

Numerical Determination of Transfer Impedance for Perforates

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ABSTRACT

A simplified method to model perforated tubes in mufflers is the equivalent transfer impedance approach. Various empirical formulas that consider the porosity, hole diameter, wall thickness, and flow type have been proposed to date. They normally work very well under the conditions that the formulas are intended for. However, there are situations that the empirical formulas may not be able to cover. In this paper, we propose a simple BEM-based numerical procedure to determine the transfer impedance from a small perforate sample, and then send the transfer impedance to the muffler BEM model for analysis purposes. Numerical results are verified in three test cases.

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INTRODUCTION

Reactive mufflers used in the automotive industry usually contain perforated tubes to guide exhaust gas with a minimum impact on pressure drop, and at the same time, dissipate sound. Sometimes, a small section of perforation may also be used as the short neck passage to the Helmholtz resonator enclosing the perforated tube. Due to the large number of small holes on a typical perforated tube, it is not desirable to model the detailed geometry of every single hole in a complex muffler design. A simplified modeling approach is the "membrane analogy", in which the perforated tube is replaced by an equivalent membrane with a transfer impedance boundary condition that relates the pressure jump across the membrane to the normal particle velocity. Various empirical formulas based on experiments have been given in the literature for acoustical engineers to directly use the transfer impedance in their models [1, 2, 3, 4, 5]. Since many empirical formulas already consider the porosity, wall thickness, hole diameter, and even the flow effect, no detailed perforate geometry modeling is required in the muffler analysis process.

The boundary element method (BEM) has been successfully used in conjunction with the perforate empirical formulas to model mufflers with complex internal geometry [6, 7]. One major advantage of the BEM is that only the surface needs to be modeled. A perforated tube is replaced by a single layer of mesh at its mid surface. This is indeed consistent with the membrane analogy. The transfer impedance based on one of the empirical formulas is then applied as the boundary condition on the mid surface. In the direct mixed-body BEM [6, 7], elements used on the mid surface of the perforated tube are designated as the "P" elements. On the other hand, the BEM may also be used in a hard way if all the geometry details are to be modeled. In other words, both sides of a perforated tube can be meshed even though the tube is very thin, and each hole also has a small depth (which is the wall thickness of the tube) with boundary elements attached to its interior side. These boundary elements that cover all

physical surfaces of a perforated tube are no different than the boundary elements used on the chamber surface, inlet or outlet. In the direct mixed-body BEM, these elements are the so-called "R" (regular) elements. If such a direct and brute-force approach is adopted, no empirical formula is needed because the model simply relies on the Helmholtz equation and the BEM solver to run its course. If the holes are not too small and there is a minimum amount of flow, this direct approach may still work fine. However, the computational cost is extremely high due to the large number of elements involved. In addition, the Helmholtz equation does not include any viscous or flow effects. Therefore, the direct modeling approach will not work if the holes get too small, such as in the case of micro perforated panels (MPP).

Although the empirical formulas normally work very well under the conditions that they are intended for, there are always fuzzy situations that the empirical formulas may not be able to cover. Examples include non-circular holes, holes with louvers, perforate patterns with a mix of different types of holes, and holes with a varying diameter along the depth (wall thickness). In this paper, we propose a simple BEM-based technique to numerically "measure" the transfer impedance under those potential fuzzy situations. It is similar to a very recent viscous fluid mechanics model using the finite-element method (FEM) by Carbajo et al. [8] for micro perforated panels. Basically, we prepare a small sample of the perforate pattern, usually in the form of a rectangular plate, and place the sample in a "virtual impedance tube" with an anechoic termination applied to its end. The details of every single hole are fully modeled by the BEM. This is indeed the same brute-force approach mentioned in the previous paragraph except that the computational cost is cut to a minimum due to the small sample size. Once the transfer impedance is extracted from the small sample, it can be used in the muffler BEM model as if it were obtained from an empirical formula.

In this paper, we first illustrate the proposed numerical “measurement” technique. Three test cases are given to validate the proposed technique. The transmission loss (TL) results are compared to available experimental TL data, as well as numerical results based on the empirical formulas, the direct modeling, and the zero-thickness thin-body modeling. The zero-thickness thin-body modeling also models every single hole in the BEM but the tube is assumed to have no thickness. In the direct mixed-body BEM, the zero-thickness, non-perforate elements are designated as the “T” (thin-body) elements [6, 7]. The “T” elements are normally used to model extended inlet/outlet tubes, internal connecting tubes, flow plugs, and thin baffles. Wall thickness in those internal thin components is usually not important acoustically. However, it may play a crucial role in perforated tubes. The results from this study may shed some light on how these different types of modeling techniques should be used in the future.

