

Cessna Citation X Engine Model Identification from Flight Tests

Georges Ghazi, Ruxandra Botez, and Joseph Messi Achigui
École de Technologie Supérieure

ABSTRACT

During aircraft development, mathematical models are elaborated from our knowledge of fundamental physical laws. Those models are used to gain knowledge in order to make decisions in all development stages. Since engine model is one of the most important items in aircraft simulation, the aviation industry has recently developed a high interest on them. With the power capacities development in the last years, numerical simulations have been widely used for predicting engine response. In this paper, a methodology to identify an engine model from flight tests is presented. A Cessna Citation X Level D Flight Simulator designed and manufactured by CAE Inc. was used to sample the engine thrust force data. More than 500 flight tests were made for different flight conditions expressed in Mach numbers ($M = 0$ to $M = 0.9$), altitudes ($h = 0$ ft to $h = 50,000$ ft) and different throttle positions (idle to maximum). The engine mathematical model was developed from analysis and comparison of several existing engine models in the literature. Two estimation algorithms were created to identify the parameters defining the model. To validate the model, simulations were performed and compared. Results have shown that the obtained model was accurate and could be further used to estimate the engine thrust forces for any flight conditions.

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INTRODUCTION

Since turbofan engine modeling and validation are the most complex items in aircraft simulation, the aviation industry has recently developed a high interest on them. Turbofan engine models are used to gain knowledge in order to make better decisions within the development stages. In the early design phase, they are used to estimate the airplane behavior and performance [1]. Subsequently, they are used to develop flight controls that will maintain the aircraft speeds during the climb, cruise and descent phases. Engine models are therefore vital for the development of a new aircraft. Nevertheless, in most cases, not all engine data are available for the aircraft preliminary design. Therefore, it is very difficult to obtain accurate models to estimate the engine performance. For this reason, numerous researchers have focused their work to develop generic engine models from minimum available data.

With the power capacities development in the last years, numerical simulations have been widely used for predicting engine response. These simulations have allowed a deeper engine dynamics and steady state behavior understanding [1, 2]. Based on these observations, several authors have developed generic models that predict with high accuracy the engine performance. These models are used to estimate the aircraft behavior [3], to predict the thrust force for each flight condition [4] or to create performance databases (PDBs) [5, 6, 7, 8]. These models may be complex, based on the engine aerodynamics and thermodynamics analysis cycle; or simplistic, adapted to a specific flight phase [9, 10].

Yadav *et al.* in [11] presented an aero-thermodynamic model for turbofan engine digital simulation. The authors used the state variable and control volume approaches to develop an aero-thermodynamic model for a twin-spool mixed flow turbofan engine. The proposed engine model was capable of predicting its performance at the early design stage. Simulation results for operating lines of the fan, duct and the main compressor were compared with results obtained using the software Gas Turbine Simulation Program (GSP). In general, good results were obtained.

Similarly, Rory *et al.* in [12] also used thermodynamics equations to propose new techniques to develop a dynamic turbofan engine model. The fundamental thermodynamics equations were used to model each component of the engine separately in Matlab/Simulink (The MathWorks, Inc.), and then they were combined to investigate its behavior. Special attention was given to improve the physical model by eliminating algebraic constraints. The new approach proposed by these authors has reduced the simulation time.

May *et al.* [13] at NASA Glenn Research Center have developed generic high-bypass ratio, twin-spool commercial turbofan engine model called C-MAPSS40k. The proposed simulation model was able to predict the behavior and performance of many turbofan engines situated in the 40,000 lb thrust class. It was built based on dynamic flight test data and engine simulation models developed in-house. In addition to providing a high-fidelity simulation, the model also included a realistic engine controller that was

representative of those used in industry. However, although this model may be used to simulate the engine dynamic behavior, its main goal was to enable new control system development.

These examples have shown the extraordinary progress that has been made in numerical modeling in recent years. In addition to providing a good accuracy, the models presented in the previous paragraphs were able to give complete information about the engine state variables. However, regarding research in the aerospace domain, such as aircraft stability analysis [14, 15], trajectory optimization [16, 17, 18, 19, 20, 21, 22] or aerodynamic model identification from flight tests [23, 24, 25], only the engine thrust force and fuel flow are important. Thus, the proposed models by NASA, Rory and Yadav were found to be inappropriate or too much complex for these studies. For this reason, several authors have developed different techniques to estimate the maximum thrust force of a turbofan from simple equations or from existing models.

