

Comparative Assessment of Frequency Dependent Joint Properties Using Direct and Inverse Identification Methods

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ABSTRACT

Elastomeric joints are utilized in many automotive applications, and exhibit frequency and excitation amplitude dependent properties. Current methods commonly identify only the cross-point joint property using displacement excitation at stepped single frequencies. This process is often time consuming and is limited to measuring a single dynamic stiffness term of the joint stiffness matrix. This study focuses on developing tractable laboratory inverse experiments to identify frequency dependent stiffness matrices up to 1000 Hz. Direct measurements are performed on a commercial elastomer test system and an inverse experiment consisting of an elastic beam (with a square cross section) attached to a cylindrical elastomeric joint. Sources of error in the inverse methodology are thoroughly examined and explained through simulation which include ill-conditioning of matrices and the sensitivity to modeling error. The identified translational dynamic stiffness and loss factor values show good agreement between the two identification methods, though challenges remain for the rotational and coupling stiffness terms. The experimental methods are applied to two different elastomeric materials of the same geometry.

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INTRODUCTION

Elastomeric joints are commonly used to connect two substructures providing isolation and/or motion control to a system. Current methods used to identify joint properties commonly employ a cross-point measurement where a uniaxial displacement excitation is applied to the joint component under preload [1, 2]. This method requires the component to be tested with a specified input displacement or force amplitude in a step-sine sweep fashion [2, 3]. Typical machines used for these measurements consist of commercial elastomer test systems [4] where its actuator excites the joint with a displacement input and measures the force response with a load cell [1, 2]. These measurements are often limited in frequency range (less than 1000 Hz) due to frame and fixture resonances and capabilities of the machines [1, 2]. Due to these characteristics common to direct measurements, the test can often be time consuming and only able to determine single stiffness terms in the joint stiffness matrix. Inverse methods have been developed as another approach to characterize joint stiffness properties [1, 5, 6, 7].

Kim and Singh [1], incorporate two inertial elements connected to each end of a joint to identify frequency dependent dynamic stiffness matrices using the measured mobilities of the overall system and rigid body system theory. This method shows promising results in identifying high frequency stiffness properties of the joint; however, the experimental configuration is not capable of incorporating preload [1].

Other methods [5, 6, 7] incorporate the use of elastic beam theory in place of the rigid inertia elements and thus permit incorporating preload into the identification experiment. Hong and Lee [6] develop a method in which an unconstrained structure (base structure) and an experimental setup (base structure connected to joints) is used to identify joint properties. By acquiring the complete dynamic compliance matrices of the two models, the identified joint properties can be found by subtraction of the inverse of the two compliance matrices [6]. The method appears useful to identify multidimensional frequency dependent properties under a variety of preload conditions [7]. Although such formulations exist in open literature, this article serves to fill the void of a lack of experimental validation where an elastic beam element is incorporated in the setup to identify frequency dependent properties of elastomeric joints.

PROBLEM FORMULATION

This study extends the prior work of Hong and Lee [6] by conducting benchmark experiments that contain joints with frequency dependent properties. The joints considered are composed of a cylindrical elastomeric material with steel caps at each end. Two joints of different material are analyzed. The scope is focused on identifying results in the frequency domain up to 1000 Hz.

The specific objectives of this study are as follows: 1) Design tractable laboratory experiments that allow for a comparison between inverse and direct identification methods; 2) Conduct numerical verification simulations to examine sources of error common to the

inverse approach; and 3) Conduct benchmark validation experiments to identify the joint properties. The experimental validation study intentionally examines the translational stiffness term and leaves the rotational and coupling terms for future research.

An elastic (beam) element is intentionally selected for the experimental configuration. Prior research [1] has used rigid inertial element attachments to the elastomeric component; however, such a configuration does not permit incorporating preload into the experiment. The formulation is general in nature and applicable to linear elastic jointed structures. As a first step, the experiment did not include preload and is reserved for future research.

