

Virtual Dominant Sensor for Observability Improvement in Rotating Systems

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ABSTRACT

Spectral maps and order tracks are tools which are susceptible to improper sensor location on rotating machinery and to measurement noise. On a complex/large rotating system, the major behavior in a particular direction cannot be observed by using standard digital signal processing averaging techniques on different sensor outputs. Also, measurement noise cannot be reduced by applying averaging - due to the slew rate of the system.

A newly developed technique tested on experimental data, is presented which uses singular value decomposition (SVD) as its basis to improve the observability of rotating systems. By using data acquired from multiple accelerometers on a machine, singular values - obtained from a SVD of the cross-power matrix at each 2-D point in the frequency-RPM domain - can be plotted in a color-map format similar to a RPM spectral map. These "Singular value maps" can be used to find the best possible sensor combination on a complex rotating system like a powertrain and to observe the system as one unit in a particular direction retaining major and inherent dynamic activity and minimizing the measurement noise.

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INTRODUCTION

For the purposes of product development, health monitoring, diagnostic testing and modal analysis, rotational systems can be tested and analyzed using several numerical techniques. Accurate extraction of relevant data is the basis of every numerical procedure, the results of which are vital to further decisions in the design stage of a product, or to assess the overall health of the system.

But, all of these techniques and procedures, process data which has been acquired from various sensors on the system - often, without prior knowledge whether the sensor location is active or not, i.e. whether the sensor in consideration depicts the true dynamic behavior of the machine or not. Hence, purely based on sensor location, a large vulnerability can loom over further analysis and decision-making.

For example, a particular order tracking method will process any data which is fed into the numerical formulation and produce results without considering the physical relevance of the sensor. There is a possibility, that due to the improper location of the sensor on a complex rotating system, a particular dynamic characteristic can be misrepresented. This can lead to wrong interpretations of the health or behavior of the system, leading to a cascading effect which can magnify the possibility of a catastrophic failure.

A new technique is presented in this research, which aims to inform the user of the physical relevance of a particular sensor assembly or tri-axial sensor(s), by considering only the major and active sources on a system.

Using the concept of singular value decomposition (SVD), on the response cross-power spectrum matrices - calculated using the frequency domain complex response, at each frequency of vibration and each RPM, for every sensor used in the analysis - singular values can be calculated and plotted similar to a RPM spectral map. The resulting singular value plots, or maps, will contain and depict the cross-contributions from every sensor, thereby quantifying the entire sensor assembly.

The SVD approach also leads to reduction of measurement noise in the system, since the retention of dominant response (achieved by the SVD process) from multiple sensors acts similarly as results produced by averaging in modal analysis measurements [2][3][16].

An example is discussed which discusses the inspiration and the need to improve observability in rotating systems. Consider a RPM spectral map of acceleration data from an un-axial accelerometer (+Y direction) placed on an engine during a test-cell run-up. (Time domain data provided by ACS Inc. [24])

Suppose, the order of interest is the 12th order (blue highlight in [Figure 1](#)), which is the fundamental order of a particular component. Observing the order cut of the 12th order in the +Y direction, in [Figure 2](#), an amplitude dip in the RPM range of 1500-1600 can be seen.

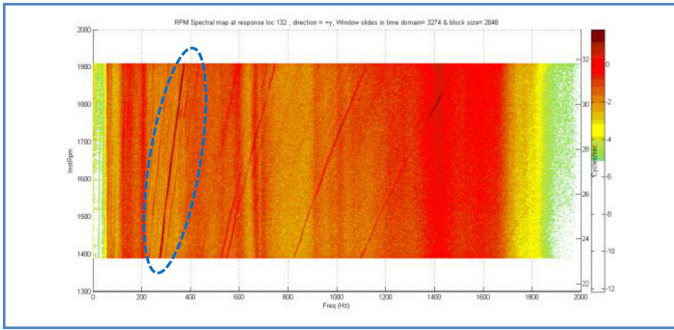


Figure 1. Overhead spectral map, loc 132, +Y dirn¹

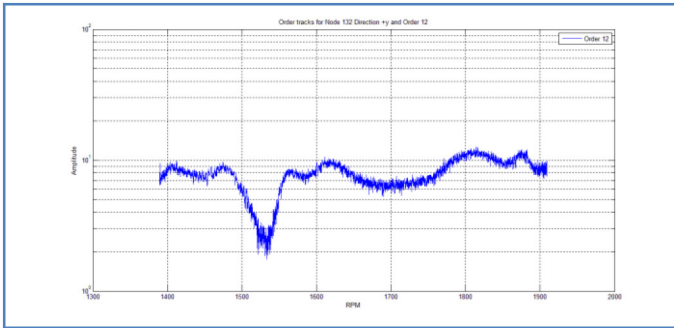


Figure 2. Kalman filter order track (wf=10,000), 12th order, loc 132, +Y dirn

If the amplitude range of this entire order history is below the intended target, it can be concluded that the dynamic behavior of this component is within the acceptable limits.