Wanner in [9] and Mattingly in [10] have presented different empirical equations that were able to predict the maximum thrust force for various altitudes and Mach numbers. These models assumed that the maximum thrust force is mainly function of the air density and Mach number. Contrary to Mattingly, the model proposed by Wanner also included the throttle position. According to the author, the thrust force variation due to throttle position was assumed to be a linear function. Although these models were not very accurate, they could be used for a preliminary aircraft performance analysis.

Matlab/Simulink (TheMathWorks, Inc.) team has proposed a generic model for high bypass ratio turbofan. The model estimate the thrust force and the fuel flow at a specific throttle position, Mach number and altitude. It took into account the maximum static thrust, the specific fuel consumption and the fastest engine time constant at sea level. The thrust and the fuel flow were computed using unitless heuristic parameters that are mainly function of throttle position and Mach number. As the model proposed by Wanner, the thrust variation due to the throttle position was also assumed to be linear.

Rodriguez and Botez presented in [26] a new generic model that was able to predict the maximum turbofan engine thrust within the whole flight envelope. The main advantage of this model was the consideration of the bypass ratio. Contrary to the previous models, the one proposed by Rodriguez included the bypass ratio in its equations. However, the author did not take into account the variation of the thrust due to the throttle position. A linear variation as proposed by Wanner can be used in order to extend the use of the model for any flight phase.

Gong et Chan assumed in [27] that the engine thrust behavior for all flight conditions remained constant for two engine of the same class. According to the authors, for a specific thrust rating, the corrected thrust (defined as the ratio between the net thrust and the pressure ratio) was a function of Mach number. Thus, using an existing engine model, it was possible to predict any engine performance by scaling the existing one. The methodology was used in order to obtain more accurate climb trajectories, and promising results were obtained. In addition to provide good results, such an engine model could be scaled in order to better understand or predict other engine model behavior.

The main objective of this paper is to provide a turbofan generic engine model that is efficient, flexible and responsive for the Laboratory of Applied Research in Active Control, Avionics and Aero-Servo-Elasticity (LARCASE) team. This model can then be used to help the researchers to improve their algorithms for trajectories optimization [16, 17, 18, 19, 20, 21, 22], parameters identification from flight tests [28, 29, 30, 31, 32] and control design system [33, 34, 35, 36].

The proposed model was formulated using several generic engine models data available in the literature. The parameters of the model were estimated using System Identification Method from flight tests performed on a Cessna Citation X Level D Aircraft Research Aircraft Flight Simulator (RAFS) designed and manufactured by CAE Inc. According to the Federal Aviation Administration (FAA, AC 120-40B), the level D is the highest certification level that can be delivered by the certification authorities for the flight dynamics modeling.



Figure 1. Level D Cessna Citation X Flight Simulator

This paper is arranged as follows. First, a methodology to obtain a generic model that can estimate the thrust force for any flight condition is presented. Secondly, the identification method used to tune the different parameters of the proposed model is described. Finally, simulation results are discussed and presented.

SYSTEM IDENTIFICATION METHOD

System identification methods have been widely used in the aerospace industry [37-38]. System identification can be defined according to Zadeh as in [39]:

“System identification is the determination, on the basis of observation of input and output, of a system within a specified class of system to which the system under test is equivalent.”

In others words, “system identification” includes the model structure definition and the estimation of parameters defining the model, as shown is [Figure 2](#).

As seen in [Figure 2](#), the identification process is divided in two steps. In a first step, a *mathematical model* that described the studied system (i.e. engine) was defined. Then, in a second step, an *estimation algorithm* that tunes the parameters defining the model was developed.

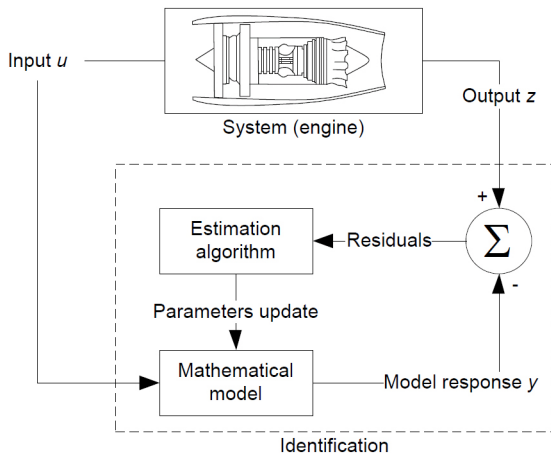


Figure 2. System identification principles

ENGINE MATHEMATICAL MODEL

The goal of this section is to present a mathematical engine model that can be used to estimate its thrust force for any flight conditions. A block diagram for the engine with its inputs and outputs is shown in Figure 3.

