

Modelling and Evaluation of Aircraft Contrails for 4-Dimensional Trajectory Optimisation

Yixiang Lim, Alessandro Gardi, and Roberto Sabatini
RMIT University

ABSTRACT

Contrails and aircraft-induced cirrus clouds are reputed being the largest components of aviation-induced global warming, even greater than carbon dioxide (CO_2) exhaust emissions by aircraft. This article presents a contrail model algorithm specifically developed to be integrated within a multi-objective flight trajectory optimization software framework. The purpose of the algorithm is to supply to the optimizer a measure of the estimated radiative forcing from the contrails generated by the aircraft while flying a specific trajectory. In order to determine the precise measure, a comprehensive model is employed exploiting the Schmidt-Appleman criterion and ice-supersaturation regions. Additional parameters such as the solar zenith angle, contrail lifetime and spread are also considered. The optimization of flight trajectories encompassing such contrail model allows for selective avoidance of the positive radiative forcing conditions, such as only avoiding persistent contrails, or contrails which lead to negative radiative forcing. The model assesses the radiative forcing associated with 4-Dimensional (4D) trajectories in a 4D weather field, encompassing both the local time-of-day and the contrail lifetime. Some preliminary algorithm validation activities are presented, including a simulation case study involving a medium-range domestic flight of a turbofan aircraft from Melbourne to Brisbane.

CITATION: Lim, Y., Gardi, A., and Sabatini, R., "Modelling and Evaluation of Aircraft Contrails for 4-Dimensional Trajectory Optimisation," *SAE Int. J. Aerosp.* 8(2):2015, doi:10.4271/2015-01-2538.

INTRODUCTION

With the currently foreseen growth of air traffic globally, the effects of aircraft condensation trails (contrails) are predicted to become significant by 2050. Currently, the physics behind the formation of contrails is relatively well understood. However, research regarding the persistence or dissipation timescales of contrails is still ongoing, with particular focus on their evolution into cirrus clouds, since these clouds have been shown to contribute significantly to global warming. The transition into cirrus is a combination of the macro-physical processes such as wind shear (which increase contrail spread), and the microphysics such as the ageing and growth of the ice particles. Contrail-based trajectory optimization is a relatively recent field of research, and work done so far has focused mainly on contrail avoidance. A review [1] noted that a general reduction in flight altitudes might not be the most effective solution due to the geographical, seasonal and diurnal variations in contrail formation regions. For example, contrails are avoided in the tropical regions by flying at a lower altitude while an increase in flight altitudes in temperate regions is required for contrail avoidance. Alternative solutions, such as shifting the air traffic densities towards sunrise and sunset, when the cooling effect of contrails is the strongest, have been suggested in other studies [2, 3]. A 4D-trajectory optimization models for contrail avoidance has been developed by [4]. This model designates persistent contrail regions as regions that both satisfy the Schmidt-Appleman criterion and are supersaturated with respect to ice, and subsequently a contrail formation cost is associated with such a region. However, not all persistent contrail regions should be

penalized, as the radiative impact is also dependent on the time of day and geographic location of the contrail regions. A new scheme is presented in this paper that uses existing contrail models to compute the associated Radiative Forcing (RF) based on a set of weather, aircraft and trajectory inputs and attempts to address any possible implications arising from the adoption of such a comprehensive system. The model is specifically designed to be integrated in a Multi-Objective Trajectory Optimization (MOTO) software framework, also under development. The MOTO software is conceived for the implementation in ground-based and airborne Air Traffic Management (ATM) and Avionics systems currently under development as part of this research [5, 6, 7, 8, 9], addressing the strategic and tactical online operations. The contrail model will be exploited both as stand-alone and within the MOTO software to determine and optimize the formation, persistency and radiative contribution of contrails by a number of simulated flights under different weather conditions at various times of the day. Some results are included, compared and discussed here, with particular focus on the trade-offs between the environmental impact of aircraft emissions and contrail formation.