But, if instead of a uni-axial accelerometer, a tri-axial accelerometer is placed on the same point and the test is repeated with the same sweep rate and conditions, the comparison of order histories in the +X, +Y and +Z directions shows considerable activity in the RPM range mentioned above.

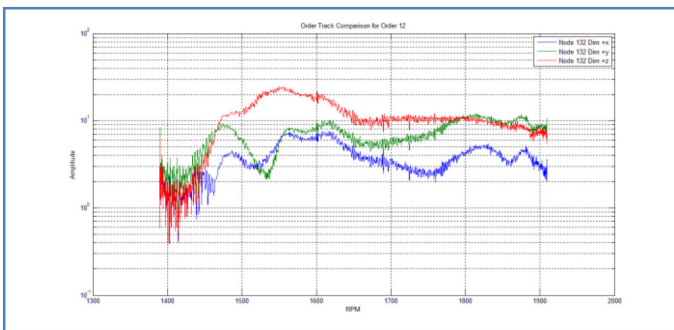


Figure 3. Comparison of order tracks from +X(red), +Y(green) and +Z(blue) dirns, 12th order, loc 132

Considering the peak observed in the 1500-1600 RPM range, for +Z direction, it is possible that the amplitude is above the acceptable limit and indeed further work is required to reduce the vibration response of the system at that location.

Cases similar to these are abundantly possible on a complex assembly. Like directional variability of a location on a system, it is also possible to have variability between different points on a component.

1. Magnitude scaling through-out the paper is in logarithmic format

BACKGROUND

RPM spectral maps are frequently used in analyzing data for rotating systems analysis. The calculated FFT (Fast Fourier Transform) spectrum of the measured data is plotted against the vibration frequency (X-axis) at each instantaneous RPM (Y-axis) [1]. Analyzing response spectrum or the auto-power spectrum at different speeds is important because often, a single RPM value is not enough to understand the mechanical signature of a defect or a dynamic signature of a component [14].

Using existing frequency domain data, calculated during spectral mapping, the generation of singular value maps is similar to that of a RPM spectral map.

For m sensors used on a machine, a typical response vector in the frequency domain, at each RPM, is a column vector of size $\{m \times 1\}$. The response vector is complex vector as each term is estimated via FFT.

$$R_{k,n} = \left(\begin{bmatrix} a_1 + i b_1 \\ a_2 + i b_2 \\ \vdots \\ a_m + i b_m \end{bmatrix}_{m \times 1} \right)_{k,n} \quad (1)$$

Where,

$R_{k,n}$ = response vector at the k^{th} RPM and n^{th} frequency of vibration

m = number of sensors used

a_j and b_j = cosine and sine fourier coefficients respectively, of the response for sensor j ,

i is the imaginary number.

The cross-power of this column vector $R_{k,n}$ is a square matrix of size $[m \times m]$, having auto-power terms in the diagonal and cross-power terms in the off-diagonal positions.

$$C_{k,n} = R_{k,n} \times R_{k,n}^H = \left(\begin{bmatrix} (a_1 + i b_1) \times (a_1 - i b_1) & \cdots & (a_1 + i b_1) \times (a_m - i b_m) \\ \vdots & \ddots & \vdots \\ (a_m + i b_m) \times (a_1 - i b_1) & \cdots & (a_m + i b_m) \times (a_m - i b_m) \end{bmatrix}_{m \times m} \right)_{k,n} \quad (2)$$

Where,

$R_{k,n}^H$ is the hermitian of the response vector $R_{k,n}$

For the cross-power matrix C at each RPM and each frequency of vibration, a singular value decomposition procedure can be performed.

$$[U]_{k,n} \times [\Sigma]_{k,n} \times [V]_{k,n}^H = SVD([C]_{k,n})$$

Where,

At each RPM k and frequency n .

$[U]$ = left singular matrix

$[V]$ = right singular matrix

$[\Sigma]$ = singular value diagonal matrix

$$[[\Sigma]_{k,n}]_{m \times m} = \left(\begin{bmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_m \end{bmatrix}_{m \times m} \right)_{k,n} \quad (4)$$

The singular values, σ_1 to σ_m will exist at each point in the RPM vs frequency map and have an associated magnitude. Singular values are real scalar values. Similar to a color map, or a spectral map, all m sets of singular values can be plotted in m maps.

