

Development of a Coupled Air and Particle Thermal Model for Engine Icing Test Facilities

Tadas P. Bartkus
Ohio Aerospace Institute

Peter Struk
NASA John Glenn Research Center

Jen-Ching Tsao
Ohio Aerospace Institute

ABSTRACT

This paper describes a numerical model that simulates the thermal interaction between ice particles, water droplets, and the flowing air applicable during icing wind tunnel tests where there is significant phase-change of the cloud. It has been previously observed that test conditions, most notably temperature and humidity, change when the icing cloud is activated. It is hypothesized that the ice particles and water droplets thermally interact with the flowing air causing the air temperature and humidity to change by the time it reaches the test section. Unlike previous models where the air and particles are uncoupled, this model attempts to explain the observed changes in test conditions by coupling the conservation of mass and energy equations. The model is compared to measurements taken during wind tunnel tests simulating ice-crystal and mixed-phase icing that relate to ice accretions within turbofan engines. The model simulates trends that were experimentally observed, but does not fully explain the measured values. Some possible explanations for this discrepancy are offered. This model, written in MATLAB, is based on fundamental conservation laws and empirical correlations.

CITATION: Bartkus, T., Struk, P., and Tsao, J., "Development of a Coupled Air and Particle Thermal Model for Engine Icing Test Facilities," *SAE Int. J. Aerosp.* 8(1):2015, doi:10.4271/2015-01-2155.

INTRODUCTION

Several jet engine power-loss events of commercial aircraft at high altitude have been reported since the 1990's. Mason et al. [1] discussed how power-loss can result from ice crystals entering the engine core, partially melting from the warm engine airflow and refreezing on internal components. According to Mason et al. [2], the accumulation of ice in the engine, from these ice-crystals, can cause engine surge, stall, flameout, and even damage to the blades and vanes from ice shedding.

Due to these numerous power-loss events, potential ice accretion within the engine due to the ingestion of ice crystals is being investigated. To better understand this phenomenon and determine the physical mechanism of engine ice accretion, fundamental tests have been collaboratively conducted by NASA Glenn Research Center and the National Research Council of Canada (NRC) [3, 4]. NRC's Research Altitude Test Facility (RATFac) is capable of generating ice-crystal and mixed-phase cloud conditions. While conducting icing tests, the total air temperature measured at the test section was observed to decrease when the icing cloud was activated. In some cases, the temperature decrease was considerable, by as much as several degrees Celsius, and was accompanied by a measurable increase in water vapor [5, 6]. It is hypothesized that the

ice particles and water droplets thermally interacted with the flowing air causing the air temperature and humidity to change by the time it reached the test section. Since a difference of a few degrees can mark the difference between an icing event and a non-icing event, it is important to understand the cause of this temperature change so that accurate test conditions can be specified. To that end, the focus of this paper is to model the observed changes in air temperature and water vapor content due to the air-particle enthalpy and mass transfer exchange. In this paper, the term 'particle' is generically used to refer to liquid water, ice or mixed phase water.

Existing thermal models [7, 8, 9, 10, 11, 12] show little effect due to coupling of the ice/water particles with the flowing air, or do not couple, approximating the air mass as an infinite thermal reservoir with unchanging properties. The model presented in this paper attempts to explain the observed changes in test conditions by coupling the conservation of mass and energy equations for both the ice/water particle and flowing air mass. The model simulates both ice particle melting and droplet freezing, including spontaneous latent heat release during freezing for supercooled droplets. In the simulation, humidity changes are due to ice particle sublimation/deposition and water droplet evaporation/condensation, and the associated latent heating/cooling and mass transfer change the ice

water content (*IWC*) and liquid water content (*LWC*) from the moment of injection to the time the flowing mass reaches the test section. The conservation of momentum is employed to account for particle velocity. In addition, to more accurately simulate spray injection, the model incorporates a distribution of particle sizes as an initial input condition. As a result, the model predicts changes in air temperature, humidity, *IWC* and *LWC*, as well as the particle phase, size, temperature and velocity at the test section.

