

Improved Model for Coupled Structural-Acoustic Modes of Tires

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ABSTRACT

Experimental measurements of tire tread band vibration have provided direct evidence that higher order structural-acoustic modes exist in tires, not just the well-known fundamental acoustical mode. These modes display both circumferential and radial pressure variations within the tire's air cavity. The theory governing these modes has thus been investigated. A brief recapitulation of the previously-presented coupled structural-acoustical model based on a tensioned string approach will be given, and then an improved tire-acoustical model with a ring-like shape will be introduced. In the latter model, the effects of flexural and circumferential stiffness are considered. This improved model accounts for propagating in-plane vibration in addition to the essentially structure-borne flexural wave and the essentially airborne longitudinal wave accounted for in the previous model. The longitudinal structure-borne wave "cuts on" at the tire's circumferential ring frequency. Explicit solutions for the structural and acoustical modes will be given in the form of dispersion relations. The latter results will be compared with measured dispersion relations, and the features associated primarily with the higher order acoustic modes will be highlighted. Finally, the effect of tire rotational speed on the natural frequencies of these various modes types will also be discussed.

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1. INTRODUCTION

Since the first practical pneumatic tire was invented by John Dunlop over a hundred year ago, the tire structure has gone through many transformations to arrive at today's appearance. Unfortunately, this important invention also has a side effect which is tire/road noise, and this type of noise has become a major contributor to road traffic noise and vehicle in-cabin noise. So it is desirable to investigate tire/road noise mechanisms and use appropriate models to help understand and improve tire designs.

The hollowed, cylindrical profile of an undeformed tire easily suggests a shell model to us. But research shows that the width in the axle direction is not a factor that dominates tire sound radiation; thus here we reduce the tire model to a thin rotating ring structure. Kropp [1] modeled the tire as a circular ring to study structure-borne sound, and he also considered the static tension in the tire, which was accounted for by the pressurized air in the tire model, but he did not allow for the effects of rotation. He also showed that the tire behaved much like a tensioned string at low frequencies and like a thick beam at higher frequencies. Gong [2] explored the in-plane dynamics of tires on the road based on an analytical, ring-like rolling tire model. However, in his equations of motion, it was assumed that the ring was inextensible in the circumferential direction, thus neglecting the longitudinal wave in the structure. Huang and Soedel [3] first investigated the effect of rotation of the ring structure without applying the inextensional assumption, and thus they accounted for both radial and circumferential motions. Their approach is of great importance since it makes it possible to observe the tire's breathing

mode, but they failed to incorporate static tension in the ring. Later, Pinnington [4] proposed a curved belt model for a tire which included most key features such as side-wall stiffness, curvature, rotation and static pretension. Three types of structural waves could then be identified and examined from a sound radiation perspective. Kim and Bolton [5] also used a cylindrical shell to model a tire and mainly investigated the effect of rotation on wave dispersion relations. It was shown that the flexural wave speed was increased due to rotational stiffening and bifurcation of dispersion curves was also observed.

The abovementioned studies only considered the structural characteristics of tire models, although some models incorporated an effective stiffness to account for the influence of the air cavity. Kindt et al. [6] not only developed an analytical static ring model but also incorporated the air cavity by using finite element analysis. The results showed the natural frequencies of different modes and disclosed the possibility of strong coupling of the sound field in the air cavity with certain modes of the ring structure. Molisani et al. [7] coupled the air cavity with their closed form expressions for the motion of the cylindrical shell structure to model the tire in a static state without internal pressure. The study indicated that acoustic resonance had a significant impact on the structural response and thus need to be included in order to predict vehicle interior noise. However, in their case, the coupling was only "one way", which meant that the acoustic field was assumed to be the same as if there were no vibrating structure. Zhang et al. [8] studied the coupled vibration of fluid-filled cylindrical shells, but neither spin nor fluid

flow was included in their model. They compared the coupled structural-acoustic behavior with the uncoupled case to illustrate the effect of fluid loading.