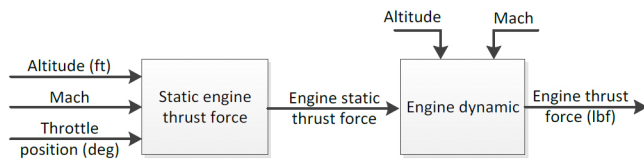


Figure 3. Engine block diagram

As illustrated, the engine model is divided into two sub-models. The first sub-model relates to the *engine static state*, while the second sub-model refers to the engine dynamics (i.e. transient state). Each sub-model is presented separately in the two following sections.

Static Engine Model

In this section, several models available in the literature are presented and compared against experimental data obtained from the Citation X level D flight simulator. This first analysis aims to determine which model gives the most accurate results.

Existing Static Models

Wanner model. Wanner assumed in [9] that the engine *thrust ratio*, defined as the maximum thrust T_{max} to T_0 ratio, can be defined as follows:

$$\frac{T_{max}(h, M)}{T_0} = k_f \rho(h) [aM]^{\lambda_f} \quad (1)$$

where $T_{max}(h, M)$ is the maximum thrust force at the current altitude h and Mach number M , $T_0 = T_{max}(0, 0)$ is the maximum thrust force at sea level and at zero speed, k_f is a constant, ρ is the air density at the current altitude, a is the speed of sound at the current altitude, and λ_f is a parameter that depends of the engine class.

Aerospace model. The aerospace model is one of the most commonly used models in the aerospace industry [40]. This model assumed that engine thrust ratio can be expressed as follows:

$$\frac{T_{max}(h, M)}{T_0} = \frac{\rho(h)}{\rho_0} \left[1 - M + \frac{M^2}{2} \right] k \quad (2)$$

where ρ_0 is the air density at the sea-level and k is the ratio of installed thrust to uninstalled thrust. The parameter k is optional and allows the consideration of the engine performance losses due to engine installation on the aircraft fuselage.

However, as all the other models presented in this section do not take into account these losses, the parameter k is assumed to be equal to 1.

Mattingly model. According to Mattingly in [10], the engine thrust ratio can be defined as follows:

$$\frac{T_{max}(h, M)}{T_0} = \left[\frac{\rho(h)}{\rho_0} \right]^{0.6} \left[0.568 + \frac{(1.2 - M)^3}{4} \right] \quad (3)$$

The model proposed by Mattingly is similar to the aerospace model. Both models assume the thrust falling due to altitude (i.e. air density) and Mach number effects are completely dissociated. The main difference of this model with respect to other models is the order of the polynomials in Mach.

Rodriguez model. Rodriguez assumed that the engine thrust ratio can be formulated as follows [26]:

$$\frac{T_{max}(h, M)}{T_0} = \frac{\rho(h)}{\rho_0} n(R) \times [k_1 + k_2 R + (k_3 + k_4 R) M] \quad (4)$$

where n is a corrective factor which depends on the bypass ratio R , and $k_i, i = \{1, 2, 3, 4\}$ are experimental corrective factors. These factors are expressed as functions of the bypass ratio R and the Mach number M .

Matlab/Simulink model. The model proposed by Matlab/Simulink to compute the engine thrust force is given in Eq. (5),

$$\frac{T_{max}(h, M)}{T_0} = \frac{P(h)}{P_0} \left[1 + \frac{1}{2} (\gamma - 1) M^2 \right]^{\frac{\gamma}{\gamma - 1}} T_{NDT}(M, \delta_{TLA}) \quad (5)$$

where P_0 is the pressure at the sea-level, P is the pressure at the altitude h , γ is the adiabatic coefficient equal to 1.4 (the air is considered as an ideal gas), δ_{TLA} is the throttle position, and T_{NDT} is a nondimensional heuristic lookup table expressed as a function of Mach number M and throttle position δ_{TLA} .

Comparisons and Analysis of Results Obtained with Five Models

To validate the models presented in the previous section, several flight tests were performed with the Citation X level D Research Aircraft Flight Simulator. These flight tests consisted in flying the aircraft at a specific altitude, putting the throttle at its maximum position, and letting the aircraft accelerate from its minimum to its maximum speed (i.e. Mach operation). For example, the thrust ratio obtained at the sea level for Mach numbers values between 0 and 0.4 is shown in Figure 4.