THERMAL MODEL DESCRIPTION

The thermal model simulates an icing wind tunnel by calculating particle and air properties using the conservation of mass, momentum and energy equations. The ice, water, and humid air mass in an icing tunnel is broken into fundamental control volume units. For a cloud containing uniformly sized particles, the fundamental unit is composed of a single ice or water particle and a parcel of humid air. For an icing cloud containing a spectrum of particle sizes, the fundamental unit contains the particle distribution and corresponding air parcel. The particle(s) and the parcel of air are each treated as separate control volumes where energy and mass transfer between each other. The energy and mass in each fundamental control volume are maintained as constant. Details of the control volumes are described later. The model simulates particles on the icing tunnel centerline, providing thermodynamic conditions at the test section.

Assumptions

The thermal model operates under the following assumptions:

1. Air and particle flow are one dimensional where the geometry of the icing tunnel is constant in the direction of flow.
2. Air and water vapor are treated as ideal gases.
3. Air is well mixed, meaning that temperature, water vapor content and all thermodynamic properties are homogeneous within the air control volume at any time (or axial location).
4. The fundamental control volume is adiabatic and mass is maintained as constant.
5. Supersaturated water vapor content in air does not occur.
6. Particles are evenly spaced within the air from the injection point to the test section.
7. All particles are perfectly spherical.
8. Particle aggregation / coalescences and breakup through collision are negligible.
9. Particle size distribution is characterized as discrete diameters.
10. Particles are injected in the direction of the flow and remain entrained with negligible gravity affects.
11. Temperature is uniform within the particle.
12. Water droplets can be supercooled to a homogenous crystallization temperature that is a function of particle diameter.
13. The latent heat release stage during supercooled freezing occurs instantaneously.
14. Mixed phase particles are spatially homogeneous in water/ice content. This means that evaporation and sublimation occurs from a mixed phase particle surface and the rate is determined by the water/ice content ratio. This applies to the reverse process of condensation and deposition as well.
15. Evaporation, sublimation, condensation and deposition occur at the particle surface at particle temperature.
16. Air velocity and pressure do not change due to heat or mass transfer.
17. Heat transfer at the tunnel walls is negligible.
18. The flow of particles and air is a continuous stream. This means that while following a particular set of particles within a reference control volume, faster moving particles and air that exit the reference control volume are replaced by thermodynamically identical particles and air from a neighboring control volume that is upstream of the reference control volume. The reference control volume is centered on the slowest moving particle in distribution of particles.

To validate Assumption 11, that temperature is uniform within the particle, the Biot number, Bi , is calculated. The Biot number is a ratio of convective heat transfer rate across the particle boundary to the rate of conductive heat transfer within the particle. A Bi value that is below a critical value of 0.1 verifies the use of the lumped capacitance method where particle temperature is uniform within the particle.

$$Bi = \frac{hL_c}{k_p} = \frac{hd}{6k_p} < 0.1 \quad \text{for spheres}$$

(1)

In the numerator, h is the convective heat transfer coefficient, L_c is the critical length which equals $1/6^{\text{th}}$ of the particle diameter, d , for a sphere. The particle's thermal conductivity is represented by k_p . The largest Bi values will occur with large particles ($d = \sim 200$ microns) and high slip velocities ($U = \sim 30$ m/s). Using these extreme values along with other appropriate properties, $Bi = \sim 0.015$, much less than the critical value needed to validate use of lumped capacitance.

Thermal Model Algorithm

The thermal model was written in MATLAB version R2014a. The model solves the differential equations of mass, momentum and energy transfer. All conservation equations presented in the following model formulation section are solved using MATLAB's built-in ODE45 solver. The system of differential equations is solved by first inputting the initial conditions. Time marching methods are employed by the MATLAB solver such that each iterative solution is solved with *numerical* relative and absolute tolerances of 10^{-12} . *Physical* accuracy of the solution is dependent on the accuracy of all the property values input into the model. For all simulations presented in this paper, the energy transferred between the air and particle(s) is balanced to the order of 10^{-4} . The mass transfer between the air and particle(s) initial and final states is balanced to the order of 10^{-15} .