As, singular values are arranged in descending order in the $[\Sigma]$ matrix, the first few singular values, which are significantly larger in magnitude than the lower ones, reveal the rank of the base $[C]$ matrix, hence indicating the magnitude of important information [2][16]. It can also be said that, the rank denotes the number of linearly independent characteristics present in the base matrix. With respect to vibration response analysis, the dominant response is captured by the largest (first) singular value, since the right and left vectors are unitary, i.e they are normalized to orthogonal with unit length [2].

Due to the retention of the dominant response (or dominant information in the base matrix), any random noise collectively present in the response vector, will not contribute to the dominant singular values that will be further used for analysis. Thus, the SVD approach can result in noise-reduction in rotating systems analysis when common DSP (Digital Signal Processing) averaging methods cannot be used due to the machine's slew rate.

Also, the base matrix being the cross-power matrix of the response vector, the entire dominant response of the sensor assembly is captured in the singular value map. The SVD procedure condenses the data generated by all possible cross-contributions.

Singular Value Decomposition and Vibration Analysis

Singular value decomposition has been used in Modal analysis and Rotating systems analysis to extract underlying structural behavior from large sets of measured data [2][3][9].

In modal parameter estimation, equation condensation transforms the measured data to match the number of unknowns in the parameter estimation algorithm using SVD. Similarly, coefficient condensation uses a transformation matrix resulting from a SVD procedure to reduce the number of physical co-ordinates to an approximate number of effective modal frequencies [2].

The CMIF function (Complex Mode Indicator Function) uses SVD of the Frequency Response Function (FRF) matrix at each spectral line to provide a set of singular values which can be plotted together. The peaking singular values, indicating the presence of a mode, denote the magnitude by which it is excited [3].

Blough, et al in 1999, developed a technique based on singular value decomposition, for rotating systems analysis, which enabled the independent operating shape determination based on order track measurements [6][9].

The decomposition is done by performing a SVD of the operating shapes estimated at each order at each RPM [6][9].

$$[U] \times [\Sigma] \times [V]^H = SVD(\{O_1\} \quad \{O_2\} \quad \dots \quad \{O_m\}) \quad (5)$$

Where,

$\{O_k\}$ = Operating shape for the k^{th} order at a particular rpm

The left singular vectors in the $[U]$ matrix, represent the set of linearly independent operating shapes corresponding to the singular values. The singular values at each rpm, will identify how well excited each linearly independent operating shape is at that rpm value.

The techniques described, throw light on the possibility of condensing large measured data sets or degrees of freedom into combined, representative models, using singular value decomposition.

This research attempts to use SVD to spatially 'average', measurement data on rotating machinery to result in a condensed virtual signal representative of the dominant response of the system.

EXAMPLE WITH INTERPRETATION

The following example is used to illustrate the application of singular value maps. The data was acquired on a leaf blower, driven by a variable AC motor (for speed sweeps), driving the single crank-piston assembly which drove the multi-blade rotor. A linear speed sweep from 0 to 3500 RPM was used.

A total of 8 accelerometers (PCB 100 mV/g) were placed on different locations on the system. Overhead RPM spectral maps, with a logarithmic magnitude scale (mV), are depicted in the figures below.

Considerable smearing at high RPM and frequency is visible from each map, although at different areas.

Using the singular value map concept explained previously, 8 separate SV maps can be plotted. The first two significant singular value maps are plotted below.