In the work described in this paper, a ring model was employed to model the tire: the model included curvature, side-wall stiffness, pretension and rotation. The key features of the air cavity were also included and were fully coupled with the ring structure so that the effect of rotation on the coupled structural-acoustic modes could be modeled. Not only were the fundamental acoustical modes investigated, but the higher order acoustical modes, which display radial nodal surfaces in the air cavity, are also described in detail.

2. Model Description and Theory

2.1. Tensioned String Model Review

A tensioned string model was described previously by Cao and Bolton [9] to simulate the acoustical-structural interaction in a coupled rotating tire. However, the tire was “unwrapped” as if it was a straight duct composed of a flexible top boundary and a hard-walled bottom boundary with a uniform flow within it: see Figure 1. Since it was modeled as a two-dimensional case, the flexible boundary could be modeled as, a tensioned string instead of a membrane.

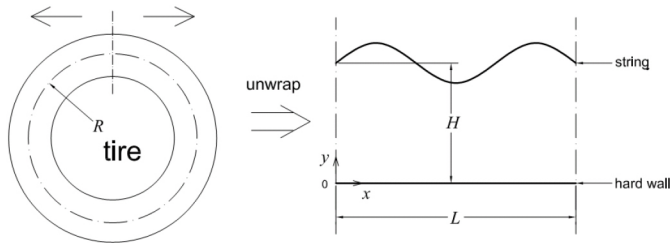


Figure 1. A tensioned string model of tire.

The obtained results showed the 1st acoustical, circumferential mode as expected from the work of Sakata, et al. [10]. Moreover, the model provided results for higher acoustical modes and structural modes. Those results turned out to be very helpful in setting up the new model and for verifying the validity of the improved ring-like tire model.

2.2. Introduce the Improved Ring Model

The model being studied in this paper is shown in Figure 2 and the ring rotates at the same angular velocity as if the tire were rolling on a flat surface at a constant speed v_0 without being subject to any deformation. The model is composed mainly of three parts: a rigid inner ring structure which is supported by spokes connected to the tire spindle, a flexible outer ring structure which is used to simulate the vibrational behavior of the tire and the pressurized air cavity between the two ring structures. In addition, distributed radial and tangential stiffnesses, which model the tire sidewall, are applied directly to the flexible ring structure so it will be possible to observe the modes in which the ring vibrates as a rigid body with respect to the wheel. In addition, the pressurized air within the tire is modeled as a sheared flowing fluid since the tire is considered to be spinning: i.e., the fluid rotates at the same angular velocity as the tire, but the

fluid at different radii have different absolute velocities. We also assumed there was no viscosity in the fluid. The specifications of the tire model are listed in Table 1.

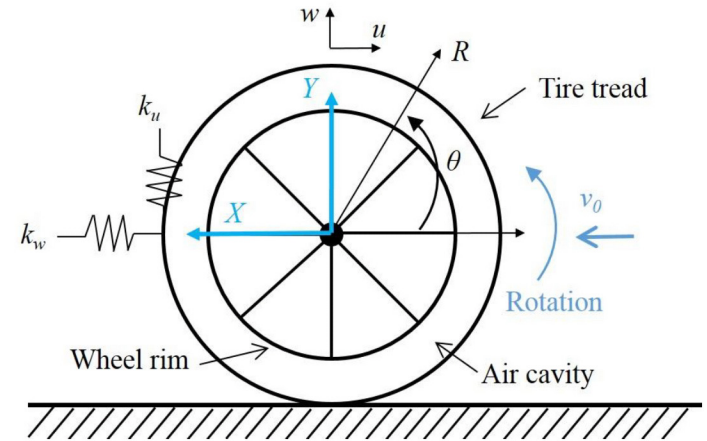


Figure 2. Coupled tire model with ring structure and air cavity.

Table 1. Model parameters.

