

# The Jet Fuel Hydrodynamic Cavitation Bubble Size with Cavitation Power and Energy from Rayleigh-Plesset Equation

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## ABSTRACT

Cavitation erosion in aircraft engine and control systems is a major concern in hydrodynamic power units. In developing turbulent flow of low pressure and high velocities, a certain amount of cavitation erosion is not unusual. Cavitation can occur with the presence of fuel vapor or air bubbles dissolved in the fuel tank that are transported through the system. Cavitation erosion is caused by collapse of the bubble, which occurs violently and creates a pressure shock wave of fluid. Striking a solid surface, the shock wave can cause progressive damage if it persists. A kinetic cavitation power rate is developed to make a meaningful estimation of the cavitation erosion rate theoretically, which then can be validated with laboratory experiments. Theoretically, we manipulate parameters such as bubble size, collapse pressure, and energy for a given fuel system design, finding variation within each component of the system. However, cavitation erosion rates vary wildly even when relative developments and comparisons are made. These variations are attributed to the simplicity of governing equations, boundary condition settings, and bubble sizing and geometry assumption differences of given derivation formulations. The jet fuel's bubble size, collapsing pressure, and surrounding fluid pressure, influenced by fuel properties due to temperature variation, appear to be the key factors in cavitation power and energy when predicting component life and degradation of efficiency in engine control systems.

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## INTRODUCTION

Cavitation is a very important phenomenon, which can highly influence the performance of a hydraulic machine. The occurrence of bubbles in aerospace machinery is innumerable and their role in aircraft engine and control systems is one of fundamental importance. The incidence of cavitation in an aircraft engine and control system can lead to unexpected degradation in engine system performance and/or damage to fuel system components. The cavitation damage can range from a relatively minor amount of pitting after years of service to catastrophic failure in a relatively short period of time. Cavitation is a two stage phenomena with liquid flow. The first stage is the formation and growth of vapor (or gas) bubbles in the liquid as the fluid passes through the trim, and the pressure is reduced below the fluid's vapor pressure. The second stage is the collapse of the vapor bubbles as the fluid passes through the vena contracta, and the surrounding pressure recovers and increases above the vapor pressure. The collapsing bubble is particularly important because of the noise and material damage that can be caused by the bubble's high

collapsing velocity, pressure, shock wave, and temperature. The dynamics of jet fuel bubbles are interesting and varied, and can differ greatly from other fluids. One can then expect the behavior of bubbles within an aviation kerosene fluid to differ drastically from a Newtonian fluid, like water. This study was aimed at gaining a better understanding of the aspects of jet fuel cavitation power and energy in aircraft fuel systems which are different than other industries' applications. The jet fuels in modern commercial aircraft engine control systems are often characterized by temperature, normally between  $-65^{\circ}\text{F}$  to  $300^{\circ}\text{F}$  ( $-54^{\circ}\text{C}$  to  $149^{\circ}\text{C}$ ), and pressure which can be up to 5000 psia (34.5 MPa). Such wide ranges of temperature and working pressure application can lead to a wide range of fuel property and impact to the cavitation power and energy. The prediction of the rapidly collapsing bubble dynamic cavitation power and energy from jet fuels can then be used for the assessment of cavitation erosion in the hydraulic machine in order to determine the point of best efficiency, or when the erosion is caused by collapse of individual cavitation bubbles without strong interaction.

The formation of individual bubbles and the subsequent development of bubble clouds are directly related to reductions in pressure to a critical value, which in turn is associated with dynamic effects. The importance of bubble dynamics and cavitation has been a subject of intense research for decades. The first resolute theoretical study of bubble dynamic cavitation was started by Lord Rayleigh [1] at beginning of last century in 1917. A large number of publications have followed in subsequent decades. Plesset [2] derived a Rayleigh-Plesset equation from the Navier-Stokes equations under the assumption that spherical symmetry add surface tension and viscosity from Rayleigh[1]. Keller and Kolodner [3] proposed the modified equations neglecting thermal, viscous, or surface tension effects. Benjamin and Ellis [4] observed a more complex dynamic occurrence during collapse near a wall in which the bubble no longer remains spherical. W. Lauterborn and H. Bolle [5] and A. Shima, K. Takayama, Y. Tomita, and N. Miura [6] investigated jet velocities during bubble collapse, which can typically reach 100 m/s. A. Philipp and W. Lauterborn [7] found that following jet impact, the subsequent toroidal dynamics can produce large pressure shock waves, which also contribute to cavitation damage. There are several cavitation and bubble dynamics text books such as C. Brenner [8] and H. Kim & G. Chahine [9] and many research published technical papers such as [10, 11, 12, 13]. Yuri T. Didenko & Kenneth S. Suslick [14] observed that during a single bubble collapse less than 1 millionth of the bubble's energy is converted into light and a thousand times more energy goes into chemical reactions. But the largest part of the sonic energy is converted into mechanical energy, causing shock waves and motion in the surrounding liquid. Theoretical and numerical studies are, for the most part, restricted to spherical bubble dynamics and the solution of the governing non-linear Ordinary Differential Equation (ODE). In reality, bubbles rarely remain spherical, which makes the analytical solution and experiments more difficult to validate. While a multitude of numerical methods have been developed to simulate non-spherical Newtonian dynamics, numerical studies into non-spherical viscoelastic dynamics are sadly lacking. Therefore, the bubble dynamic process which receives the greatest development in advanced Computational Fluid Dynamic (CFD) is arguably the most fascinating. However, the CFD analysis accuracy is restricted by the complexity of governing equation algorithms, meshing methodologies as Eulerian fixed mesh or Lagrangian moving mesh techniques, meshing grid size, finite time step, initial condition of non-spherical bubble sizing, and geometric variation in near wall bubble collapse.

