

Determination of the Transfer Matrix for Isolators Using Simulation with Application to Determining Insertion Loss

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ABSTRACT

Transmissibility is the most common metric used for isolator characterization. However, engineers are becoming increasingly concerned about energy transmission through an isolator at high frequencies and how the compliance of the machine and foundation factor into the performance. In this paper, the transfer matrix approach for isolator characterization is first reviewed. Two methods are detailed for determining the transfer matrix of an isolator using finite element simulation. This is accomplished by determining either the mobility or impedance matrix for the isolator and then converting to a transfer matrix. It is shown that results are similar using either approach. In both cases, the isolator is first pre-loaded before the transfer matrix is determined. The approach to find isolator insertion loss is demonstrated for an isolator between two plates, and the effect of making changes to the structural impedance on the machine side of the isolator by adding ribs is examined. After which, the value of isolator insertion loss as a metric is examined for a case where multiple isolators are used on a construction cab. The results suggest that the metric has limited value at lower frequencies but may be appropriate at higher frequencies.

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INTRODUCTION

The goal of vibration isolation is to reduce the vibration level at some receiver point by introducing an impedance change in the vibration path. This is normally accomplished by utilizing a compliant element such as a spring or rubber isolator. Isolation is normally effective over a wide frequency range. However, it is well known that energy propagates effectively at the structural modes of isolators, which are sometimes referred to in industry as surge frequencies. As pressures for quieter products mount, structure-borne energy propagation through isolators has become a concern especially at higher frequencies.

Isolators are often modeled as a linear component termed a dynamic stiffness (k_d) defined as

$$k_d = \frac{j\omega F}{v_1 - v_2} \quad (1)$$

where v_1 and v_2 are the velocities on the inlet and outlet sides of the mount respectively and F is the dynamic force. Note that this definition of dynamic stiffness assumes that the force is equivalent on both sides of the isolator neglecting the inertia effects of the isolator.

While still a linear approach, the transfer matrix approach provides a more complete description of an isolator and takes into account inertial effects. The force and vibration on one side of the isolator (F_1 and v_1) are related to that on the other (F_2 and v_2) using the four pole parameters (a_{11} , a_{12} , a_{21} , and a_{22}). This can be expressed as

$$\begin{Bmatrix} F_1 \\ v_1 \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{Bmatrix} F_2 \\ v_2 \end{Bmatrix} \quad (2)$$

The four pole parameters are complex and frequency dependent. Fig. 1 shows a schematic of an isolator with forces (F_1 and F_2) and velocities (v_1 and v_2) identified along with the direction convention.

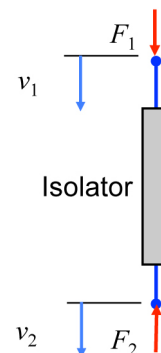


Figure 1. Schematic illustrating mount with force and velocity variables.

It appears that Molloy [1] was the first to suggest the use of four-pole parameters to determine the properties of isolators. After which, Snowdon [2] further developed the idea and applied it to vibration isolation developing the four-pole parameters for a number of simple mechanical idealizations. The theory of which is virtually identical to that described by Munjal [3] for acoustic waveguides (1-D systems). Dickens and Norwood [4-5] developed an experimental two mass

approach for determining the four pole parameters of an isolator. Measurements were performed twice using two different receiver side floating masses. The approach was general and applicable to simple and asymmetric isolators.

Kim and Singh [6-7] have researched elastomeric isolators using a similar but more sophisticated mobility matrix approach. In their efforts, a multi-axial model was used for the isolator and analytical results were compared with measurement.

This paper will focus on the transfer matrix approach. One advantage of the transfer matrix approach is that more representative metrics can be used to assess effectiveness of an isolator. Transmissibility or transmissibility ratio is the metric that is most commonly used for assessing isolators. It is defined as the ratio between magnitudes on the input and output sides of an isolator, and may be defined in terms of either displacements or forces [8]. While useful and simple to measure, transmissibility is a property of the isolator, source, and receiver system. On the other hand, the transfer matrix of an isolator is primarily a property of the isolator alone though it will also depend on the pre-load.

