

Evaluation of Spring Properties of Tire Sidewall under Changes in Inflation Pressure

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ABSTRACT

One of the elements of tire stiffness is sidewall stiffness. This stiffness, which influences tire vibration characteristics, is also an important design parameter for carrying the vehicle body. Tire is one of pressure vessels and inflation pressure is dominant in sidewall stiffness. Thus, tire sidewall stiffness is decided from the tension of inflation pressure and the structural dynamic, including the properties of the rubber material.

To reveal the dynamic characteristics of tire sidewall stiffness, this study describes differences in stiffness due to inflation pressure. It can be expected that variation of inflation pressure is monitored from the axle vibration response during vehicle traveling in the future. That is because the relationship of the vibration characteristics and the inflation pressure of tire are derived by sidewall stiffness. First, we derive a formula for sidewall stiffness based on the structural dynamics of Akasaka's theory. This formula includes two terms dependent on the inflation pressure and the rubber material. Second, sidewall stiffness is identified from experiments on the conditions that result from differences in inflation pressure. The sidewall stiffness parameter identified is calculated from a theoretical equation of natural frequency by tire ring model and experimental data. Finally, sidewall stiffness was evaluated by separating the terms dependent on the inflation pressure and the rubber material.

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INTRODUCTION

Recently, the automotive industry has taken a proactive stance on environmentally friendly product design and development [1, 2, 3]. Car manufacturers have focused on the development of hybrid and electric vehicles to reduce emissions, as well as the noise of the air intake and exhaust systems. Hence, vibration noise from tires, including road noise, has become more noticeable overall. Thus, tire manufacturers have sought to develop low-rolling-resistance tires to address this problem, even though grip performance is lower with such tires. Alternatively, improved grip performance will speed up tire wear. These are the generally known facts of tire performance [4-5]. Tire performance is influenced by inflation pressure and depends on the mechanical characteristics of the tire.

In this study, we review the relationship between tire mechanical characteristics and inflation pressure. Tire mechanical characteristics evaluation includes sidewall stiffness. This stiffness, which influences tire vibration characteristics, is also an important design parameter for supporting the vehicle body. Tire is one of pressure vessels and inflation pressure is dominant in sidewall stiffness. Thus, tire sidewall stiffness can be said to derive from the tension of inflation pressure and the structural dynamic characteristics, including the properties of the rubber material. To reveal the dynamic characteristics of tire sidewall stiffness, this study describes stiffness under different inflation pressures. First, a formula for sidewall stiffness is derived

from Akasaka's theory based on structural dynamics. This formula has terms dependent on the inflation pressure and the rubber material. Second, sidewall stiffness is identified from experimental conditions that vary inflation pressure. Identified sidewall stiffness parameters are calculated from theoretical equations of natural frequency by tire ring model and experimental data. The tire ring model is easily understandable for a relationship of the structural design parameters, the mechanical properties of the tire and the physical phenomena. Results indicate that sidewall stiffness has a linear relationship to inflation pressure. This means the formula for sidewall stiffness can be separated into terms dependent on the inflation pressure and others. We define terms dependent on the inflation pressure and the rubber material in the formula of sidewall stiffness. Finally, we evaluate sidewall stiffness by separating inflation pressure and structural dynamics.

STRUCTURAL DYNAMICS OF TIRES

Tire Model and Spring Characteristics of Sidewall

Tire structure is known for a laminated style with a shell structure of rubber that is strengthened by the number of ply. One tire stiffness index is the spring constant in three directions shown in Figure 1[6].

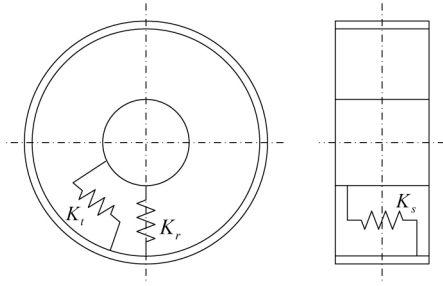


Figure 1. Spring bedded ring model of radial tire. This model has a three-directional spring in in the sidewall. Researchers involved with Akasaka used this model in some of the earliest studies, including those on automobile kinematic performance.

The tire model shown in the figure was used by Akasaka, and consists of a rigid body tread model; the spring of each sidewall is denoted as follows - spring of lateral direction is K_s , radial direction is K_r , and rotational direction is K_t [7]. Akasaka developed a theoretical formula of the spring constant of sidewall using a netting theory as a decoding structure consisting of code and rubber. Coordinates of a sidewall [7-8] are shown in Figures 2 and 3 at an inflation pressure.