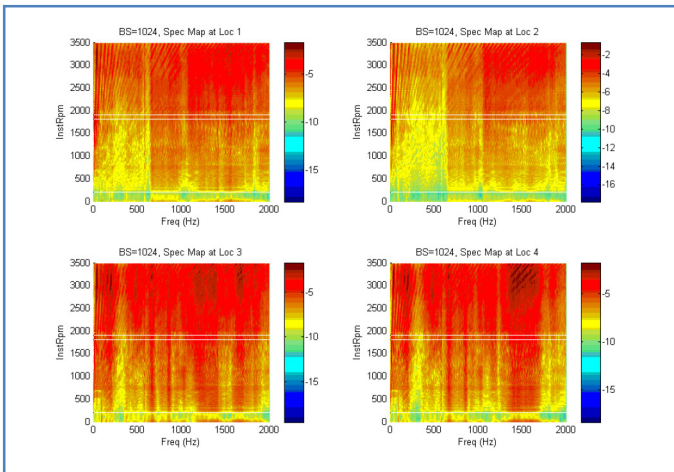


Figure 4. Overhead RPM spectral maps, loc 1 to 4, Leaf blower system

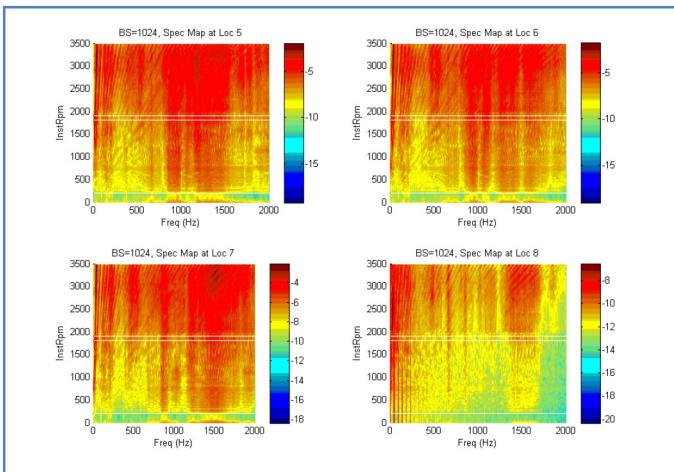
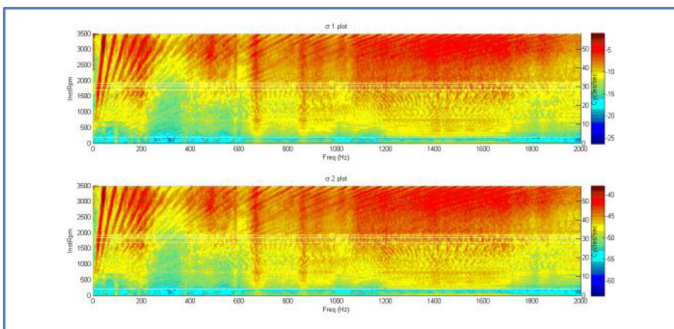


Figure 5. Overhead RPM spectral maps, loc 5 to 8, Leaf blower system

Figure 6. Singular value plots, σ_1 (top), σ_2 (bottom)

Noise Reduction

Except for the smearing effect which is observed at higher frequencies and higher RPMs in FFT based spectral maps (which is due to the spreading of the energy content of orders or harmonics over more spectral lines, because of their rapid change in frequency [14]) there is considerable reduction of measurement noise due to the retention of only important or major information, like order lines, by the singular value decomposition procedure on the cross-power matrices.

Noise reduction is possible because noise is not consistently present at the same frequency content in all sensors. One degree of freedom may have different susceptibility to noise at different conditions as compared with another measurement degree of freedom. This difference becomes advantageous during the singular value decomposition procedure, as the same noise information is not present in all sensors. On the other hand, the true mechanical signature of the machine, like harmonics related to number of blades, firing order or mechanical faults, are present throughout, if not at a constant amplitude. The contribution from these characteristics will hence be present at a significant magnitude in the cross power matrix and eventually would be depicted in the singular value plot, while noise is “averaged” out to produce a virtually noise-free spectral map.

Rank Information from Maximum Magnitude

The maximum magnitude of the σ_2 plot is to the order of 10^{-38} . The maximum magnitudes of the remaining significant singular values were found to be around this range as well. In total, it was shown from the singular value maps, that σ_1 map had a maximum value of the order of 10^{-1} while σ_2 to σ_5 had an order of magnitude around -38 to -40 . The remaining three singular value plots contained information to the order of -60 and beyond and hence can be considered insignificant. Only σ_1 and σ_2 are plotted above, as the remaining three plots were similar to the second singular value plot.

In this case, it can be said that the rank is 5 due to the presence of 5 significant singular values. However, the difference in magnitude between the first and second singular value is about 30 orders of magnitude. Hence, the effective rank is closer to 1.

In essence, it can be said that one virtual sensor is depicting the entire machine's inherent dominant dynamic response in one signal.

Virtual dominant sensor

Depending on the structure being analyzed, the singular value map concept can be applied on either: 1) Response data from one location, 3 principal directions or 2) Response data from different locations.

If the first case is analyzed, the single “virtual sensor” will be depicting the major contributions from the directional sensors (which acquire data via a tri-axial accelerometer). It can be also said that the virtual sensor will be “observing” the corresponding location in a complete sense, taking into consideration the major sources of vibration and dynamics from all three directions and filtering out, or “averaging” out the noise. Data which is significantly present for 2 or all sensors, but not in a considerable magnitude, will eventually be depicted in σ_2 plots or even the remaining ones.

In the second case, the virtual dominant sensor concept can be most useful when the structure or component is small enough, or in case of large sensor sets, data from only one direction is used. The procedure will be similar to equation or coefficient condensation in modal analysis. In this case, the spatial domain is being condensed to the