Ring density	$\rho = 1200 \text{ kg/m}^3$
Air density	$\rho_0 = 1.24 \text{ kg/m}^3$
Outer radius	$r_1 = 0.3 \text{ m}$
Inner radius	$r_2 = 0.2 \text{ m}$
Ring thickness	$h = 0.008 \text{ m}$
Tire inflation pressure	$p_0 = 10300 \text{ Pa}$
Radial stiffness	$k_w = 1 \times 10^5 \text{ N/m}$
Tangential stiffness	$k_u = 1 \times 10^5 \text{ N/m}$
Young's modulus	$E = 4.8 \times 10^8 \text{ Pa}$

2.3. Rotating Ring Theory

Since the tire is spinning at a constant rate, it can be observed from a co-moving Cartesian coordinate without complicating the problem. Following from Huang and Su's theory [11] of a pretensioned ring rotating about a fixed center, the equations of motion that were used to represent the radial and tangential motion in the ring structure, are

$$\begin{aligned} \frac{Eh^3}{12R^4} \left(\frac{\partial^3 w}{\partial \theta^3} - \frac{\partial^2 u}{\partial \theta^2} \right) - \frac{Eh}{R^2} \left(\frac{\partial w}{\partial \theta} + \frac{\partial^2 u}{\partial \theta^2} \right) - \frac{\sigma_\theta h}{R^2} \left(2 \frac{\partial w}{\partial \theta} + \frac{\partial^2 u}{\partial \theta^2} \right) \\ + \left(\frac{P_0}{R} + k_u \right) u + \rho h \left(\frac{\partial^2 u}{\partial t^2} + 2\Omega \frac{\partial w}{\partial t} + 2\Omega \frac{\partial^2 u}{\partial \theta \partial t} \right) = f_u \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{Eh^3}{12R^4} \left(\frac{\partial^4 w}{\partial \theta^4} - \frac{\partial^3 u}{\partial \theta^3} \right) + \frac{Eh}{R^2} \left(w + \frac{\partial u}{\partial \theta} \right) + \frac{\sigma_\theta h}{R^2} \left(2 \frac{\partial u}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2} \right) \\ + \left(\frac{P_0}{R} + k_w \right) w + \rho h \left(\frac{\partial^2 w}{\partial t^2} - 2\Omega \frac{\partial u}{\partial t} + 2\Omega \frac{\partial^2 w}{\partial \theta \partial t} \right) = f_w \end{aligned} \quad (2)$$

where u is the tangential displacement, w is the radial displacement, θ is the angle of the ring span and Ω is the angular velocity of the ring during rotation. Also, f_u and f_w are external loadings applied to the structure in the tangential and radial directions respectively. As for the constants in the equations, E is the Young's modulus of the flexible ring from

its center to its neutral axis and E is the Young's modulus. The other important parameters are p_o , the internal pressure of the air cavity and $\sigma_\theta h$ the pretension force in the ring which includes both the static inflation pressure and dynamic centrifugal force: i.e.,

$$\sigma_\theta h = \rho h R^2 \Omega^2 + p_o R. \quad (3)$$

To solve for the uncoupled ring vibration, it is a standard procedure to assume harmonic solutions for u and w of the following form:

$$w = \alpha e^{-jk_\theta \theta} e^{j\omega t}, \quad u = \beta e^{-jk_\theta \theta} e^{j\omega t} \quad (4,5)$$

where k_θ is the circumferential wave number, and α and β are the amplitudes of each motion. Usually, f_u and f_w are also set to be zero since no external excitation exists in this model. By substituting these solutions into the equation of motion, we can derive two characteristic equations containing k_θ and ω as the two variables: i.e.,

$$j\left(\frac{Eh^3}{12R^4}k_\theta^3 + \left(\frac{Eh}{R^2} + 2\frac{\sigma_\theta h}{R^2}\right)k_\theta + 2\rho h\Omega\omega\right)\alpha + \left(\left(\frac{Eh^3}{12R^4} + \frac{Eh}{R^2} + \frac{\sigma_\theta h}{R^2}\right)k_\theta^2 + \left(\frac{p_o}{R} + k_u\right) - \rho h\omega^2 + 2\rho h\Omega\omega k_\theta\right)\beta = 0 \quad (6)$$

$$\left(\frac{Eh^3}{12R^4}k_\theta^4 + \frac{\sigma_\theta h}{R^2}k_\theta^2 + \left(\frac{Eh}{R^2} + \frac{p_o}{R} + k_w\right) - \rho h\omega^2 + 2\rho h\Omega\omega k_\theta\right)\alpha - j\left(\frac{Eh^3}{12R^4}k_\theta^3 + \left(\frac{Eh}{R^2} + 2\frac{\sigma_\theta h}{R^2}\right)k_\theta + 2\rho h\Omega\omega k_\theta\right)\beta = 0 \quad (7)$$

which, for the sake of brevity, can be expressed in matrix form as,

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (8)$$

If there is to be a non-trivial solution to the latter equation, the determinant of the matrix must be zero. A single characteristic function with two variables, k_θ and ω , can thus be obtained if we expand the determinant. In one case, we can fix the frequency at a certain value and solve the characteristic equation in terms of k_θ , thus allowing the complex dispersion relation to be found. Alternatively, the order of the circumferential mode, in other words, k_θ , can be fixed, in which case, the equation can be solved for ω : i.e., the natural frequency. For this uncoupled situation, either case can be easily solved numerically since the equation is a polynomial of either order 6 for k_θ or order 4 for ω .

2.4. Rotating Air Cavity

Since the fluid in the cavity is assumed to be rotating at the same rate as the ring, it is necessary to identify a wave equation that applies to a moving medium in circular coordinates, as in our new model. First, consider the circular air cavity model. The air cavity is encapsulated in a circular duct with sheared flow. The flow is rotating with the tire at the same angular velocity, which means that the flow velocity of

the air differs with respect to the radius. The circular coordinate has a radial direction r and a circumferential direction, θ . The air in contact with tire structure has tangential velocity v_θ , which is also the vehicle operating speed. The inner circular boundary has radius r_o while the outer circular boundary has radius R .

The work developed next is essentially a review of Fernandez's development of a similar model [12]. The sheared flow has a circumferential velocity

$$v_\theta = \frac{v_o}{R} r \quad (9)$$

where r ranges from r_o to R . After adapting Morse and Ingard's wave equation with flow from *Theoretical Acoustics* [13] to the above described circular coordinates and after substituting in the circumferential velocity, v_θ , we obtain our wave equation:

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} = \frac{1}{c_o^2} \left(\frac{\partial^2 \psi}{\partial t^2} + 2 \frac{v_o}{R} \frac{\partial^2 \psi}{\partial t \partial \theta} + \frac{v_o^2}{R^2} \frac{\partial^2 \psi}{\partial \theta^2} \right). \quad (10)$$

Here, the velocity potential, ψ , is introduced. Because of the simple relation between velocity potential and particle velocity, the form of the coupling described in the next section will be greatly simplified. Note that the pressure is related to velocity potential as,

$$p = -\rho_o \left[\frac{\partial \psi}{\partial t} + \vec{v}_{flow} \cdot \nabla \psi \right] \quad (11)$$

where the flow velocity vector is

$$\vec{v}_{flow} = (0, v_\theta). \quad (12)$$

Such a wave equation which describes the wave propagation within the rotating air cavity results in a model that can be used to study the flow effect on the natural frequencies of the rotating tires. To solve this wave equation, we can assume a factored harmonic solution: i.e.,

$$\psi(r, \theta, t) = g(r) e^{-jk_\theta \theta} e^{j\omega t}. \quad (13)$$

Then, substitute this assumed solution into Equation 10 and rearrange to obtain the Bessel equation

$$r^2 \frac{\partial^2 g}{\partial r^2} + r \frac{\partial g}{\partial r} + \left(r^2 \left(\frac{\omega}{c_o} - \frac{M}{R} k_\theta \right)^2 - k_\theta^2 \right) g = 0 \quad (14)$$

where M is the Mach number. By denoting the expression in the parenthesis as κ ,