UNIAXIAL CHARACTERIZATION AND PARAMETER IDENTIFICATION USING THE DIRECT METHOD

The joint is a cylindrical geometry with an elastomeric material and steel caps on opposing ends as shown in Figure 1a and Figure 1b. The joint rotational, θ_1 and θ_2 , and translational, X_1 and X_2 , degrees of freedom are considered; thus, the joint stiffness matrix is given as

$$\begin{bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{bmatrix} = \begin{bmatrix} \tilde{k}_{F_1 X_1} & \tilde{k}_{F_1 \theta_1} & \tilde{k}_{F_1 X_2} & \tilde{k}_{F_1 \theta_2} \\ \tilde{k}_{M_1 X_1} & \tilde{k}_{M_1 \theta_1} & \tilde{k}_{M_1 X_2} & \tilde{k}_{M_1 \theta_2} \\ \tilde{k}_{F_2 X_1} & \tilde{k}_{F_2 \theta_1} & \tilde{k}_{F_2 X_2} & \tilde{k}_{F_2 \theta_2} \\ \tilde{k}_{M_2 X_1} & \tilde{k}_{M_2 \theta_1} & \tilde{k}_{M_2 X_2} & \tilde{k}_{M_2 \theta_2} \end{bmatrix} \begin{bmatrix} X_1 \\ \theta_1 \\ X_2 \\ \theta_2 \end{bmatrix} \quad (1)$$

where F_1 and F_2 are applied forces, and M_1 and M_2 are the moments. The stiffness terms of interest to identify consist of the translational stiffness element, $\tilde{k}_{F_1 X_1}$, the rotational stiffness element, $\tilde{k}_{M_1 \theta_1}$, and the coupling terms, $\tilde{k}_{F_1 \theta_1}$ and $\tilde{k}_{M_1 X_1}$ defined with reference to Equation (1) as

$$\tilde{k}_{F_1 X_1} = \frac{F_1}{X_1}, \quad \tilde{k}_{F_1 \theta_1} = \frac{F_1}{\theta_1}, \quad \tilde{k}_{M_1 \theta_1} = \frac{M_1}{\theta_1}, \quad \tilde{k}_{M_1 X_1} = \frac{M_1}{X_1} \quad (2 a-d)$$

with the all other boundary conditions ideally fixed. Dynamic stiffness of a damped joint is defined as

$$\tilde{k} = k' + jk'' \quad (3)$$

$$\eta = \frac{k''}{k'} \quad (4)$$

where k' is the real (storage) component of the dynamic stiffness, k'' is the imaginary component (dissipative) of the dynamic stiffness, j is $\sqrt{-1}$, and η is the loss factor defined by Equation (4).

The joints are tested on an elastomer test system [2, 4] which is a non-resonant test where input displacement is applied at one end of the specimen and then the force transmitted is measured at the other end. Since the stiffness of the joint is being pursued, the fixture itself

utilized for experimentation must be stiff relative to the joint [8]. Accordingly, aluminum is chosen as the fixture material. The fixture is composed of six blocks bolted together, as illustrated in Figure 1c. The bottom fixture is composed of blocks labeled 1, 2 and 3, and block 3 is mounted to the load cell of the test system. The top fixture, composed of block 4, provides an input displacement (X_1) to the joint which transmits a force, to blocks 1, 2, 3 and eventually the load cell of the test system (F_2). The direct test measures $\tilde{k}_{F_2 X_1}$, which is a cross-point term of the stiffness matrix Equation (1).

Figure 2 contains the experimental results of the direct cross point measurement up to 600 Hz with a frequency resolution of 25 Hz. The upper frequency in the test is 600 Hz rather than 1000 Hz as the elastomer test system fails to converge to the correct displacement amplitude beyond 600 Hz for the benchmark tests. The displacement amplitude is chosen to create a sine wave that is within 5% of its amplitude peak. At higher frequencies, the test system is less consistent in staying within this tolerance. The test is performed with a peak-to-peak input displacement amplitude of 0.05 mm. The dynamic stiffness magnitude is divided by two since there are two joints in parallel (shear).