Thermal Model Equations: Single Particle Formulation

For an icing cloud of uniform particle size, the fundamental control volume consists of just a single particle control volume and air control volume. The mass of the air control volume, m_{air} , is determined by the icing conditions and is the following equation.

$$m_{air} = \frac{\pi d^3}{6} \frac{\rho_p \rho_{air}}{LWC} \quad (2)$$

In Eq. (2), d is the particle diameter, ρ_p is the particle density, and ρ_{air} is the air density. The denominator, LWC , can be interchanged with IWC for an ice particle cloud. LWC and IWC represent water content density with units of mass of water per volume of air. The conservation of mass, momentum and energy equations are applied to the two interacting control volumes.

Conservation of Energy

The energy balance for a particle (ice particle, water droplet or mixed phase) thermally interacting with a parcel of air is the balance between the internal energy of the particle and the heat flux across the particle boundary. More specifically, the change in the particle's enthalpy, H_p , with respect to time, t , is due to the convective heat transfer, q_{conv} , and latent energy due to mass transfer, q_{latent} , and is given by the following general expression.

$$\frac{\partial H_p}{\partial t} = q_{conv} + q_{latent} \quad (3)$$

Dalton [13] is credited for starting the empirical hydrodynamic approach to the evaporation problem. He stated that the rate of mass loss from a water surface is proportional to the difference in water vapor pressure between the particle surface and the ambient air. He also stated that this rate is proportional to the wind speed. Applying Dalton's formulation to the latent energy term, as well as expanding on the internal energy and convective energy terms of Eq. (3), gives the following expression for an ice particle undergoing sublimation.

$$\rho_p C_p \frac{\pi d^3}{6} \frac{\partial T_p}{\partial t} = \pi d^2 h (T_{air} - T_p) + \pi d^2 h_m \rho_{air} L_{subl} (\omega_{air} - \omega_p) \quad (4)$$

In Eq. (4), C_p is the specific heat capacity of the particle, T_p is the particle temperature, T_{air} is the ambient air temperature, h is the heat transfer coefficient, h_m is the mass transfer coefficient, L_{subl} is the latent heat of sublimation, ω_{air} is the water vapor mass fraction in the ambient air, and ω_p is the water vapor mass fraction at the particle surface. The water vapor mass fractions are defined as the following.

$$\omega_{air} = \frac{MW_{water}}{MW_{air}} \frac{P_{wv,air}}{P_{air}} \quad (5)$$

$$\omega_{surf} = \frac{MW_{water}}{MW_{air}} \frac{P_{wv,surf}}{P_{air}} \quad (6)$$

In Eqns. (5) and (6), MW is molecular weight, P_{air} is air pressure, $P_{wv,air}$ is the ambient vapor pressure, and $P_{wv,surf}$ is vapor pressure at the particle surface (saturated). Eq. (4) applies for a fully frozen ice particle prior to any melting. For an ice particle undergoing melting, the ice/water temperature remains constant and the rate of the phase change is given by the following expression.

$$\rho_p L_{melt} \frac{\pi d^3}{6} \frac{\partial \eta_p}{\partial t} = \pi d^2 h (T_{air} - T_p) + \pi d^2 h_m \rho_{air} L_{subl/evap} (\omega_{air} - \omega_p) \quad (7)$$

In Eq. (7), L_{melt} is the latent heat of melting, $L_{subl/evap}$ is the proportional mix of the latent heat of sublimation and evaporation, and η_p is the particle melt fraction (or melt ratio). Melt fraction is the ratio of liquid water mass to the total water/ice content (0 is solid ice and 1 is fully liquid). The melt ratio term is used for both melting and freezing cases. The value of $L_{subl/evap}$ is associated with the melt fraction and is given by the following expression.

$$L_{subl/evap} = (1 - \eta_p) L_{subl} + (\eta_p) L_{evap} \quad (8)$$

For a water droplet undergoing evaporation, the formulation is similar to Eq. (4) except the latent heat of evaporation variable, L_{evap} , replaces L_{subl} .

$$\rho_p C_p \frac{\pi d^3}{6} \frac{\partial T_p}{\partial t} = \pi d^2 h (T_{air} - T_p) + \pi d^2 h_m \rho_{air} L_{evap} (\omega_{air} - \omega_p) \quad (9)$$

Equations (4), (7) and (9) are explicitly defined for ice sublimation, melting and droplet evaporation. These particle energy equations can be used for the reverse thermal processes of ice deposition, freezing and water condensation respectively. The absolute values of the corresponding latent heats of phase change (sublimation : deposition, melt : freeze, and evaporation : condensation) are equal and the direction of the latent energy due to mass transfer is determined by the water vapor mass fraction difference.