PHYSICS OF CONTRAIL FORMATION

Contrails, or condensation trails, are formed from the condensation of water vapor in the plume of a jet engine and can be the precursor to the ice clouds called cirrus, found at higher altitudes where ice supersaturation is present. The formation of these contrails is

thermodynamically triggered by the engine exhaust, which contains heat, moisture and soot. The contrail lifecycle is broken down into three phases: the jet, vortex and dispersion phases. The jet phase lasts for around 20 seconds and determines the thermodynamic formation of the contrail as the hot humid exhaust plume interacts and cools in the ambient air. The Schmidt-Appleman criterion, developed first by Schmidt (1941) then independently by Appleman (1953), is used to determine the onset of contrail formation [10]. The vortex phase follows from the jet phase and has a timescale of 2 minutes. In the vortex phase, the exhaust plume comes under the influence of the vortices generated by the aircraft, which causes a downward sinking of the contrails. The initial characteristics of the contrail (its breath, depth, ice properties, etc) can be considered to have stabilized and should be determined. A parametric model developed by [11] is used to model this phase. The dispersion phase follows from the end of the vortex phase and involves the advection, spread and eventual evaporation of the contrail. Depending on ambient conditions, this can range from a few minutes, up to hours. The integration scheme employed in [11] is adapted for use in this model, with some changes. Instead of a second-order two-step Runge-Kutta scheme, a forward Euler method is used to determine the advection of the contrail. In addition, while the terminal particle velocity V_T in the former is determined as in [12], this paper uses a simplified model as in [13]. These models are developed and implemented in Matlab, with separate functions to model each phase of the contrail lifecycle.

WEATHER DATA RETRIEVAL

Weather data is obtained from the Global Forecasting System (GFS). Forecast data is given in a 0.25° resolution, updated 4 times daily (every 6 hours), and provides a projection of up to 180 hours in 3-hour intervals. The data is further interpolated to get a better resolution in space and time. The relative humidity with respect to ice (RH_i), obtained as a function of the relative humidity with respect to water RH_w, and temperature T, both obtained from GFS

$$RH_i = RH_w \frac{p_{liq}(T)}{p_{ice}(T)}$$

(1)

where e_w and e_i are the saturation pressures for water vapor and ice respectively, based on empirical relations by (Sonntag 1996).

$$p_{liq}(T) = 100 \exp \left[\frac{-6096.9385}{T} + 16.635794 - 0.027111937T + 1.673952e^{-5}T^2 + 2.433502 \ln(T) \right]$$

(2)

$$p_{ice}(T) = 100 \exp \left[\frac{-6024.5282}{T} + 24.7219 + 0.10613868T - 1.3198825e^{-5}T^2 - 0.49382577 \ln(T) \right]$$

(3)

with T in degrees Kelvin.

CONTRAIL MODEL

The contrail model is based on the Contrail Cirrus Prediction Tool (CoCiP) developed by Schumann [11]. The algorithm proceeds as follows. An initial check based on the Schmidt-Appleman Criterion is done to flag contrail formation segments along the flight path. A contrail lifetime analysis is then run for flagged segments. The initial size and properties of the plume are computed using a parametric model for the jet and vortex phases. These are then integrated in time using another parametric model for the dispersion phase. A model based on [14] then estimates the contrail-related RF based on the solar zenith angle, the spread and the lifetime of the contrail. A block-diagram of the algorithm is represented in Fig. 1. In the operational concept depicted in Fig. 2, an aircraft undertakes a 2-hour journey, departing at 1700hrs and arriving at 1900hrs. It encounters two contrail formation regions, shown in blue. In the first region, the contrail quickly dissipates while persistence is observed in the second region. Midway during the flight, the sun sets. The sunset line is shown as a dashed-line and subsequent contrails after this are greyed out. The model computes the RF of the second contrail and flags to the 4DT optimizer the red-shaded areas above the threshold RF, marked as a dot-dashed line.