More recently, the size of the initial cavitation nuclei has been predicted empirically using the acoustic spectrometer and photography by G. L. Chahine, K. M. Kalumuck, J-Y Cheng and G. S. Frederick [15]. Also the optical method by Ming Vang Su, Douglas Todoroff, and John Cartmill [16] has been developed to estimate the energy dissipated during the bubble collapse. These molecular dynamics observations provide ways to model the interior boundary of the bubble and couple the gas dynamics with the equations governing the bubble size, wall velocity, and collapsing pressure. But these methods are varied by the laboratory systems of observing bubble size range, sampling volume, and the method of performing automatic data analysis. Nevertheless, the acoustic spectrometer

seems to be the most suitable method for measuring bubble density in the field, especially for long-term monitoring. While theoretical work has been conducted to understand how a bubble will respond to an external force, the internal dynamics of the cavitation power and energy can be integrated and estimated.

Most of the past research and technical papers have focused on using water as the fluid. There are fewer described mathematical models designed to provide efficient illustrations of the jet fuel cavitation erosion power potential using numerical modeling with a single known sized bubble. A study of diesel fuel atomization based on cavitation bubble collapse energy was presented by A. Nishmura and D.N. Assanis [17] regarding fuel spray development in direct injection diesel engines that demonstrated a mean bubble radius of 25 microns for all of their cases. I. E. Dorofeeva et. al. [18] presented experimental investigations and models of JP-8 fuel in a converging-diverging nozzle to capture the pressure drop, distribution, and shock location.

The behavior of bubbles in jet fuel is different from other types of fluids and little research has been conducted using jet fuel as the fluid. The fuel bubble dynamics are characterized by fuel properties and the damped oscillation of bubble radius and time. The bubble dynamics and near wall bubble collapse are coupled in solid material cavitation erosion damage. We first study spherical bubbles in a jet fuel fluid regime and how various modeling choices and assumptions acting upon the evolution of the bubble collapse as well as the state of the gas particle system, in the hopes of providing a more accurate description of jet fuel bubble collapse power and energy. The complexity of near wall and non-spherical bubble collapse is not presented here and can be a topic for future research.

## BUBBLE DYNAMICS AND GOVERNING EQUATIONS

Bubble dynamics is extremely difficult to model in a quantitative analysis of the fluid associated with multi-bubble cavitation. Unknown parameters include the number of active bubbles, the acoustic pressure acting on each bubble, initiation of bubble sizes, and the interaction between bubble fluid and thermal properties. In the interest of gaining more crucial insights into jet fuel bubble dynamics, the simplest bubble growth equation is introduced and extended to complex nonlinear differential governing equations to simulate bubble dynamics. Here we first demonstrate the existence of cavitation within a single cavitation bubble, and quantify the sources of energy dissipation during a bubble collapse. The homogeneous bubble growth and dynamics of collapse were examined. The free body diagram of spherical bubble net force  $F$  of bubble balance pressure vs. surface tension and radius is derived in [Figure 1](#). The behavior of a single bubble in an infinite domain of liquid and uniform temperature is assumed containing only vapor in which the interior pressure  $P_B$  will be the saturated vapor pressure  $P_v$ . The exterior liquid pressure can be a step function pressure, where starting pressure has to be less than  $P_v$  in order to produce equilibrium conditions. This spherically symmetric force balancing ( $F=0$ ) situation provides the simplest case that is amenable to analysis as derived in [Equation-1](#).

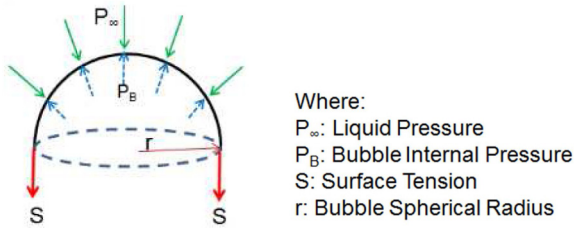


Figure 1. Bubble surface for balance free body diagram

$$F = 2\pi r S + (P_{\infty} - P_B)\pi r^2 = 0$$

$$P_B - P_{\infty} = \frac{2S}{r}$$

Eq.-(1)

Following the simplest spherical bubble force balancing, one of the recognized bubble dynamics equation derived from Rayleigh-Plesset [2] is exploited and solved with numerical methods to investigate the problem of traveling cavitation bubbles in a jet fuel fluid. The bubble dynamics model of Rayleigh-Plesset (R-P) equation is shown in Equation-(2). The neglect of viscosity and surface tension is shown in Equation (3).