The non-dimensional Nusselt number, Nu , and Sherwood number, Sh , are used to determine the heat and mass transfer coefficients in the three particle energy equations and are defined as follows.

$$Nu = \frac{hd}{k_{air}} \quad (10)$$

$$Sh = \frac{h_m d}{D_{ab}} \quad (11)$$

In the above equations, k_{air} is the thermal conductivity of air, and D_{ab} is the diffusivity of water vapor in air. The empirical heat transfer expressions for flow over a sphere [14] is as follows:

$$Nu = 2 + 0.6 Re^{1/2} Pr^{1/3}$$

Using the heat and mass transfer analogy, the Sherwood number is the following:

$$Sh = 2 + 0.6Re^{1/2}Sc^{1/3} \quad (13)$$

In the above equations, the Reynolds number, Re , Prandtl number, Pr , and Schmidt number, Sc , are defined as follows:

$$Re = \frac{\rho_{air}|v_{air}-v_p|d}{\mu_{air}} \quad (14)$$

$$Pr = \frac{C_{air}\mu_{air}}{k_{air}} \quad (15)$$

$$Sc = \frac{\mu_{air}}{\rho_{air}D_{ab}} \quad (16)$$

In the above equations, C_{air} is the specific heat capacity of air, μ_{air} is the dynamic viscosity and the difference between the air velocity, v_{air} , and the particle velocity, v_p , is also known as the slip velocity.

The energy that transfers across the particle control volume is balanced by an equivalent and opposite amount of air enthalpy change. The change in the air parcel's enthalpy, H_{air} , with respect to time is due to the convective heat transfer and the sensible energy change of the water vapor mass that has changed phase, $q_{wv,sens}$. Water vapor mass that has changed phase can be vapor that has been liberated from the particle by evaporation/sublimation, or it can refer to the water vapor in the moist air that has condensed/deposited onto the particle. The conservation of energy for the air mass is given by the following general expression.

$$\frac{\partial H_{air}}{\partial t} = q_{conv} + q_{wv,sens} \quad (17)$$

The sensible energy term in Eq. (17) is typically small compared to the convective heat transfer term, but is necessary to account for all energy being transferred. For example, a cool water droplet in warm air will evaporate, producing water vapor whose temperature is at the cool water droplet temperature. The vaporized mass will increase in temperature as it mixes with warm air. This thermal mixing situation is accounted for by the sensible energy term in Eq. (17). Expanding the above equation gives the following energy balance for air:

$$m_{air}C_{air}\frac{\partial T_{air}}{\partial t} = \pi d^2 h(T_p - T_{air}) + \frac{\partial m_{wv}}{\partial t} \int_{T_{air}}^{T_p} C_{wv} \partial T_{wv} \quad (18)$$

Where m_{wv} is the mass of water vapor that has changed phase, C_{wv} is the specific heat capacity of vapor, and T_{wv} is the vapor temperature.

Conservation of Mass

The change in particle mass, m_p , due to phase change (evaporation, sublimation, condensation or deposition) for a spherical particle is given by the following mass balance equation.

$$\frac{\partial m_p}{\partial t} = \rho_p \frac{\partial}{\partial t} \left(\frac{\pi d^3}{6} \right) = \pi d^2 h_m \rho_{air} (\omega_{air} - \omega_p) \quad (19)$$

To balance the overall system mass, the change in mass for the air mass is equal to but opposite the change in particle mass and is given by the following expression.

$$\frac{\partial m_{air}}{\partial t} = - \frac{\partial m_p}{\partial t} = \pi d^2 h_m \rho_{air} (\omega_p - \omega_{air}) \quad (20)$$