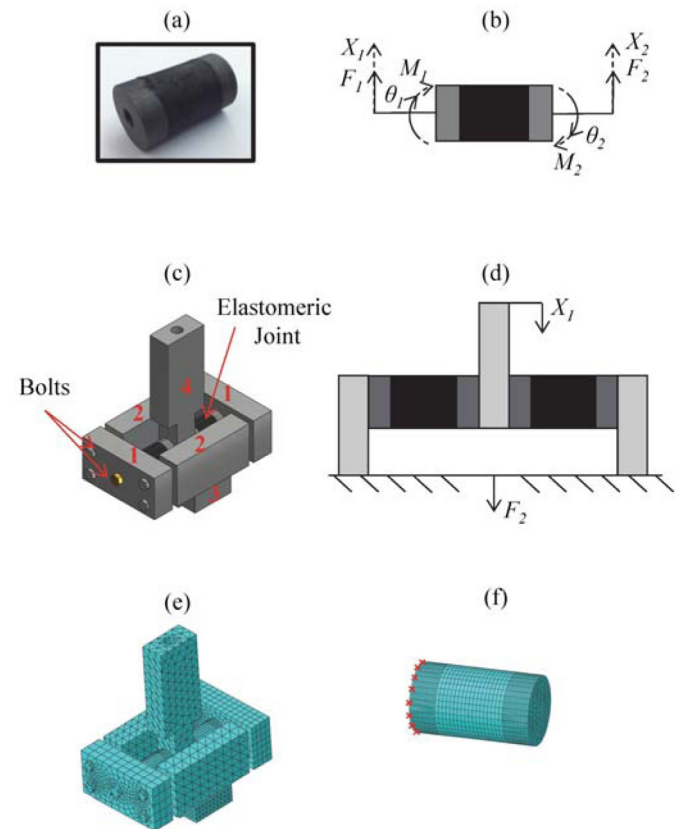


Figure 1. Direct characterization of the elastomeric joint, (a) Typical joint used in experiments, (b) Joint schematic with coordinate system, (c) CAD model of direct testing experimental fixture with labeled aluminum blocks, (d) Simplified equivalent model of direct fixture with X_1 associated with displacement of block four and F_2 is associated with force seen by the elastomer test system's load cell, (e) Finite element model of direct fixture and (f) Finite element model of joint fixed on one end and a free boundary condition on the opposing end

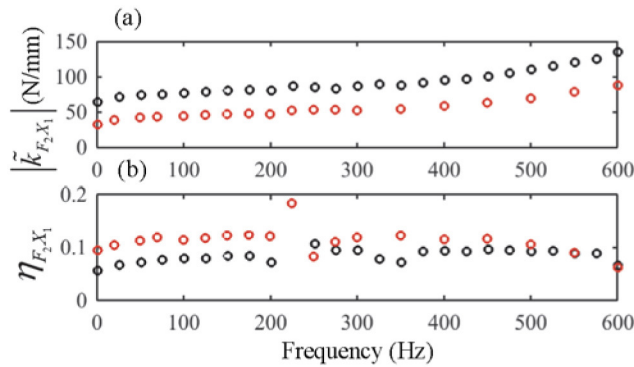


Figure 2. Measured cross-point dynamic stiffness. Magnitude in (a) and loss factor in (b). Key: ○○○○, Material A; ○○○○, Material B

MODELING THE DIRECT IDENTIFICATION EXPERIMENT USING A FINITE ELEMENT MODEL

The inverse identification experiment is constructed to identify the driving-point properties of the elastomeric joint; thus, a finite element model of the cross-point measurement of the direct experiment is used to establish a driving-point comparison. The direct uniaxial characterization experiments are used to determine the effective elastic modulus, loss factor, and density of the two elastomeric materials. A finite element model of the direct fixture is used to determine the material parameters of the joint elastic material. It is recognized that material nonlinearities are present; however, the assumption is made that at small displacements above say 50 Hz, the elastomer can be approximated as a linear elastic material with structural (hysteretic) damping proportional to stiffness. The static stiffness can be found by choosing a value of the dynamic stiffness magnitude at low frequencies. From this stiffness value, $\tilde{k}_{static}$, the modulus of rigidity is found with the following equation $\tilde{G} = (\tilde{k}_{static} L) / A$, where L is the length of the elastic material of the joint (2.54 cm) and A is the cross sectional area of a 2.54 cm diameter circle. The elastic modulus of the structure can then be found by the relationship provided by the Poisson's ratio $\tilde{E} = 2\tilde{G}(1 + \nu)$, where the Poisson's ratio, ν , is chosen as .49 for rubber. The dynamic stiffness and loss factor comparison between the experiment and FEA is displayed on Figure 3. The values found for the elastic moduli, loss factors, and densities of the joint elastic material are shown on Table 1. In order to now obtain simulated direct calculations comparable to the inverse driving point calculations, a simplified finite element model of a joint fixed on one end as displayed on Figure 1f is used in the subsequent sections.