Ungar and Dietrich [8] recommended the use of *isolator effectiveness* (E) that can also be expressed as an *isolator insertion loss* [4,10]. The primary advantage of the isolator effectiveness is that the compliance of both the source and receiver structures are also included in the assessment. Isolator insertion loss compares the unisolated and isolated responses in dB as illustrated in Fig. 2 and can be mathematically expressed as

$$IL = 20 \log_{10} |E| = 20 \log_{10} \frac{|v_f^{\text{rigid}}|}{|v_f^{\text{isolated}}|} \quad (3)$$

where v_f^{rigid} and v_f^{isolated} are the unisolated and isolated vibrations respectively. It can be expressed in terms of the four pole parameters as

$$IL = 20 \log_{10} \left| \frac{a_{11}Z_S + a_{12} + a_{21}Z_S Z_R + a_{22}Z_R}{Z_S + Z_R} \right| \quad (4)$$

where Z_S and Z_R are the mechanical impedances on the source and receiver side respectively.

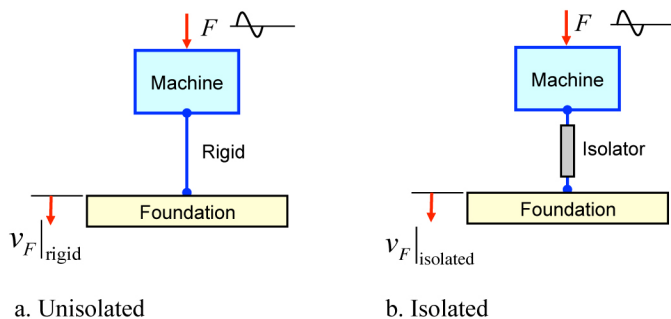


Figure 2. Schematic illustrating isolator insertion loss.

The mechanical impedance of the source or foundation (Z_S and Z_R respectively) can be determined by exciting the respective structure at the isolator attachment point and determining the response as illustrated in Fig. 3. Accordingly,

$$Z_i = \frac{F_i}{v_i} \quad (5)$$

where i refers to the appropriate attachment point on the source or receiver side. This impedance is easily determined using a structural finite element model or via measurement. All impedances in this paper were found using finite element analysis.

In this paper, two similar approaches to determine the transfer matrix of an isolator using finite element analysis are suggested. First, a static analysis is performed in order to include the stress stiffening effects of the isolator due to the static preload for the dynamic analysis that follows. This is followed by a modal analysis to determine the structural modes including the effect of the pre-load.

The transfer matrix can then be found by first finding either the mobility or impedance matrix from two successive forced response analyses with different loading conditions. It is shown that either approach produces similar results for the insertion loss especially if the insertion loss is below 40 dB.

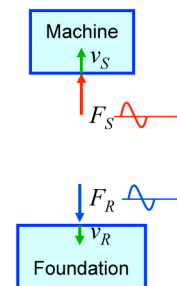


Figure 3. Schematic illustrating determination of impedances.

The methodology is applied to a spring isolator connecting two plates. One plate represents the machine side and the other more massive plate the foundation. The effect on insertion loss of adding ribs to the machine or source side is illustrated.

DETERMINATION OF THE FOUR-POLE PARAMETERS

The procedure for determining the four-pole parameters is summarized in Fig. 4. First, a static finite element analysis is conducted to deal with the static pre-load. This analysis can be linear or nonlinear. The purpose of the analysis is to update the stiffness matrix to include the pre-load (i.e., stress stiffening effects). If the pre-load does not significantly alter the structural modes of the isolator, the static analysis will be unnecessary.

This is followed by a second analysis to determine the structural modes (including the updated stiffness matrix). After which, two successive modal superposition forced response analyses are used to

determine the four-pole parameters. All dynamic analyses assume that the loaded isolator behaves in a linear fashion and that displacements are small. All analyses were performed using ANSYS Workbench [11].