Situations can occur where the air is at vapor saturation, yet the driving force for evaporation (or sublimation) still exists such that $\omega_p > \omega_{air}$. For example, a warm water droplet in saturated cold air is a situation where the vapor gradient would favor evaporation. This model assumes that supersaturation of the air does not exist (Assumption 5). A limit function, ϕ , is appended to the mass transfer term such that when the air is near saturation, it will regulate the mass transfer rate asymptotically at RH = 100%. The limit function reduces mass transfer rate at a level near saturation defined by the variable RH_{limit}. For the thermal model, mass transfer regulation was arbitrarily chosen to begin at RH_{limit} = 99%.

$$\phi = \frac{100 - RH}{100 - RH_{limit}} \quad (21)$$

The right hand term of the conservation of mass equations (Eq. (19) and Eq. (20)) is multiplied by this limit function when relative humidity is at or above RH_{limit} and $\omega_p > \omega_{air}$. Similarly, the latent energy due to mass transfer term in the particle conservation of energy equations (Eq. (4), Eq. (7), and Eq. (9)) is multiplied by this limit function for the same conditions.

Conservation of Momentum

Unlike the conservation of mass equations where the air and particle together constitute a closed system, the conservation of momentum equation is solved only in reference to the particle. The forces exerted onto the particle are the forces of drag, F_{drag} , and gravity, F_g .

$$F = F_{drag} + F_g \rightarrow m_p a = \rho_p \frac{\pi d^3}{6} \frac{\partial v_p}{\partial t} \quad (22)$$

In the above equation, a is the particle's acceleration due to the sum of all forces acting on the particle, F . The force of gravity is negligible and can be assumed to be zero.

$$F_g = 0 \quad (23)$$

The drag force can be described by the following equation.

$$F_{drag} = \frac{1}{2} \rho_{air} U^2 A C_D = \frac{1}{2} \rho_{air} (v_{air} - v_p)^2 \pi d^2 C_D \quad (24)$$

In Eq. (24), U is the slip velocity and A is the projected area of the spherical particle. Many correlations exist for the coefficient of drag for flow past a sphere. Morrison [15] provides a correlation that spans a wide range of Reynolds numbers and is given below.

$$C_D = \frac{24}{Re} + \frac{2.6\left(\frac{Re}{5.0}\right)}{1 + \left(\frac{Re}{5.0}\right)^{1.52}} + \frac{0.411\left(\frac{Re}{263,000}\right)^{-7.94}}{1 + \left(\frac{Re}{263,000}\right)^{-8.00}} + \frac{Re^{0.80}}{461,000} \quad (25)$$

Substituting and rearranging terms in Eq. (22) provides a simple differential equation for the conservation of momentum.

$$\frac{\partial v_p}{\partial t} = \frac{3}{4} \frac{\rho_{air} C_D (v_{air} - v_p)^2}{\rho_p d} \quad (26)$$

Energy Balance Equations with a Heat Source

The axial air temperature in the RATFac icing tunnel facility at NRC is not constant even in the absence of an icing cloud. A detailed description of the icing facility is given by Knezevici et al. [16]. The air temperature increases from the point of injection to test section. Without the presence of a cloud, cold air is injected into the flow stream and is mixed with warm (and moist) ambient air. The cold air jet and entrained warm air are drawn by a vacuum downstream of the test section. A computational fluid model (CFD) was previously written that approximated the air temperature and flow field profiles along the icing tunnel (these internal calculations are currently unpublished). These profiles were implemented into this paper's model. Figure 1 shows approximate values for the normalized air temperature difference between the test section air temperature, T_{test} , and the injection air temperature, T_{inj} at the tunnel centerline, $(T - T_{inj})/(T_{test} - T_{inj})$.

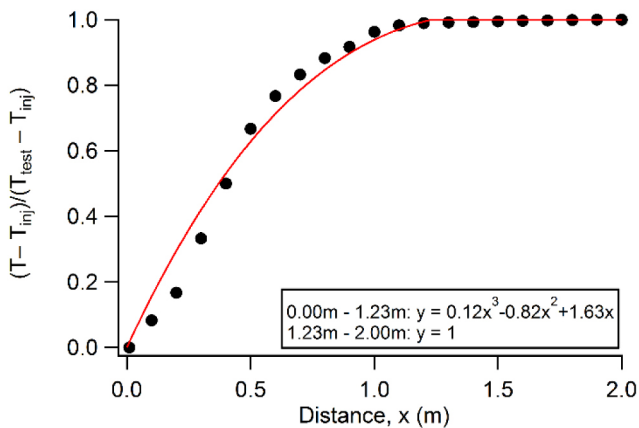


Figure 1. Normalized temperature profile along the icing tunnel centerline in the absence of a cloud, as determined from unpublished CFD calculations.