$$\frac{P_B(t) - P_{\infty}(t)}{\rho_L} = R \frac{d^2 R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 + \frac{4\nu_L}{R} \frac{dR}{dt} + \frac{2S}{\rho_L R}$$

Eq.-(2)

Where:

$P_B(t)$  is the uniform pressure within the bubble,  
 $P_{\infty}(t)$  is the external pressure infinitely far from the bubble,  
 $\rho_L$  is the constant density of the surrounding liquid,  
 $R(t)$  is the radius of the bubble at time (t),  
 $S$  is the surface tension of the bubble  
 $\nu_L$  is the constant kinematic viscosity of the surrounding liquid,

$$\frac{P_B(t) - P_{\infty}(t)}{\rho_L} = R \frac{d^2 R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2$$

Eq.-(3)

A cavitation bubble generally contains a mixture of gases, which we simplify here by dividing into two species - vapor and gas. The pressure inside the bubble requires subsequent analytical law for parameter dependences including the bubble content effect listed in Equation (4).

$$P_B(t) = P_v(t) + P_{G0} \left( \frac{T_B}{T_{\infty}} \right) \left( \frac{R_0}{R} \right)^3$$

Eq.-(4)

Where:

$P_v$ ,  $P_{G0}$  and  $R_0$  are the vapor and gas pressure and bubble radius at the initial reference state,  
 $T_B$  and  $T_{\infty}$  are the bubble and surrounding temperature.

The Rayleigh-Plesset Equation-(2) is a second order, nonlinear Ordinary Differential Equation (ODE) that can be solved for time-varying bubble radius  $R(t)$  in a straightforward manner using

standard numerical techniques by MathCad[19]. Given surrounding pressure  $P_{\infty}(t)$ , this equation can be solved to find  $R(t)$  provided with a known  $P_B(t)$  and initial conditions of bubble radius  $R(0)=R_0$  and velocity  $dR(0)/dt=0$ . The numerical solutions of R-P Equation-(2) can also be integrated to the case of applying step function in external pressure  $P_{\infty}(t)$  to simulate a known real fluid system pressure variation. In addition to the Rayleigh-Plesset equation, the bubble contents shown in Equation-(4) assumes that the bubble contains some quantity of contaminant gas whose partial pressure is  $P_{G0}$  at some reference size,  $R_0$ , and surrounding temperature,  $T_{\infty}$ . In this study, the vapor bubble contain is assumed to be a homogeneously distributed vapor, neglecting the gas contained in a uniform temperature  $P_B=P_v$ , the bubble pressure at wall  $P(R)$  can be simplified to Equation-(5) from P-R[2].

$$P(R) = P_B - \frac{4\nu_L}{R} \frac{dR}{dt} + \frac{2S}{\rho_L R}$$

Eq.-(5)

### Cavitation Power and Energy

Energy focusing during cavitation collapse is most concisely illustrated by a R-P [2] model to provide direct insight into the hydrodynamic, forced growth of the nuclei leading to the inception of cavitation implosion. We assume a bubble which is initially at rest at radius  $R=R_0$ . We neglect the small quantity of gas  $P_G$  at temperature  $T_0$ . The bubble is then subjected to a step increase in far-field drive pressure to some large value  $P_{\infty} \gg P_v$ . Since the pressure inside the bubble remains negligible until it is very small, the total kinetic energy of the collapse bubble will be equal to the work done on the collapsing cavity by the far-field pressure. The above single bubble model accounts for cavitation bubble kinetic energy, power, and aerodynamic forces during the bubble's growth and collapse on the liquid core. In the hypothetical flow of a liquid that can't withstand any surface tension, vapor bubbles appear instantaneously when the surrounding pressure  $P_{\infty}$  reaches the true vapor pressure  $P_v$ . Consider the nucleation of a free stream nucleus that travels along the streamline at time  $t$  as shown in Figure-2.

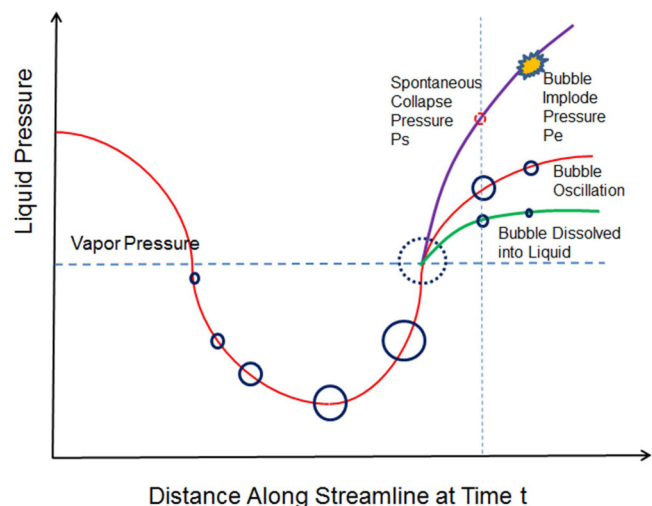


Figure-2. Bubble growth and collapse vs distance along streamline at time t and bubble pressure.