

Dynamic Analysis of Hydraulic Bushings with Measured Nonlinear Compliance Parameters

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ABSTRACT

Hydraulic bushings with amplitude sensitive and spectrally varying properties are commonly used in automotive suspension. However, scientific investigation of their dynamic properties has been mostly limited to linear system based theory, which cannot capture the significant amplitude dependence exhibited by the devices. This paper extends prior literature by introducing a nonlinear fluid compliance term for reduced-order bushing models. Quasi-linear models developed from step sine tests on an elastomeric test machine can predict amplitude dependence trends, but offer limited insight into the physics of the system. A bench experiment focusing on the compliance parameter isolated from other system properties yields additional understanding and a more precise characterization. Amplitude dependence predictions are improved in numerical simulations of the system while using the experimentally determined nonlinear compliance parameter, expanding the capabilities of reduced-order hydraulic bushing models.

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INTRODUCTION

Hydraulic bushings are widely used as compliance joints in automotive suspension because their unique, amplitude and frequency dependent dynamic properties are beneficial for both ride quality and vibration isolation [1-2]. Design features in these devices typically include pumping chambers with working fluid, flow channels, elastomeric structure, and stoppers-all of which may introduce nonlinearity into the system. As a result, hydraulic bushing configurations are often analyzed in terms of a "black box" [1, 2, 3]. The linear system theory could be used to characterize the device as a cross-point stiffness magnitude and loss factor, but only at a given displacement amplitude and preload. The transfer function method captures only the global behavior of the device and may not yield any insight regarding amplitude dependent behavior. The use of reduced-order models [4, 5, 6, 7, 8, 9, 10] to analyze hydraulic bushings offers useful simulation results and improved insight into the physics of the device, though prior work on such models has largely been limited to linear system theory [4, 5, 6, 7, 8]. Limited effort has focused on adding a single nonlinear element to more precisely describe the physics of a design feature [9, 10]. Feature-based linear or nonlinear models are often useful to designers, as the link may be clearer between design parameters and resulting properties than with a "black box" or detailed finite element model.

PROBLEM FORMULATION

The purpose of this paper is to develop feature-based models (based on reduced order dynamic system theory) which can predict unknown amplitude dependent dynamic stiffness behavior of hydraulic bushings. Model development in this paper assumes steady-state sinusoidal excitation at low frequencies (up to on the order of 50 Hz) and only one

device with a single fluid path (inertia track) is considered. The first objective is to use a reduced order model framework to create linear (I) and quasi-linear (II) models with empirical parameters for sinusoidal loading. Second, a method of measuring fluid compliance is introduced and nonlinear models (III A-C) are developed which include nonlinear fluid resistance and compliance elements. Finally, the effectiveness of these models to predict amplitude dependence in stiffness magnitude and loss angle spectra are evaluated. The schematic for a simplified reduced-order hydraulic bushing model is displayed in [Figure 1](#), showing all of the relevant parameters, and [Figure 2](#) shows measured static stiffness curve. Typical measured production bushing dynamic stiffness magnitude and loss angle curves are shown in [Figure 3](#). [Table 1](#) presents an overview of each model proposed in this paper.

LINEAR MODEL (I)

A Laplace domain representation of the system dynamics can be useful to characterize the spectral properties of the device. This assumes a linear time-invariant system and carries many assumptions with it, including the principle of superposition and a steady-state input. The dynamic system shown in [Figure 1](#) is characterized by the following transfer function in the Laplace domain (s), based on a reduced order linear system:

$$K_d(s) = \frac{F_T}{X}(s) = \frac{a_2 s^2 + a_1 s + a_0}{b_2 s^2 + b_1 s + b_0}, \quad (1-a)$$

$$a_2 = \left(k_r + \frac{A_1^2}{C_1} + \frac{A_2^2}{C_2} \right) I_i, \quad (1-b)$$

$$a_1 = \left(k_r + \frac{A_1^2}{C_1} + \frac{A_2^2}{C_2} \right) R_i, \quad (1-c)$$