Figure 1 shows the computational results where the test section air speed is Mach = 0.2 with a total pressure of 43644 Pa. The distance between cold air injection and test section is approximately 2 m. While the values obtained from the fluid model are specific to the conditions run, for the purposes of this paper, the general normalized temperature profile was used as an approximation for different injection and test section temperatures.

A parcel of air - without any particles - heated by a source, q_{source} , is given by the following expression.

$$\frac{\partial H_{air}}{\partial t} = m_{air} C_{air} \frac{\partial T_{air}}{\partial t} = q_{source} \quad (27)$$

To convert the above equation from a stationary air mass to one that simulates an air mass moving in the x -direction, the following manipulation can be done.

$$\frac{\partial H_{air}}{\partial t} = m_{air} C_{air} \frac{\partial T_{air}}{\partial x} \frac{\partial x}{\partial t} = m_{air} C_{air} \frac{\partial T_{air}}{\partial x} v_p = q_{source} \quad (28)$$

In Eq. (28), x is distance and $\partial T_{air}/\partial x$ can be derived from the computational fluid model that approximated the temperature profile. $\partial T_{air}/\partial x$ is the derivative of the polynomial fit in Figure 1. The term $\partial x/\partial t$ is the velocity of the reference frame, which is the particle velocity. A constant value of $\partial T_{air}/\partial x = 0$ was prescribed after the distance where the normalized temperature surpassed a value of 1.

To approximate this changing temperature air profile in the icing tunnel, the source term is added to the air energy equation.

$$\frac{\partial H_{air}}{\partial t} = q_{conv} + q_{wv,sens} + q_{source} \quad (29)$$

Mass Balance Equations with a Mass Source

As was the case with the icing tunnel temperature profile, the humidity profile changes. At the icing facility, warm humid air is entrained by a cold dry air stream and mixes. A parcel of air - without any particles - gaining water vapor mass from a source is given by the following simple expression.

$$\frac{\partial m_{air}}{\partial t} = \frac{\partial m_{wv}}{\partial t} = \dot{m}_{source} \quad (30)$$

Again, to convert the above equation from a stationary air mass to one that simulates an air mass moving in the x -direction, the following manipulation can be done.

$$\frac{\partial m_{wv}}{\partial t} = \frac{\partial m_{wv}}{\partial x} \frac{\partial x}{\partial t} = \frac{\partial m_{wv}}{\partial x} v_p = \dot{m}_{source} \quad (31)$$

A water vapor profile for the icing facility, in the absence of a cloud, was not previously modelled. As an approximation, the normalized profile determined for the air temperature is used analogously for the water vapor mass profile. Therefore, in the above equation, $\partial m_{wv}/\partial x$ can be derived from the water vapor mass profile.

A water vapor mass source term, m_{source} , is added to the air mass balance equation to approximate the changing humidity along the length of the icing tunnel.

$$\frac{\partial m_{air}}{\partial t} = -\frac{\partial m_p}{\partial t} + \dot{m}_{source} \quad (32)$$

Flow Field Profile

The air velocity is not constant from the point of air injection to the test section. A flow field was calculated for the icing tunnel in the previously written computational fluid model. A normalized velocity profile at the icing tunnel centerline is given in Figure 2. The expression in Figure 2 is valid only when the injection velocity, v_{inj} , equals the test section velocity, v_{test} , with values near Mach = 0.2.