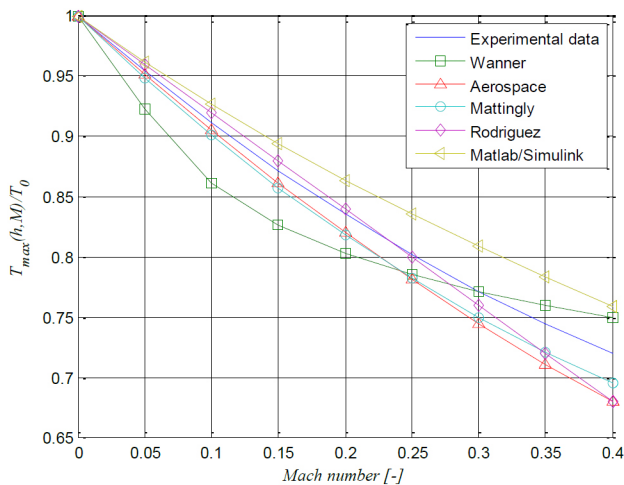


Figure 4. $T_{max}(h, M)/T_0$ ratio variation with Mach number at sea level

In Figure 4, the experimental data correspond to the engine thrust ratio measured from the RAFS, while the other data correspond to the estimation of the engine thrust ratio obtained from the 5 models mentioned in the previous section. All models provided very good results. Indeed, the maximum error was obtained with the model proposed by Wanner, approximately 6% at Mach 0.1 with respect to 3.4% at Mach 0.4 for the model proposed by Mattingly.

This analysis was repeated for 6 altitudes in order to obtain an overall tendency of the error. Figure 5 shows the maximum relative error for each model for altitudes from 5,000 ft to 30,000 ft. This error was calculated with respect to the experimental flight test data.

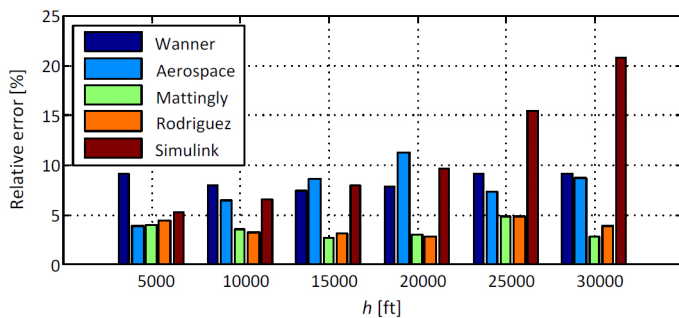


Figure 5. Maximum relative error with respect to altitude

In general, several observations can be made. First, among all the models, the one proposed by Mattingly seemed to give the best results with an average relative error of 3.4%. Concerning the other models, it was possible to see that the errors tend to increase with the altitude, which was probably due to the dissociation of the Mach and

altitude effects on the thrust falling. Finally, it was also noticed that the bypass ratio R had no influence on the results. Indeed, only the model proposed by Rodriguez considered this parameter, and yet, even following the consideration of R , the results obtained with the Rodriguez model were still close to results obtained with the other models as seen on Figure 5.

Proposed Static Engine Model

Following the previous results analysis, it has been concluded that there were three main external variables affecting the thrust variation (air density, Mach number and altitude) and one internal variable (throttle position). Based on these observations, the engine static model described in Eq. (6) has been proposed:

$$\frac{T(h, M, \delta_{TLA})}{T_0} = \bar{\sigma}(h, M) \times \tilde{T}(h, M, \delta_{TLA}) \quad (6)$$

where $T(h, M, \delta_{TLA})$ is the engine static thrust force at the altitude h , the Mach number M and the throttle position δ_{TLA} , $\bar{\sigma}$ is the relative air density ratio defined as:

$$\bar{\sigma}(h, M) = \frac{\rho(h)}{\rho_0} \left[1 + \frac{(\gamma - 1)}{2} M^2 \right]^{\frac{1}{\gamma - 1}} \quad (7)$$

and $\tilde{T}(h, M, \delta_{TLA})$ is a polynomial defined as:

$$\tilde{T}(h, M, \delta_{TLA}) = \sum_{i=0}^n \sum_{j=0}^m \tilde{T}_{ij}(h) M^i \delta_{TLA}^j \quad (8)$$

where $\tilde{T}_{ij}(h)$ are the polynomial coefficients depending on the altitude h , and $\{n, m\}$ represent the order of $\tilde{T}$ with respect to Mach number and throttle position.

According to several authors [9, 41], the variation of thrust due to the throttle position may be considered as linear. However, most of flight tests performed on the flight simulator have shown that the variation of thrust due to throttle position was piecewise linear or quadratic. For this reason, the order of the polynomial with respect to throttle position (m) was set at 2.

Similarly, as the model proposed by Mattingly gave the best results expressed as smallest relative errors (see Figure 4), the order of the polynomial with respect to the Mach number (n) was set at 4. A higher polynomial order has been considered for our proposed engine model in Eq. (8) compared to the one proposed by Mattingly in order to improve the results.

Transient Engine Model

In the previous section, a static model of the engine was developed. In this section the engine transient model is presented.