$$a_0 = k_r \left(\frac{1}{C_1} + \frac{1}{C_2} \right) + \frac{(A_1 - A_2)^2}{C_1 C_2}, \quad (1-d)$$

$$b_2 = I, \quad (1-e)$$

$$b_1 = R, \quad (1-f)$$

$$b_0 = \frac{1}{C_1} + \frac{1}{C_2}, \quad (1-g)$$

where, K_d is dynamic stiffness, F_T is the transmitted force, X is the displacement amplitude, k_r is the rubber path stiffness, A_1 and A_2 are the effective pumping areas of each chamber, C_1 and C_2 are the fluid compliances of each chamber, I_i is the fluid inductance of the inertial track, R_i is the fluid resistance of the inertial track, and a_j and b_j represent transfer function coefficients of the numerator and denominator respectively. Damping associated with the rubber path is assumed to be insignificant. A frequency (f) domain representation is generated using the following relation, $s = i2\pi f$.

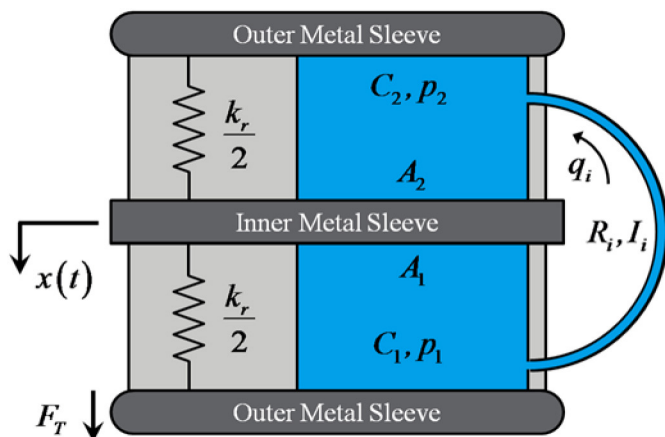


Figure 1. Schematic of generic hydraulic bushing with single inertia track. Here, k_r denotes the stiffness of the elastomeric structure, C_1 and C_2 denote the fluid compliances of the pumping chambers, A_1 and A_2 represent the effective pumping areas of each chamber, I_i is the fluid inductance of the inertia track, and R_i is associated fluid resistance. The system is excited through a displacement input on the inner sleeve, $x(t)$, and the output is the transmitted force through the outer sleeve, F_T . Additional state variables for the fluid system include p_1 and p_2 , which are the pressures in each chamber, and q_i which is the volume flow rate in the inertia track.

In previous work [4, 5, 6, 7, 8], linear model parameters have been established through a combination of direct calculation and empirical estimation. The static stiffness curve, constitutive equations, and fluid mechanics principles of fluid resistance (related to viscosity) produce most of the parameters, but others are heavily geometrically dependent, and must therefore be estimated given the specific materials and configuration of a particular bushing design.

Furthermore, since the form of the transfer function has been specified, a curve-fitting procedure may be used to estimate the more difficult parameters, such as fluid compliance, C , fluid resistance, R , and effective pumping area, A . For this analysis, some simplifications are made based on the assumption of symmetry in the bushing, i.e. that $C_1 = C_2 = C$, $A_1 = A_2 = A$. A nonlinear constrained optimization using an interior-point algorithm served to optimize the coefficients of the normalized transfer function to minimize the error between the curve-fit and the measured dynamic stiffness curve. The cost function, E , is defined as the sum of the magnitude of error at each frequency, f_j .