The coordinate axis r is set to pass through point D . If point A is at the center of the tread, point B is the bead, point C is the greatest width of the tire, and point D is end of tread, then these points sit on a code. φ_D is the angle made by the tangent line at point D and axis r ; α is the angle made by rotational direction and code from a lateral view. At the radial tire, $\alpha = 90^\circ$. Sidewall cross section and code tension is defined in the following equation. Length of code is shown in equation (1), and coordinate point B in equation (2). In these equations, A and B are shown in equation (3), curvature of the sidewall in equation (4), and code tension in equation (5).

$$L = \int_{r_B}^{r_D} \frac{B}{\sqrt{B^2 - A^2}} dr \quad (1)$$

$$z_B = \int_{r_B}^{r_D} \frac{A}{\sqrt{B^2 - A^2}} dr \quad (2)$$

$$A = (r^2 - r_C^2) \sin \varphi_D \frac{r \sin \alpha_D}{\sqrt{r^2 - C^2}} \quad (3)$$

$$B = (r_D^2 - r_C^2), \because C = r \cos \alpha = \text{const}$$

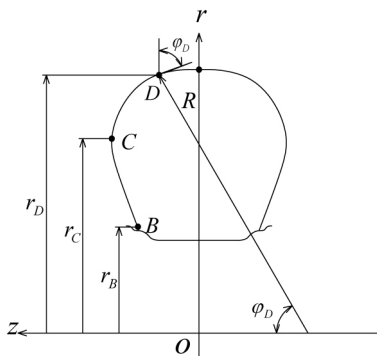


Figure 2. Sidewall cross section. Length of code L , coordinate z of each point and code tension t were obtained by considering netting theory of code from this figure as a membrane structure.

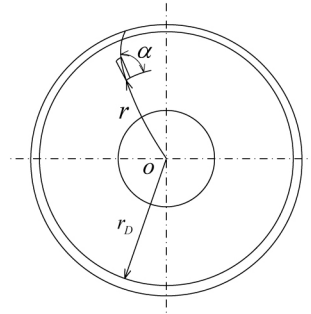


Figure 3. Direction of rotation. The figure shows the code of a sidewall bent in the rotational direction of the burden of torque. Hence, angle α results from relationship between rotational direction and bending code.

$$\kappa = \frac{1}{R} = \frac{2r \sin \varphi_D}{r_D^2 - r_m^2} \quad (4)$$

$$t = \frac{\pi P}{N} \frac{(r_D^2 - r_m^2)}{\sin \varphi_D \sin \alpha_D} \quad (5)$$

Based on above equations, Akasaka obtained spring constants by applying posits of code non-extension and the condition of fixed point B . Spring constants were derived from force impressed upon the sidewall cross section in each direction, and displacement due to that force.

Characteristics of Sidewall Spring

Lateral Spring Constant

As shown in Figure 4, to obtain the lateral displacement δz_B , the distributed force q was added in the lateral direction of point B . Lateral spring constant of the code was derived from the relationship between q and δz_B . Additionally, the lateral spring constant of the rubber was obtained by adapting the principle of least potential energy to energy of strain, extension, and potential-drop. The lateral spring constant [7,9] is shown in equations (6), (7), (8).

The characteristics of the rubber are shown in equation (8) (Young's modulus is E_r , content rate is V_r , Poisson's ratio is ν_r and thickness is h).

$$K_s = -\frac{2r_C}{\xi r_B} P + \frac{q}{\xi \delta r_C} \quad (6)$$

$$\xi = \frac{\partial z_B}{\partial r_C} + \frac{\partial z_B}{\partial \varphi_D} U_0, \because U_0 = \frac{\delta \varphi_D}{\delta r_C} \quad (7)$$

$$q = \frac{2\delta r_C}{\xi r_B} \int_{r_B}^{r_D} \left\{ \frac{E_r h^3 r}{3(1-\nu_r^2)} \zeta_1^2 + \frac{E_r h}{V_r(1-\nu_r^2)} \frac{\zeta_2^2}{r} \right\} ds$$

$$\because \zeta_1 = \frac{\delta \kappa}{\delta r_C}, \zeta_2 = \frac{\delta r_F}{\delta r_C}$$

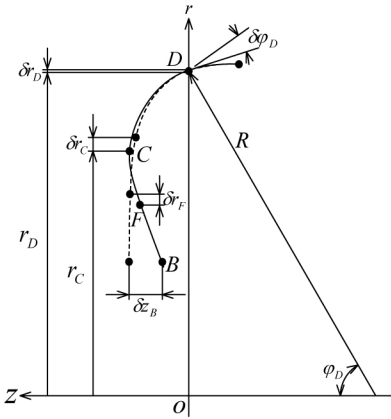


Figure 4. Deformed cross section of radial tire due to lateral force q ; hence, displacement δz_B , δr_p , δr_c , δr_D , and displacement angle $\delta \varphi_D$ resulting from q .

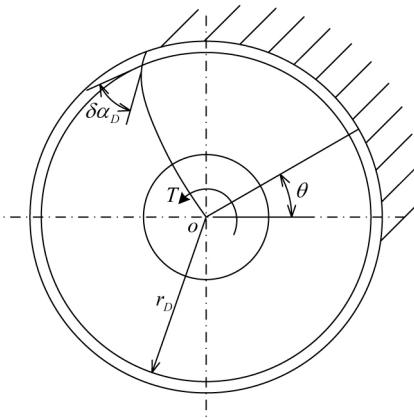


Figure 5. Rim rotation angle θ of radial tire due to rotation torque T ; hence, displacement angle $\delta \alpha_D$ resulting from T .

Rotational Spring Constant

As shown in Figure 5, to obtained rotational displacement θ , the torque of code T_c was added on the wheel rim. The rotational spring constant of the code was derived from the relationship between T_c and θ . Additionally, the rotational spring constant of the rubber was derived from the relationship of the torque of rubber T_r and the strain of rotational direction γ . The rotational spring constant [7,10] is shown in equations (9), (10), (11), (12). (Modules of rigidity is G_r)

$$K_t = \frac{R}{2\pi r_B^3} \quad (9)$$

$$R = \frac{dT_c}{d\theta} + \frac{T_r}{\theta} \quad (10)$$

$$\frac{dT_c}{d\theta} = \frac{\frac{\partial T_c}{\partial r_c} V_1 + \frac{\partial T_c}{\partial \varphi_D} V_2 + \frac{\partial T_c}{\partial \alpha_D}}{\frac{\partial \theta}{\partial r_c} V_1 + \frac{\partial \theta}{\partial \varphi_D} V_2 + \frac{\partial \theta}{\partial \alpha_D}} \quad (11)$$

$$\frac{T_r}{\theta} = \frac{1}{\int_{r_B}^{r_D} \left(\frac{1}{4\pi r^3} \cos \varphi_D h V_r / G_r \right) dr} \quad (12)$$

Radial Spring Constant

As shown in Figure 6, to obtained radial displacement δr_D , the force H was added on point D. The radial spring constant of the code was derived from the relationship between H and δr_D . Additionally, the radial spring constant of the rubber was obtained by adapting the principle of least potential energy to the energy of strain, extension, and potential-drop. The radial spring constant [7] is shown in equations (13), (14), (15), (16).

$$K_r = 2 \times \frac{H}{\delta r_D} \frac{N}{2\pi r_B} + \frac{q}{\delta r_D} \quad (13)$$

$$H = \left(\frac{\partial t_r}{\partial r_c} U_1 + \frac{\partial t_r}{\partial r_D} + \frac{\partial t_r}{\partial \varphi_D} U_2 \right) \delta r$$

$$\because U_1 = \frac{\delta r_c}{\delta r_D}, U_2 = \frac{\delta \varphi_D}{\delta r_D} \quad (14)$$

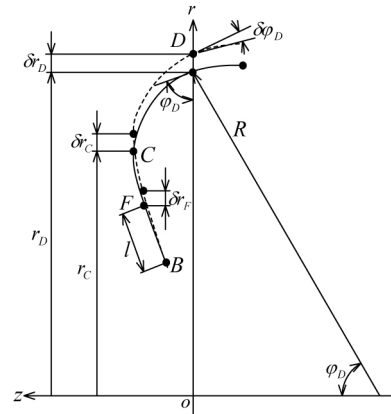


Figure 6. Deformed cross section of the radial tire due to radial force q ; displacement δr_p , δr_c , δr_D , and displacement angle $\delta \varphi_D$ resulting from q .

$$t_r = t \cos \varphi_D \quad (15)$$

$$q = \frac{2\delta r_c}{r_B} \int_{r_B}^{r_D} \left\{ \frac{E_r h^3 r}{3(1-\nu_r^2)} \eta_1^2 + \frac{E_r h}{V_r(1-\nu_r^2)} \frac{\eta_2^2}{r} \right\} ds$$

$$\because \eta_1 = \frac{\delta r_c}{\delta r_D}, \eta_2 = \frac{\delta r_F}{\delta r_D} \quad (16)$$

Theoretical Formula of Spring Constant of Sidewall about Inflation Pressure p

The theoretical formula of the lateral spring constant about P was derived from equations (6), (7), (8).