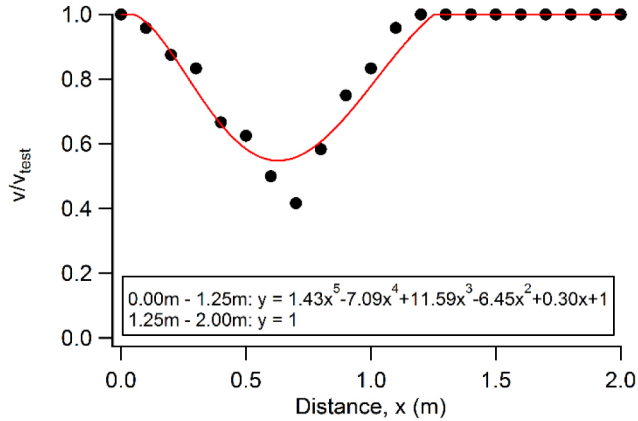


Figure 2. Normalized velocity profile along the icing tunnel centerline as determined from unpublished CFD calculations.

A simple differential equation can be written to approximate the changing flow field air velocity.

$$\frac{\partial v_{air}}{\partial t} = \frac{\partial v_{air}}{\partial x} \frac{\partial x}{\partial t} = \frac{\partial v_{air}}{\partial x} v_p \quad (33)$$

In the above equation, $\partial v_{air}/\partial x$ is the derivative of the polynomial fit in Figure 2. The term $\partial x/\partial t$ is the velocity of the reference frame, which is the particle velocity. A constant value of $\partial v_{air}/\partial x = 0$ was prescribed after the distance where the normalized velocity surpassed a value of 1.

Supercooling Formulation

Freezing of liquid water can occur in two different processes: (1) normal freezing and (2) supercooled freezing. Normal freezing occurs with tap water, for example, which contains mineral impurities. These act as nucleation sites for freezing to occur at the equilibrium freezing temperature. At atmospheric pressures, city water freezes at 273.15 K. Supercooling is the process of cooling a liquid below its equilibrium freezing point without it becoming a solid. De-ionized water or cloud droplets can be supercooled to their homogeneous crystallization temperature ($\sim -40^\circ\text{C}$) before crystallization occurs.

The process of normal freezing in air can be described in 3 stages:

1. Sensible cooling of liquid water until water temperature reaches 273.15 K.

2. Latent freezing where heat is transferred from the particle at a constant temperature of 273.15 K until it has completely frozen.
3. Sensible cooling of the solid ice particle until the temperature is reduced to the wet-bulb temperature.

The process of supercooling, also outlined by Hindmarsh et al. [8], can be described in 4 stages for a particle and follows Figure 3.

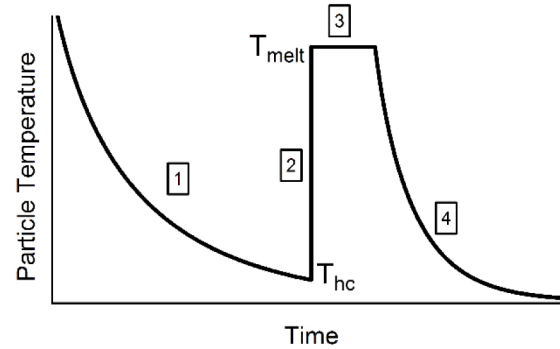


Figure 3. The four stages of the supercooled freezing process which includes (1) sensible liquid cooling, (2) latent heat release (ideal) (3) latent freezing, (4) and sensible ice cooling.

1. Sensible cooling of liquid water to below the equilibrium freezing temperature until homogeneous crystallization temperature is reached.
2. Latent heat release of supercooled water where rapid crystallization of a particle occurs, along with an increase in particle temperature. The particle will partially freeze and will have reached its equilibrium freezing temperature of 273.15 K.
3. Latent freezing where heat is transferred from the partially frozen particle at a constant temperature of 273.15 K until it has completely frozen.
4. Sensible cooling of the solid ice particle until the temperature is reduced to the wet-bulb temperature.

An expression for the homogenous crystallization temperature, T_{hc} , was derived from data given by Heverly [17] and is given below.

$$T_{hc} = 7.2015 \ln(d) + 214.64 \quad (34)$$

During the latent heat release stage, the fraction of the particle that crystallizes will depend on the amount of sensible heat lost during supercooling, $H_{sens,super}$. The melt ratio of the particle resulting after the latent heat release is found using the following formulas.

$$H_{sens,super} = m_p \int_{T_{hc}}^{273.15} C_p dT_p \quad (35)$$

$$\eta_p = 1 - \frac{H_{sens,super}}{m_p L_{melt}} \quad (36)$$