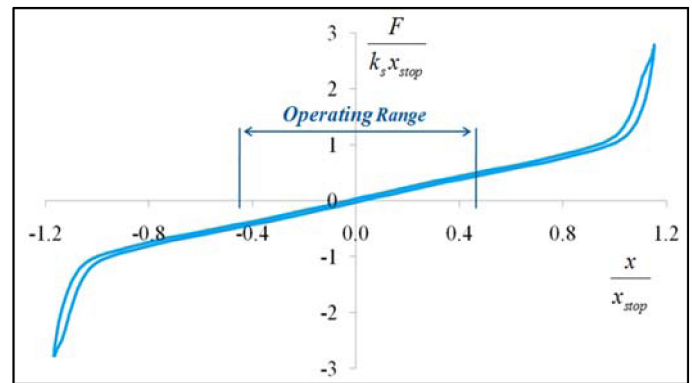


Figure 2. Static stiffness curve of hydraulic bushing example. The operating range of the bushing is within the linear region, but a nonlinear stopper region also exists for motion control. Here, the displacement is normalized by the displacement at which the stopper engages, and the applied force is normalized by the linearized static stiffness and the stopper engagement displacement.

$$E = \sum_j \left\| \tilde{K}_{d,model}(f_j) - \tilde{K}_{d,measured}(f_j) \right\|^2. \quad (2)$$

The algorithm adjusts a set of parameters in order to minimize E . Choosing the system parameters, C , R , I , and A yields very inaccurate results, presumably due to the extreme range of their values. Choosing the coefficients of the transfer function has proved to be a more effective method. Each coefficient must be positive to make physical sense, so a constraint is added to the algorithm: $a_j, b_j > 0$ for all j . The optimization algorithm requires an initial guess, so one must be iteratively supplied until order-of-magnitude transfer function coefficients are found. When the algorithm has optimized all six coefficients, an excellent approximation of the measured dynamic stiffness results. Figure 4 shows the results of this procedure. However, assuming symmetry in the hydraulic bushings, only four unknowns exist in the theoretical model. This means that two equations must be eliminated to fit the system dynamics to the transfer function fit. This is accomplished by averaging the redundant coefficient equations to produce the model parameters with minimal error introduced.

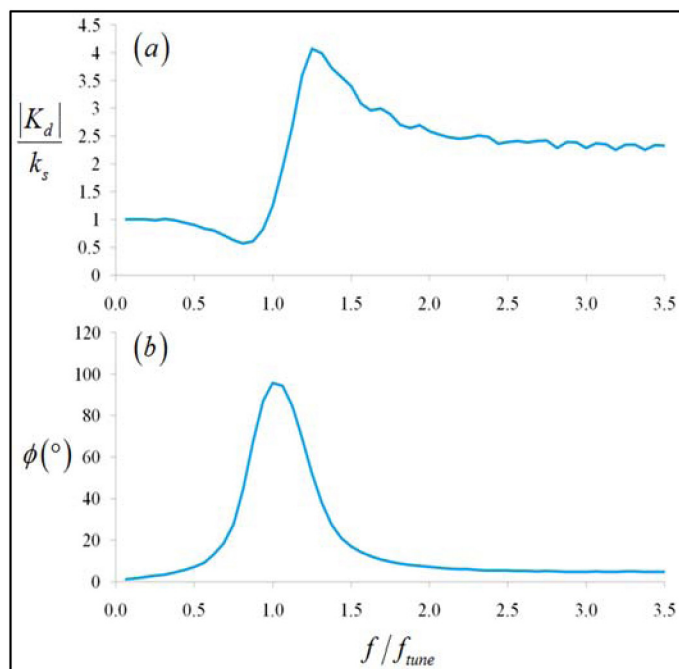


Figure 3. Dynamic stiffness spectra for an example hydraulic bushing, including (a) stiffness magnitude and (b) loss angle. Here, the dynamic stiffness magnitude is normalized by the static stiffness, k_s , and the frequency is normalized by the frequency at which the loss angle is maximized for a 0.1 mm excitation amplitude, f_{tune} . All excitation amplitudes in this paper are peak-to-peak.

Table 1. Hydraulic bushing models.

