

Bivariate "Cut-Glue" Approximation of Strongly Nonlinear Mathematical Models Based on Experimental Data

Rudolf Neydorf
Don State Technical University

ABSTRACT

Researchers meet the difficulties of experimental and computer modeling of a statics and dynamics of aircrafts connected with their essential nonlinearity. This is due to the fact that the aerodynamic effects of the interaction complex aircraft designs or their models with air environment generate abrupt changes of the character of the some dependencies. Aerodynamic coefficients in the model of interaction can be obtained only or by full-scale tests or by computer simulations. Therefore, the construction of mathematical models of the objects is associated with the mathematical processing of the points of the experimental data. In this case, the experimentally obtained dependence is usually essentially nonlinear up to piecewise, or even discontinuous nature. Approximation of such dependencies, even with the use of spline functions, is very difficult and is associated with large errors. The solution to this problem was proposed by the author and was reported to the ASME Congress in November 2014 and published in the Proceedings of the Congress in its final form. In that work possibility of "approximating&multiplicative&additive" processing of dot experimental data for creation of unified mathematical model of the studied object or the phenomenon completely is mathematically proved. The offered method is called "Cut-glue" approximation as it is based on "cutting" of the well approximated intervals of the modelled dependence and their "gluing" in the one analytical function. However, this problem has been solved only for the univariate functions case. In this paper the author presents the solution of a problem of the bivariate «Cut-glue» approximation, which significantly expands the scope of application of the method. The creation examples of mathematical models are given. Fragments of the flying devices using the aerostatic flight principle are modeled. Examples show that models even piecewise dependences represent the unified analytical functions. However we can approximate their forms to piecewise so, how it is necessary for the accuracy of the description of experimental data. It is shown that combined application of the "Cut-glue" method of approximation and the piecewise description of separate intervals of the modelled experimental dependence by methods of the regression analysis considerably increases the accuracy of the mathematical dependence description in general. For bivariate models the effect of application of a method becomes stronger, because the error of the description of the significantly nonlinear bivariate dependences by regression methods much more, than univariate dependences.

CITATION: Neydorf, R., "Bivariate "Cut-Glue" Approximation of Strongly Nonlinear Mathematical Models Based on Experimental Data," *SAE Int. J. Aerosp.* 8(1):2015, doi:10.4271/2015-01-2394.

INTRODUCTION

In articles [1,2] the new and perspective method of creation of mathematical models on the basis of experimentally found dependences is mathematically strictly reasonable. Such dependences are presented by usually discrete data set. Creation of mathematical models with use of such data is strongly complicated if dependence has piecewise structure. In similar cases it is applied or piecewise approximation [3], or a spline approximation [4, 5, 6, 7]. Both methods exclude possibility of analytical transformation of the constructed models. For elimination of this shortcoming by the author the method called by it "cut-glue" approximation [1,2] was offered. This method allows presenting piecewise dependence unital analytical function. Function has the additive structure uniting multiplicative members. These members are formed by product of two functions. It is the main approximating function and the support "cutting-out" function. The first provides approximation of dependence on an interval between break points. The second function

without essential distortions of the first function "cuts out" from the first the set interval. Besides, it does zero values of the multiplicative member behind borders of the "cut-out" interval. Addition of multiplicative members gives the smooth function approximating piecewise dependence with a necessary accuracy.

However brought to [1,2] mathematical solution of a problem of approximation it is applicable only to one-dimensional dependences. It allows building the approximating functions of one independent variable. Practical problems of modeling quite often face need of approximation of multidimensional dependences [3,5,7]. Strong example of a practical task in which it is necessary not only piecewise, but also multidimensional approximation, is a problem of mathematical modeling of aerodynamic properties of aircraft [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. So in fig. 1 3D graphic of dependence of axial effort F_x on the propeller speed ω and speed v of the stream running on it is submitted [8].

Dependence is constructed and has to be described by mathematical model in rather wide range of a variation of variables: $\omega \in [0,70]m/s$; $v \in [500,3000]rpm$. The regression analysis shows (it and is visually visible) that by known methods it is impossible to describe a little precisely all dependence one function of two variables.

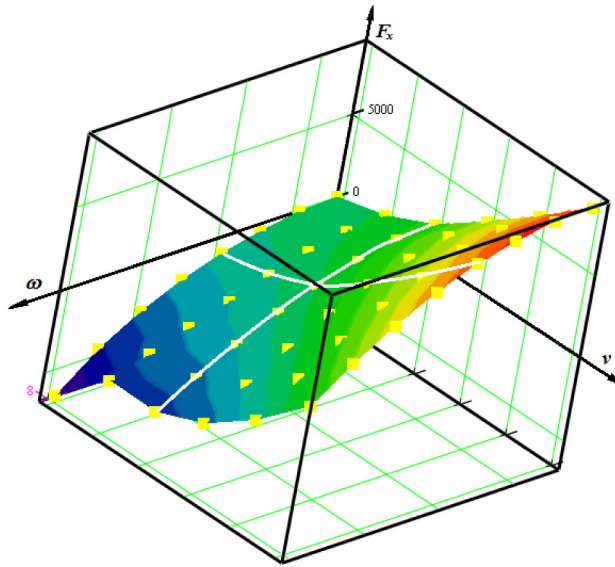


Figure 1. Dependence of axial effort on the propeller from the speed of its rotation and speed of a counter flow