Table 1. Identified material properties and parameters of the two materials

Material Property	Material A	Material B
Density, ρ (kg/m ³)	1300	1100
Elastic Modulus, $\tilde{E}$ (MPa)	15.38	8.38
Structural Damping Coefficient	0.06	0.12

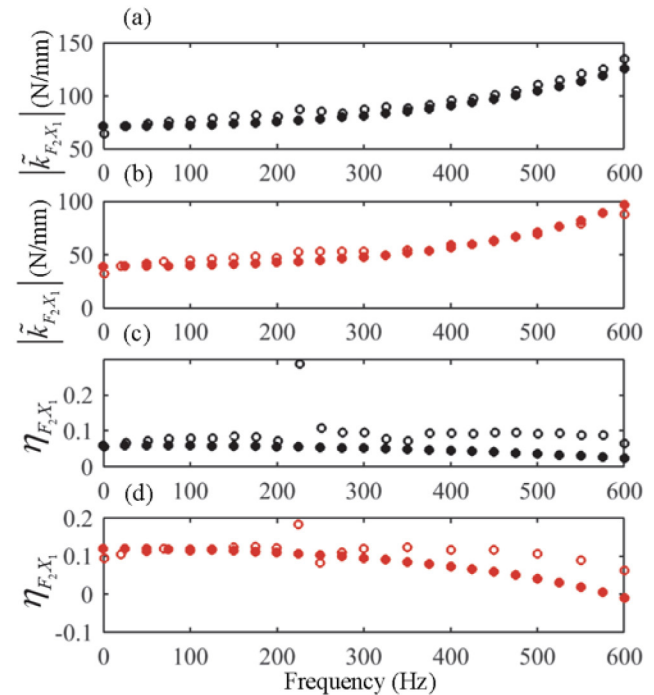


Figure 3. FEA approximation of experimental cross point dynamic stiffness magnitude (a, c) and loss factor (b, d) results for Material A and Material B. Key: (Material A) ●●●●, FEA; ○○○○, Experiment; (Material B) ●●●●, FEA; ○○○○, Experiment.

INVERSE METHOD FOR IDENTIFICATION

The method described by Hong et al. [7] requires three dynamic stiffness matrices to identify the unknown joint properties. The three dynamic stiffness matrices consist of the dynamic stiffness matrix of the constrained structure (D^*), unconstrained structure (D^o) and the unknown joint properties (D^u) as illustrated on Figure 4. To summarize the method, if one had the dynamic stiffness matrix of the constrained and unconstrained structure, the unknown joint properties could be found from the subtraction of the two matrices. In most applications, the constrained structure is not completely known. This requires the need to estimate unobserved frequency response functions. Joint locations are typically unobserved coordinates due to obstructions and lack of experimental equipment; yet, these frequency response functions are required for estimation of the unknown joint properties in the identification formula. This leads to this method requiring two steps. First, one must use the estimation formula for unobserved frequency response functions. The second step is to use the identification formula for unknown joint properties. The next two sections summarize the derivation for these two equations. The mathematical formulation is implemented into a Matlab script [9].

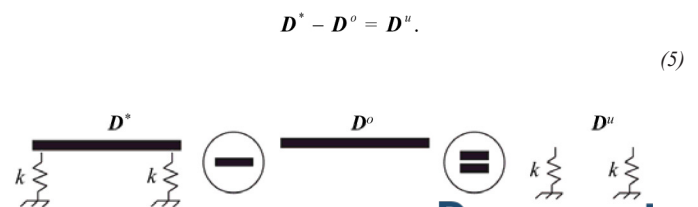


Figure 4. Simplified illustration of the identification method and the corresponding dynamic stiffness matrices where k is the stiffness element.