Model Designation	Model Type	Parameter(s)	Method
I	Linear	Empirical	Curve-fit measured data
II	Quasi-linear	Empirical	Curve-fit measured data
III-A	Nonlinear	Empirical/Theoretical	Theoretical resistance formulation
III-B	Nonlinear	Empirical/Theoretical	Theoretical resistance + Compliance formulation I from bench experiment
III-C	Nonlinear	Empirical/Theoretical	Theoretical resistance + Compliance formulation II from bench experiment

Linear system models with empirical parameters produced by this method may offer good approximations of hydraulic bushing dynamic properties at a given amplitude and preload. Nevertheless, the model will not be effective at other amplitudes, and still less accurate for other types of inputs which may have a larger dynamic range [10]. Linear models (say, based on a 0.1 mm amplitude test) should always produce the same dynamic stiffness curve regardless of excitation amplitude. This demonstrates the need for more advanced models which may predict amplitude dependent results.

QUASI-LINEAR MODEL (II)

Quasi-linear models are developed by a similar method. However, unlike the linear model, the quasi-linear model employs amplitude dependent parameters. Figure 5 shows the amplitude dependence of a production hydraulic bushing. There are significant differences in

peak loss angle and dynamic stiffness amplification between 0.1 and 1.0 mm (peak to peak) excitation amplitudes, although the tuning frequency at which the loss angle is maximized only varies slightly.

Curve-fitting the model parameters to each measured spectrum produces amplitude-dependent bushing properties. Table 2 shows the change in each parameter for three excitation amplitudes. In these results, the inertance remains constant. This is because the mass effects of the fluid should not be influenced by frequency dependent behavior up to about 50 Hz, so a theoretical value of inertance is used based on turbulent flow assumption [12]. Physical intuition suggests that the fluid resistance would increase with increased amplitude, since more fluid is pumped between the chambers. Analysis indicates that the resistance parameter is most affected by a variation in amplitude, and this is why previous work focused on this topic [10]. The compliance term varies slightly, but the effective pumping area increases by only five percent. These two parameters work together as there is interaction between the structural and fluid system; accordingly, the results are very sensitive to changes in such parameters. Finally, the rubber path stiffness drops off by about five percent, which is relatively small since the model is less sensitive to this parameter.

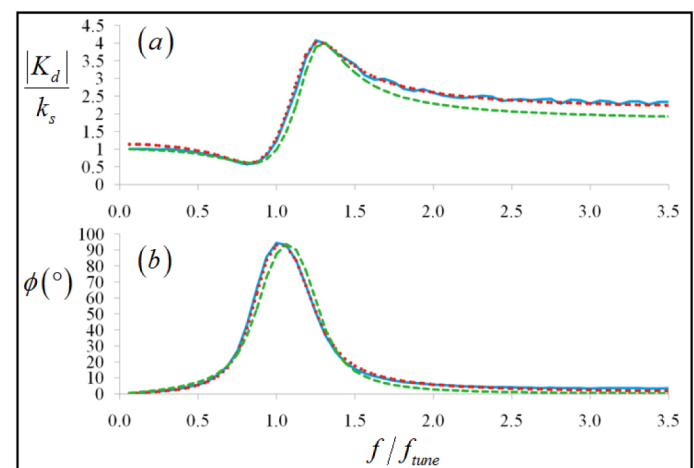


Figure 4. Verification of identified parameters for linear model I. (a) Dynamic stiffness magnitude and (b) loss angle. Key: — -measured stiffness; - transfer function coefficient fit; --- - physical parameter fit. Using the extracted parameters produces a good approximation of the measured stiffness, while just using the form of the transfer function with empirical coefficients can improve the accuracy.

Figure 6 compares the measured dynamic stiffness curves and quasi-linear model for three amplitudes, showing general agreement in both stiffness magnitude and loss angle at each amplitude. Nevertheless, the quasi-linear model is empirical in nature and provides little insight into the physics that would produce such dynamic properties. Additionally, development of quasi-linear models requires complete dynamic stiffness data at every amplitude of interest. Further, they cannot predict responses to non-sinusoidal input since the parameters (by definition) are defined for specific sinusoidal amplitudes. A more detailed investigation is therefore necessary to understand the nature and extent of the nonlinearity.