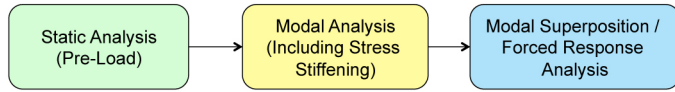


Figure 4. Flow chart illustrating analysis progression.

Mobility Matrix Approach

There are two convenient approaches for determining the transfer matrix. The first is a mobility matrix approach where the transfer matrix in Equation (2) is reconfigured as

$$\begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad (6)$$

where b_{11} , b_{12} , b_{21} , and b_{22} are mobility matrix terms. Two successive forced response analyses can be performed to determine the respective terms. The boundary conditions for the first and second analyses are

$$F_1 = 1; F_2 = 0 \quad (7)$$

and

$$F_1 = 0; F_2 = 1 \quad (8)$$

respectively. The mobility matrix terms (b_{11} , b_{12} , b_{21} , and b_{22}) can be determined from

$$\begin{aligned} b_{11} &= \left. \frac{v_1}{F_1} \right|_{F_1=1; F_2=0} & b_{12} &= \left. \frac{v_1}{F_2} \right|_{F_1=0; F_2=1} \\ b_{21} &= \left. \frac{v_2}{F_1} \right|_{F_1=1; F_2=0} & b_{22} &= \left. \frac{v_2}{F_2} \right|_{F_1=0; F_2=1} \end{aligned} \quad (9)$$

where v_1 , v_2 , F_1 , and F_2 are determined from analyses with the respective boundary conditions indicated.

The four-pole parameters can then be determined from the mobility matrix terms. This is expressed as

$$\begin{aligned} a_{11} &= -\frac{b_{22}}{b_{21}} & a_{12} &= \frac{1}{b_{21}} \\ a_{21} &= b_{12} - \frac{b_{11}b_{22}}{b_{21}} & a_{22} &= \frac{b_{22}}{b_{21}} \end{aligned} \quad (10)$$

Impedance Matrix Approach

In a similar manner, the transfer matrix terms can be determined using an impedance matrix approach. In that case, Equation (2) can be rearranged as

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} \quad (11)$$

where c_{11} , c_{12} , c_{21} , and c_{22} are the respective impedance matrix terms. Once again, two successive forced response analyses are conducted to determine the terms. The boundary conditions for the first and second analysis are

$$F_1 = 1; v_2 = 0 \quad (12)$$

and

$$F_2 = 1; v_1 = 0 \quad (13)$$

respectively. The impedance matrix terms (c_{11} , c_{12} , c_{21} , and c_{22}) can be determined from

$$\begin{aligned} c_{11} &= \left. \frac{F_1}{v_1} \right|_{F_1=1; v_2=0} & c_{12} &= \left. \frac{F_1}{v_2} \right|_{v_1=0; F_2=1} \\ c_{21} &= \left. \frac{F_2}{v_1} \right|_{F_1=1; v_2=0} & c_{22} &= \left. \frac{F_2}{v_2} \right|_{v_1=0; F_2=1} \end{aligned} \quad (14)$$

where v_1 , v_2 , F_1 , and F_2 are determined from analyses.

The four-pole parameters can then be determined from the impedance matrix via

$$\begin{aligned} a_{11} &= \frac{c_{11}}{c_{21}} & a_{12} &= c_{12} - \frac{c_{11}c_{22}}{c_{21}} \\ a_{21} &= \frac{1}{c_{21}} & a_{22} &= -\frac{c_{22}}{c_{21}} \end{aligned} \quad (15)$$

The primary difference between the two approaches is that the isolator is unconstrained if the mobility matrix approach is used, and alternately fixed on one side or the other if the impedance matrix approach is used.

Frequency Based Substructuring Approach

The transfer matrix approach can also be linked to frequency based substructuring (FBS) approaches [12, 13, 14, 15, 16, 17] sometimes referred to as transfer path analysis (TPA). In the case of FBS, isolators are commonly modeled as a dynamic stiffness if inertia effects can be neglected. However, the isolator is more properly defined as a separate dynamic system defined by the transfer functions between different isolator sides. The transfer functions can be expressed in terms of the transfer matrix terms as