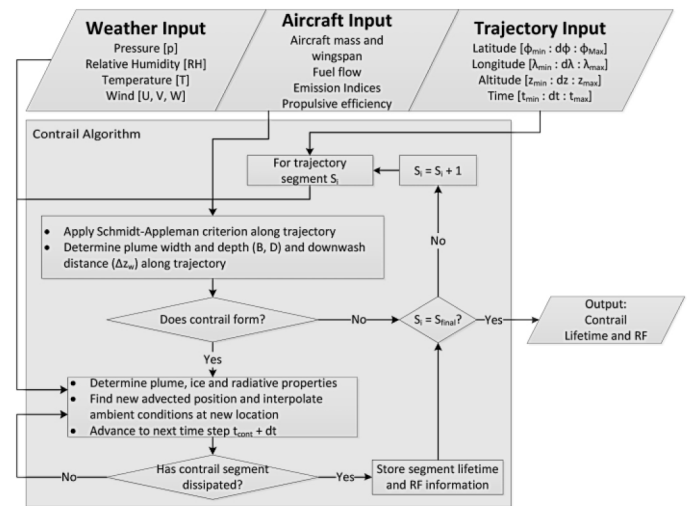


Figure 1. Block diagram of the contrail model software algorithm.

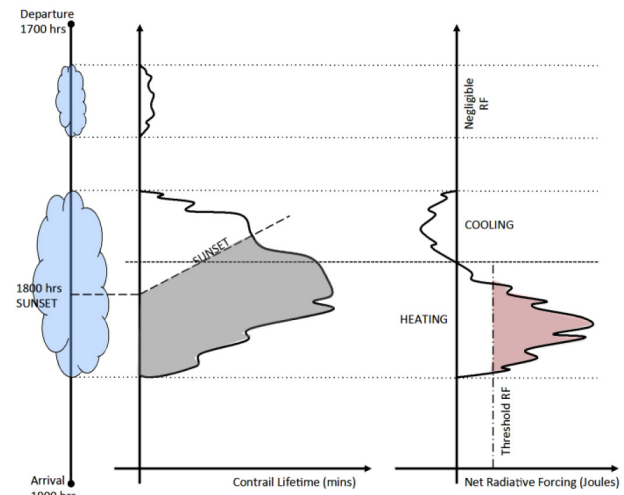


Figure 2. Operational concept of the contrail model. The timeline on the left represents the aircraft flight path. The middle graph represents the contrail lifetime and the graph on the right represents the net RF. Depending on the time of day, the RF can be positive or negative.

Jet Phase

In the jet phase, the Schmidt-Appleman Criterion is used to determine the onset of contrail formation. This is given by

$$G = \frac{EI_{H_2O} \cdot c_p \cdot p}{\varepsilon \cdot Q \cdot (1 - \eta)} \quad (4)$$

where G is the slope of the mixing line, in Pa K^{-1} , and describes the mixing of the exhaust plume with ambient air. EI_{H_2O} , Q and η are the emission index of water vapor, heat per mass of fuel and propulsive efficiency respectively and are engine characteristics, p is the ambient pressure, and c_p and ε are the specific heat capacity of air and the ratio of the molecular masses of water and air respectively.

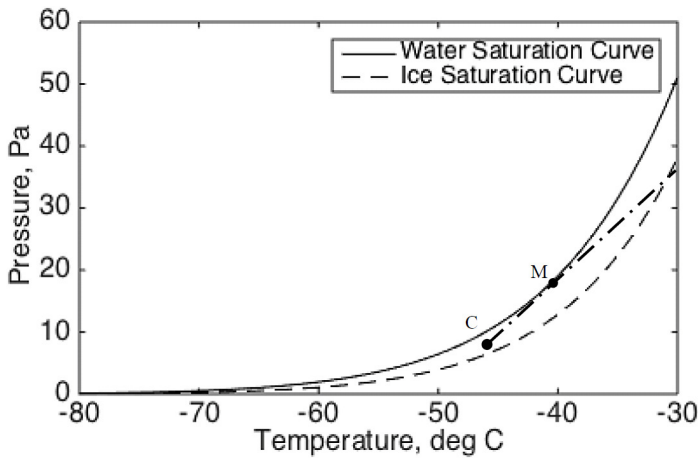


Figure 3. Mixing diagram

As the plume cools and mixes, it moves along the mixing line, as shown in Figure 3, from the top right to the bottom left, terminating at ambient conditions (Point C). The threshold conditions for contrail formation occur at Point M, where the mixing line touches the water saturation curve. Larger values of G will result in a steeper slope, leading to condensation of the exhaust plume as it crosses the water saturation curve and the formation of contrails. If ambient conditions are supersaturated with respect to ice, that is, if Point C is above the ice saturation curve, the condensed vapor will freeze into ice crystals, and the contrail might grow into cirrus given sufficient time. Reference [11] gives an approximation for the temperature at the threshold point:

$$T_{LM} = -46.46 + 9.43 \ln(G - 0.053) + 0.72 [\ln(G - 0.053)]^2 \quad (5)$$

in degrees celcius, where the subscript L denotes the threshold temperature for liquid saturation.