NUMERICAL DETERMINATION OF TRANSFER IMPEDANCE

With reference to Fig. 1, a small perforate sample is placed inside a “virtual impedance tube” with an anechoic termination. The wall thickness t and the hole spacing d of the sample are fully modeled in the BEM. A true anechoic termination is difficult to achieve in the lab, but it is very easy to apply in the numerical model. A unit-amplitude velocity is prescribed at the inlet as the source to drive the virtual measurement system.

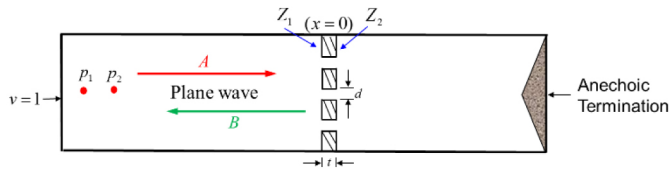


Figure 1. A perforate sample in a “virtual impedance tube”.

According to the plane-wave theory, the sound pressure p and the particle velocity u in the x direction (positive to the right) are

$$p = Ae^{-jkx} + Be^{jkx} \quad (1)$$

$$u = (1/\rho c)(Ae^{-jkx} - Be^{jkx}) \quad (2)$$

where A and B are the complex amplitudes of the incident and reflected waves, respectively, k is the wavenumber, ρ is the mean density, c is the speed of sound, and $j = \sqrt{-1}$. On the front face of the sample, the acoustical impedance is

$$Z_1 = \frac{p}{u} = \rho c(A+B)/(A-B) \quad (3)$$

if $x=0$ is set there. On the back side of the sample, the acoustical impedance is simply the characteristic impedance due to the anechoic termination. In other words,

$$Z_2 = \rho c \quad (4)$$

Therefore, the transfer impedance Z_{tr} of the perforate sample is

$$Z_{tr} = Z_1 - Z_2 = \rho c((A+B)/(A-B) - 1) \quad (5)$$

The two complex amplitudes, A and B , can be determined by placing two field points in the BEM model. Let p_1 and p_2 be the sound pressures of the two field points shown in Fig. 1. Use Eq. (1) twice to get

$$p_1 = Ae^{-jkx_1} + Be^{jkx_1} \quad (6)$$

$$p_2 = Ae^{-jkx_2} + Be^{jkx_2} \quad (7)$$

The matrix form of two equations above is

$$\begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} = \begin{bmatrix} e^{-jkx_1} & e^{jkx_1} \\ e^{-jkx_2} & e^{jkx_2} \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} \quad (8)$$

After that, the two unknowns A and B can be solved by a simple matrix inverse:

$$\begin{Bmatrix} A \\ B \end{Bmatrix} = \begin{bmatrix} e^{-jkx_1} & e^{jkx_1} \\ e^{-jkx_2} & e^{jkx_2} \end{bmatrix}^{-1} \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} \quad (9)$$

With A and B solved, the transfer impedance Z_{tr} of the perforate sample is calculated by Eq. (5). This transfer impedance can then be used later in the muffler BEM model that uses the same perforate pattern.

EMPIRICAL FORMULAS

There have been various empirical formulas available to date for calculating the transfer impedance of perforates under different conditions. In the following, we list the ones that we use in this study. None of these formulas listed below has the Mach number because our proposed method for comparison is based on the Helmholtz equation, which does not include any flow effect.

The simplest empirical formula is the classical formula given by Sullivan and Crocker in 1978 [1]. It uses the porosity σ (ratio of the open area to the total area) as the only input parameter in the formula. That is

$$\xi = (1/\rho c \sigma)(2.4 + j0.02f) \quad (10)$$

where $\xi = Z_{tr}/\rho c$ is the dimensionless transfer impedance, f is the frequency in Hz, σ is the porosity, and ρc is the characteristic impedance in Rayl. Sullivan and Crocker [1] also included a more detailed version that incorporates hole diameter d_h and wall thickness t in the formula. The more detailed version can also be found in Munjal's book [9]: