lower sheets are assumed to be isotropic linear elastic materials with the Young's modulus E_u , shear modulus G_u , and Poisson's ratio ν_u for the upper sheet material, and the Young's modulus E_l , shear modulus G_l , and Poisson's ratio ν_l for the lower sheet material. The upper and lower sheets have the nominal thicknesses t_u and t_l , respectively. The thickness and modulus ratios δ and η are defined, respectively, as

$$\delta = t_u/t_l \quad (1)$$

and

$$\eta = E'_u/E'_l \quad (2)$$

where

$$E'_u = E_u/(1-\nu_u^2) \quad (3)$$

and

$$E'_l = E_l/(1-\nu_l^2) \quad (4)$$

Figure 4 shows a two-dimensional model of two infinite strips made of dissimilar materials with different thicknesses and connection under plane strain conditions. The strip model can be used to represent the left or right portion of the specimen provided that the weld width w is large compared to the thicknesses of the upper and lower sheets. The normal structural stresses σ_x^u and σ_x^l for the strip model with respect to the Cartesian coordinate system are schematically shown in Figure 4.

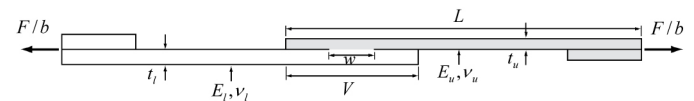


Figure 3. A schematic of a lap-shear specimen. The applied force per unit width, F/b , is shown as the bold arrows.

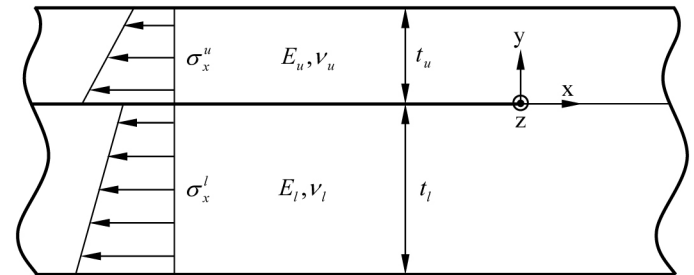


Figure 4. A two-dimensional model of two infinite strips made of dissimilar materials with different thicknesses and connection under plane strain conditions.

Figure 5 shows a schematic of the strip model near the crack tip. The normal stresses σ_{ui} , σ_{uo} , σ_{li} , σ_{lo} , σ_{ui}^* , σ_{uo}^* , σ_{li}^* and σ_{lo}^* represent the normal structural stresses at the inner (*i*) and outer (*o*) surfaces of the upper (*u*) and lower (*l*) strips, respectively. Note also that the normal structural stresses σ_{ui}^* , σ_{uo}^* , σ_{li}^* and σ_{lo}^* can be derived from the normal structural stresses σ_{ui} , σ_{uo} , σ_{li} and σ_{lo} based on the equilibrium equations, the linear strain distribution, and the strain continuity condition along the bond line as presented in Zhang [3] and Suo and Hutchinson [5].

The asymptotic in-plane stress field around an interface crack tip is an oscillatory field that can be characterized by a complex stress intensity factor $\mathbf{K}$ ($= K_1 + iK_2$, $i = \sqrt{-1}$) (Rice and Sih [6]). The stresses σ_y and τ_{xy} at a small distance r ahead of the interface crack tip are characterized by $\mathbf{K}$ as

$$\sigma_y + i\tau_{xy} = \frac{K_1 + iK_2}{\sqrt{2\pi r}} \left(\frac{r}{t} \right)^{i\varepsilon} \quad (5)$$

where the bimaterial constant ε is defined as

$$\varepsilon = \frac{1}{2\pi} \ln \frac{\kappa_u / G_u + 1 / G_l}{\kappa_l / G_l + 1 / G_u} \quad (6)$$

Here, κ_u and κ_l are defined as

$$\kappa_u = 3 - 4\nu_u \quad (7)$$

and

$$\kappa_l = 3 - 4\nu_l \quad (8)$$

In Equation (5), t represents a characteristic length [3, 5, 7]. In this investigation, t is taken as the smaller value of the upper sheet thickness t_u as in Suo and Hutchinson [5] and Zhang [3]. It should be noted that when the two materials are identical, $\varepsilon = 0$. In this case, K_1 and K_2 in Equation (5) for the interface crack become the conventional stress intensity factors K_I and K_{II} , respectively.

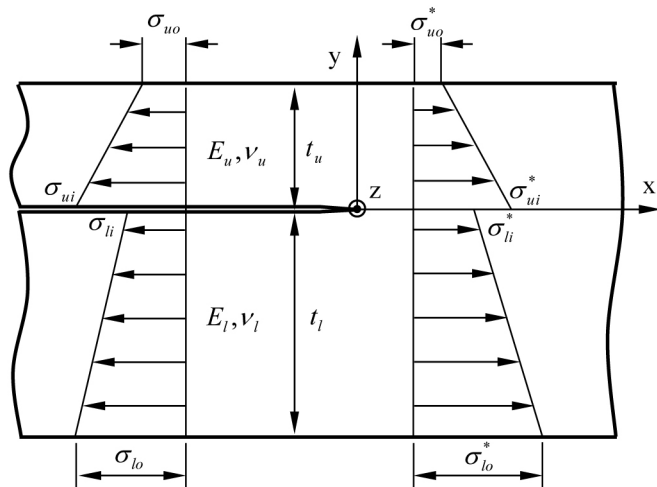


Figure 5. A schematic showing the strip model near the crack tip. The normal stresses σ_{ui} , σ_{uo} , σ_{li} , σ_{lo} , σ_{ui}^* , σ_{uo}^* , σ_{li}^* and σ_{lo}^* represent the normal structural stresses at the inner (i) and outer (o) surfaces of the upper (u) and lower (l) strips, respectively.

Based on the J integral solution for the strip model and the analytical solutions for interface cracks in Suo and Hutchinson [5], Zhang [3] derived the stress intensity factor solutions K_1 and K_2 (as defined in Equation (5)) for welds joining sheets of different materials and thicknesses in terms of the structural stresses σ_{ui} , σ_{uo} , σ_{li} and σ_{lo} as

$$K_1 = \frac{\cosh(\pi\varepsilon)\sqrt{t_u}}{2\sqrt{3(1+\eta)(1+4\eta\delta+6\eta\delta^2+3\eta\delta^3)(1+\tan^2\omega)}} \times \left\{ \left[\frac{(1+4\eta\delta+9\eta\delta^2+6\eta\delta^3)\tan\omega}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} - \sqrt{3} \right] \sigma_{ui} - \left[\frac{(1+4\eta\delta+3\eta\delta^2)\tan\omega}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} + \sqrt{3} \right] \sigma_{uo} + \eta \left[\frac{\delta(1-2\eta\delta-3\eta\delta^2)\tan\omega}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} + \sqrt{3}(2+\delta) \right] \sigma_{li} - \eta\delta \left[\frac{(1+4\eta\delta+3\eta\delta^2)\tan\omega}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} + \sqrt{3} \right] \sigma_{lo} \right\} \quad (9)$$

and

$$K_2 = \frac{\cosh(\pi\varepsilon)\sqrt{t_u}}{2\sqrt{3(1+\eta)(1+4\eta\delta+6\eta\delta^2+3\eta\delta^3)(1+\tan^2\omega)}} \times \left\{ \left[\frac{1+4\eta\delta+9\eta\delta^2+6\eta\delta^3}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} + \sqrt{3}\tan\omega \right] \sigma_{ui} - \left[\frac{1+4\eta\delta+3\eta\delta^2}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} - \sqrt{3}\tan\omega \right] \sigma_{uo} + \eta \left[\frac{\delta(1-2\eta\delta-3\eta\delta^2)}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} - \sqrt{3}(2+\delta)\tan\omega \right] \sigma_{li} - \eta\delta \left[\frac{1+4\eta\delta+3\eta\delta^2}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} - \sqrt{3}\tan\omega \right] \sigma_{lo} \right\} \quad (10)$$

Note that the expressions shown in Equations (9) and (10) are for welds joining sheets with $\delta \leq 1$. For welds joining sheets with $\delta > 1$, one should rotate the strip model by an angle of 180° to represent the same physical system but with $\delta < 1$. The values of the angular quantity ω in Equations (9) and (10) can be found in Suo and Hutchinson [5] and Zhang [3].

The normal structural stresses for the left and right cracks in a lap-shear specimen under plane strain conditions can be obtained using a simple beam bending theory. For the left crack, the normal structural stresses are

$$\sigma_{ui} = 0 \quad (11)$$

$$\sigma_{uo} = 0 \quad (12)$$

$$\sigma_{li} = 4 \frac{F}{bt_l} \quad (13)$$

$$\sigma_{lo} = -2 \frac{F}{bt_l} \quad (14)$$

For the right crack, the normal structural stresses are

$$\sigma_{ui} = 4 \frac{F}{bt_u} \quad (15)$$

$$\sigma_{uo} = -2 \frac{F}{bt_u} \quad (16)$$

$$\sigma_{li} = 0 \quad (17)$$

$$\sigma_{lo} = 0 \quad (18)$$

The normal structural stresses in Equations (11), (12), (13), (14), (15), (16), (17), (18) can then be substituted into Equations (9) and (10) to calculate K_I and K_2 for both cracks in a lap-shear specimen of different materials and thicknesses. It should be noted for the right crack, the signs for the stress intensity factor K_2 solutions are changed to be consistent with those of the left crack.

Under plane strain conditions, for an interface crack between two dissimilar linear elastic materials, the J integral is related to the stress intensity factors K_I , K_2 and K_3 as [6]

$$J = \frac{K_I^2 + K_2^2}{\cosh^2(\pi\epsilon)E^*} + \frac{K_3^2}{2G^*} \quad (19)$$

where E^* and G^* are

$$\frac{1}{E^*} = \frac{1}{2} \left(\frac{1}{E'_u} + \frac{1}{E'_l} \right) \quad (20)$$

and

$$\frac{1}{G^*} = \frac{1}{2} \left(\frac{1}{G_u} + \frac{1}{G_l} \right) \quad (21)$$

Here, K_3 is the mode 3 stress intensity factor. Since $K_3 = 0$ at the left and right cracks in a lap-shear specimen under plane strain conditions, Equation (19) can be reduced to

$$J = \frac{K_I^2 + K_2^2}{\cosh^2(\pi\epsilon)E^*} \quad (22)$$

For welds joining sheets of identical materials ($\epsilon = 0$, $\eta = 1$), Equation (22) becomes

$$J = \frac{K_I^2 + K_{II}^2}{E'} \quad (23)$$

and Equations (9) and (10) become

$$K_I = \frac{\sqrt{t_u}}{2\sqrt{2(1+\delta)(1+3\delta+3\delta^2)(1+\tan^2\omega)}} \times \left\{ \left[\frac{1+3\delta+6\delta^2}{\sqrt{3}(1+\delta)} \tan\omega - 1 \right] \sigma_{ui} - \left[\frac{1+3\delta}{\sqrt{3}(1+\delta)} \tan\omega + 1 \right] \sigma_{uo} + \left[\frac{\delta(1-3\delta)}{\sqrt{3}(1+\delta)} \tan\omega + 2 + \delta \right] \sigma_{li} - \delta \left[\frac{1+3\delta}{\sqrt{3}(1+\delta)} \tan\omega + 1 \right] \sigma_{lo} \right\} \quad (24)$$

$$K_{II} = \frac{\sqrt{t_u}}{2\sqrt{2(1+\delta)(1+3\delta+3\delta^2)(1+\tan^2\omega)}} \times \left\{ \left[\frac{1+3\delta+6\delta^2}{\sqrt{3}(1+\delta)} + \tan\omega \right] \sigma_{ui} - \left[\frac{1+3\delta}{\sqrt{3}(1+\delta)} - \tan\omega \right] \sigma_{uo} + \left[\frac{\delta(1-3\delta)}{\sqrt{3}(1+\delta)} - (2+\delta)\tan\omega \right] \sigma_{li} - \delta \left[\frac{1+3\delta}{\sqrt{3}(1+\delta)} - \tan\omega \right] \sigma_{lo} \right\} \quad (25)$$

NORMALIZED ANALYTICAL SOLUTIONS BASED ON THE BEAM BENDING THEORY

The stress intensity factor solutions for dissimilar welds based on the beam bending theory can be written in the normalized forms as

$$K_I = k_1(\delta, \eta) \frac{F}{b\sqrt{t_u}} \quad (26)$$

and

$$K_2 = k_2(\delta, \eta) \frac{F}{b\sqrt{t_u}} \quad (27)$$

where k_1 and k_2 are the dimensionless geometric functions for a given set of δ and η . For welds joining sheets of identical materials ($\epsilon = 0$ and $\eta = 1$), Equations (26) and (27) become

$$K_I = k_I(\delta) \frac{F}{b\sqrt{t_u}} \quad (28)$$

and

$$K_{II} = k_{II}(\delta) \frac{F}{b\sqrt{t_u}} \quad (29)$$

where k_I and k_{II} are the dimensionless geometric functions for a given δ .

The in-plane effective stress intensity factor $K_e (= \sqrt{K_I^2 + K_{II}^2})$ and J integral solutions can be written in the normalized forms as

$$K_e = k_e(\delta, \eta) \frac{F}{b\sqrt{t_u}} \quad (30)$$

and

$$J = j(\delta, \eta) \frac{1}{E'_u} \left(\frac{F}{b\sqrt{t_u}} \right)^2 \quad (31)$$

where k_e and j are the dimensionless geometric functions for a given set of δ and η . k_e and j can be expressed by k_1 and k_2 as

$$k_e = \sqrt{k_1^2 + k_2^2} \quad (32)$$

and

$$j = \frac{(1 + \eta)(k_1^2 + k_2^2)}{2 \cosh^2(\pi \varepsilon)} \quad (33)$$

For welds joining sheets of identical materials ($\varepsilon = 0$ and $\eta = 1$), Equations (30) and (31) become

$$K_e = k_e(\delta) \frac{F}{b\sqrt{t_u}} \quad (34)$$

and

$$J = j(\delta) \frac{1}{E'} \left(\frac{F}{b\sqrt{t_u}} \right)^2 \quad (35)$$

where

$$k_e = \sqrt{k_I^2 + k_{II}^2} \quad (36)$$

and

$$j = k_I^2 + k_{II}^2 \quad (37)$$

COMPUTATIONAL MODEL

In order to validate the analytical stress intensity factor solutions, two-dimensional finite element analyses for welds in lap-shear specimens of selected different material combinations and thickness ratios were conducted under plane strain conditions. In the finite element analyses, the materials are assumed to be linear elastic isotropic materials. The commercial finite element program ABAQUS [8] is employed to perform the computations. Second-order quarter point crack-tip elements with collapsed nodes are used to model the $1/\sqrt{r}$ singularity near the crack tip. Here, r represents the radial distance to the crack tip. The stress intensity factor and J integral solutions are directly computed by ABAQUS. The computational stress intensity factor solutions are obtained based on the interaction integral method for cracks under mixed-mode loading conditions (Shih and Asaro [9]).

It should be noted that K_1^A and K_2^A of the complex stress intensity factor $\mathbf{K}^A (= K_1^A + iK_2^A)$ obtained directly from ABAQUS are defined such that the stresses σ_y and τ_{xy} at a small distance r ahead of an interface crack tip are characterized by $\mathbf{K}^A$ as

$$\sigma_y + i\tau_{xy} = \frac{K_1^A + iK_2^A}{\sqrt{2\pi r}} r^{i\varepsilon} \quad (38)$$

The K_1 and K_2 solutions as defined in Equation (5) are related to the K_1^A and K_2^A solutions as defined in Equation (38) as

$$K_1 = K_1^A \cos(\varepsilon \ln t) - K_2^A \sin(\varepsilon \ln t) \quad (39)$$

and

$$K_2 = K_1^A \sin(\varepsilon \ln t) + K_2^A \cos(\varepsilon \ln t) \quad (40)$$

In this investigation, the computational K_1 and K_2 solutions for welds joining two sheets of dissimilar materials are obtained from Equations (39) and (40) with the K_1^A and K_2^A solutions computed by ABAQUS.

The schematic of the finite element model is shown in Figure 3. For the finite element analyses, the length L ($= 100$ mm), the overlap length V ($= 40$ mm), the specimen width b ($= 30$ mm) and the weld width w ($= 9$ mm) are based on the lap-shear specimens used in Franklin [1]. The upper sheet thickness t_u is 3 mm for different thickness ratios. In this investigation, the thickness ratios, 0.1, 0.2,