Such dependences coordinate-wise break into m and n of intervals, and build $m \times n$ piecewise two-dimensional mathematical models, or two-dimensional splines. However, if in further researches analytical transformation of mathematical model is necessary, the problem of receiving the unitary analytical two-dimensional approximating function becomes actual problem. Thus, there is a question of an opportunity of creation of the two-dimensional function which is similarly had in [1,2] one-dimensional function. In particular, there is a question of possibility of the solution of this task of the dot data displaying dependence of some output size on two independent arguments by the “cut-glue” approximation method.

PROBLEM SETTING

In this research the distribution task on a two-dimensional case of “cut-glue” of construction developed in [1,2] approaches unitary analytical, at least once of the differentiable function describing any of dependences is set, including also piecewise and broken lines. Thus, it is supposed to create “cut-glue” an approximating method of construction continual mathematical models according to experimentally obtained data, including dependences, explosive on a derivative. The method has to be based on multiplicative “cutting” of sites of the modelled surface on the coordinates of two arguments having the general borders with the next sites. The unitary two-dimensional function describing dependence has to turn out additive “gluing” of the received multiplicative members. Such problem definition is ideologically similar to one-dimensional option of “cut-glue” approximation.

ESSENCE OF ONE-DIMENSIONAL MODIFICATION “CUT-GLUE” APPROXIMATION METHOD

According to [1], the essence of one-dimensional “cut-glue” approximation method consists in representation of the approximated piecewise dependence by the following expression:

$$f(x) = \sum_{i=1}^n f_i(x), \quad (1)$$

where function

$$f_i(x) = \varphi_i(x) \cdot \lambda(x, S_i) \quad (2)$$

is called “interval excision function” (IEF); $\varphi_i(x)$ - analytical function which approximates i interval of all experimentally set dependence; $\lambda(x, S_i)$ - purpose-built for the developed method one-dimensional “multiplicatively cutting” function (1MCF); $S_i = \{x_{i-1}, x_i, \varepsilon_i\}$ - set of adjusting parameters of MCF (the left and right borders of the cut-out interval and approximation error setting ε_i of the interval [1]. In article [1] function was called simply as the MCF as the multidimensional case wasn't considered.

Function $\lambda(x, S_i)$ is defined, according to [1], the following expression:

$$\lambda(x, x_{i-1}, x_i, \varepsilon_i) = \frac{\left[x - x_{i-1} + \sqrt{(x - x_{i-1})^2 + \varepsilon_{i-1}^2} \right] \cdot \left[x_i - x + \sqrt{(x_i - x)^2 + \varepsilon_i^2} \right]}{4 \cdot \sqrt{[(x - x_{i-1})^2 + \varepsilon_{i-1}^2] \cdot [(x_i - x)^2 + \varepsilon_i^2]}} \quad (3)$$

Researches showed that the most effective option of use of parameter ε_i is the condition

$$\forall i, j \in [1, n] \rightarrow \varepsilon_i = \varepsilon_j = \varepsilon. \quad (4)$$

Function (3) possesses a number of properties which provide it effective implementation of a problem of multiplicative “cutting” of any sites of the approximating function - $\varphi_i(x)$. These properties are investigated and proved in article [1]. In brief they are reduced to the following listed below six properties.

1. Function (3) is symmetric concerning the middle of range - $[x_{i-1}, x_i]$.
2. Function (3) has the unique maximum in a point $x_m = \frac{(x_{i-1} + x_i)}{2}$, being in the middle of range $[x_{i-1}, x_i]$ and depending on parameters $S_i = \{x_{i-1}, x_i, \varepsilon_i\}$:

$$\lambda(xm, x_{i-1}, x_i, \varepsilon) = 0,25 + \frac{0,5}{\sqrt{1 + \frac{4 \cdot \varepsilon^2}{(x_{i-1} - x_i)^2}}} + \frac{0,25}{1 + \frac{4 \cdot \varepsilon^2}{(x_{i-1} - x_i)^2}}, \quad (5)$$

and two zero infimums

$$\left. \begin{aligned} \lim_{x \rightarrow -\infty} \lambda(x, x_{i-1}, x_i, \varepsilon) &= 0; \\ \lim_{x \rightarrow \infty} \lambda(x, x_{i-1}, x_i, \varepsilon) &= 0. \end{aligned} \right\} \quad (6)$$

3. Function (3) in the range $[x_{i-1}, x_i]$ can accept the value as much as close to unit, that is

$$\forall x, x_{i-1}, x_i : x_{i-1} < x_i, x \in [x_{i-1}, x_i] \rightarrow \lim_{\varepsilon \rightarrow 0} \lambda(x, x_{i-1}, x_i, \varepsilon) = 1. \quad (7)$$

4. Function (3) in the ranges $[-\infty, x_i]$ and $[x_i, \infty]$ can accept the value as much as close to zero.

And

$$\begin{aligned} \forall x, x_{i-1}, x_i, \varepsilon : x_{i-1} < x_i, x \notin [x_{i-1}, x_i] \rightarrow \\ \rightarrow \lim_{x \rightarrow -\infty} \lambda(x, x_{i-1}, x_i, \varepsilon) = \lim_{x \rightarrow \infty} \lambda(x, x_{i-1}, x_i, \varepsilon) = \\ \lim_{\varepsilon \rightarrow 0} \lambda(x, x_{i-1}, x_i, \varepsilon) = 0. \end{aligned} \quad (8)$$

5. Function (3) in boundary points accepts values

$$\forall x_{i-1}, x_i : x_{i-1} < x_i \rightarrow \lambda(x_{i-1}, x_{i-1}, x_i, \varepsilon) = \lambda(x_i, x_{i-1}, x_i, \varepsilon) = 0,5 \quad (9)$$

at any ε and with condition 3.

6. Function (3) infinite number of times is continuously differentiated, as well as any function with a fractional exponent of degree.

The last property is extremely important for use of approximating mathematical model in analytical researches. Such researches can be connected with the solution of tasks of the analysis of properties of model, identification of its parameters, forecasting of qualitative and quantitative behavior of object, synthesis of the control laws and other tasks.

DEVELOPMENT OF TWO-DIMENSIONAL MODIFICATION “CUT-GLUE” APPROXIMATION METHOD

The author has some experience of the mathematical description of multidimensional dependences for models as aircraft [8, 9, 10, 11, 12], and the environment of their flights [16, 17, 18, 19]. This experience showed that on small intervals of a variation of entrance variables rather exact approximation of experimental data is provided by the classical regression analysis (CRA). Thus in most cases it is

sufficient to use a universal polynomial form of the description [1, 2, 8, 10, 11, 16, 17, 18, 19]. At enough skilled data of CRA gives the chance to reach compromise between repetition accuracy mathematical model of experimental points and smoothness of behavior of the approximating function on interdot intervals [1, 2, 8, 9, 10, 11, 20, 21, 22, 23]. Moreover, if high precision of experimental measurements is provided, between points of a break of the modelled dependence its rather smooth sites can be received. In this case CRA can provide a zero error of the description of skilled points at a choice of the corresponding order of the approximating polynomial [1, 2, 8, 20, 21, 22, 23]. Then high precision of approximation will be provided also by result of application of the “cut-glue” method if it manages to be generalized on a multidimensional case. In the present article the problem of the two-dimensional approximating description of experimental dependences is solved.

Polynomial mathematical dependence of an output variable from entrance variables is explicit function y of independent arguments. Therefore the standard form of the approximating function for 2 entrance x_1 and x_2 variables by analogy with (1) can be the following:

$$y = f(x_1, x_2), \quad (10)$$

where

$$f(x_1, x_2) = \sum_{j=1}^m \sum_{i=1}^n f_{ij}(x_1, x_2). \quad (11)$$

Then two-dimensional IEF can be multiplicative created by analogy with (2) as follows:

$$f_i(x_1, x_2) = \varphi_{ij}(x_1, x_2) \cdot \lambda(x_1, x_2, S_{ij}), \quad (12)$$

where $\varphi_{ij}(x_1, x_2)$ - two-dimensional analytical function which approximates a patch on i interval of experimental dependence for argument x_1 , and on j its interval for x_2 ; $\lambda(x_1, x_2, S_{ij})$ - MCF for two-dimensional modification of “cut-glue” method (2MCF); $S_{ij} = \{x_{1,j-1}, x_{1,j}, x_{2,j-1}, x_{2,j}, \varepsilon_{1j}, \varepsilon_{2j}\}$ - set of adjusting parameters 2MCF (the left and right borders of the cut-out patch and approximation error settings $\varepsilon_{1j}, \varepsilon_{2j}$ on variables x_1 and x_2).

Function 2MCF $\lambda(x_1, x_2, S_{ij})$ is based as a multiplicative form of two 1MCF on each argument in a form (3)

$$\lambda(x_1, x_2, S_{ij}) = \lambda(x_1, S_{ij}) \cdot \lambda(x_2, S_{ij}). \quad (13)$$

At such structure the form (13) inherits the properties similar to properties 1 - 6 for (3) on each coordinate x_1 and x_2 .

Only property 2 changes.