The critical temperature at ambient conditions (Point C) can then be found using the criterion given by [11] based on curve fitting where T and T_{LM} are given in degrees celcius

$$T_{LC} = T_{LM} - (1 - RH_w) \frac{p_{liq}(T_{LM})}{G} - \Delta T_c \quad (6)$$

$$\Delta T_c = F_1 RH_w [W - F_2 (1 - W)] \quad (7)$$

$$W = (1 - RH_w^2)^{x_2} \quad (8)$$

$$F_1 = x_1 + x_3 \ln(G), \quad F_2 = \left[\frac{1}{4} - \left(RH_w - \frac{1}{2} \right)^2 \right]^4 \quad (9)$$

Where $x_1 = 5.686$, $x_2 = 0.3840$, $x_3 = 0.6594$. The criterion for contrail formation is thus

$$dT = T - T_{LC} < 0 \quad (10)$$

Vortex Phase

The parametric model developed by [11] was used to estimate the initial depth and width of the contrail. This is done by first computing the maximum downward displacement Δz_w and then scaling it to obtain the initial depth. The initial width is then parameterized from the depth, dilution and fuel flow. The parameters that determine the initial size of the contrail are:

Wake vortex separation $b_0 = \pi S_a / 4$,

Initial circulation $\Gamma_0 = 4 M_a g / (\pi S_a \rho_{air} V_a)$,

Effective time scale $t_0 = 2\pi b_0^2 / \Gamma_0$,

Initial velocity scale $w_0 = \Gamma_0 / (2\pi b_0)$,

Normalized dissipation rate $\epsilon^* = (\epsilon b_0)^{1/3} / w_0$.

These are dependent on the inputs

Wing span S_a ,

Aircraft mass M_a ,

True air speed V_a ,

Air density ρ_{air} ,

Brunt Vaisala frequency N_{BV} ,

Turbulent kinetic energy dissipation rate ϵ .

Schumann's parameterization distinguishes between strongly and weakly stably stratified conditions. If $N_{BV} t_0 \geq 0.8$:

$$\Delta z_w = 1.49 \frac{w_0}{N_{BV}} \quad (11)$$

else, with $\epsilon^* \leq 0.36$:

$$\frac{\Delta z_w}{b_0} = 7.68(1 - 4.07 \epsilon^* + 5.67 \epsilon^{*2})(0.79 - N_{BV} t_0) + 1.88 \quad (12)$$

with Δz_w as the maximum sinking.

The initial downward displacement is set to

$$\Delta z_1 = C_{z1} \Delta z_w, \quad C_{z1} = 0.25 \quad (13)$$

where the subscript 1 denotes the end of the vortex phase. The initial depth is set to

$$D_1 = C_{D0} \Delta z_w, \quad C_{D0} = 0.5 \quad (14)$$

and the initial width is

$$B_1 = N_{dil}(t_0) m_F / [(\pi/4) \rho D_1] \quad (15)$$

with the m_F being the fuel flow in kg m^{-1} and the dilution N_{dil} of the wake vortex formation being

$$N_{dil}(t) \approx 7000(t/t_s)^{0.8} \quad (16)$$

with $t_s = 1s$. The initial mass mixing ratio of ice I and the ice particle number N are also prescribed for the vortex phase.

$$I_1 = I_0 - \Delta I_{ad} \quad (17)$$

$$I_0 = \frac{EI_{H2O} m_F}{(\pi/4) \rho D_1 B_1} + q_0 - q_s(p_0, T_0) \quad (18)$$

$$\Delta I_{ad} = \frac{R_0}{R_1} \left[\frac{p_{ice}(T_0 + \Delta T_{ad})}{p_1} - \frac{p_{ice}(T_0)}{p_0} \right] \quad (19)$$

Here, q_0 is the specific humidity of ambient conditions at stage “0” (the start of the vortex phase) and q_s is the specific humidity at saturation point. p_1 is the ambient pressure at stage “1”, R_0 and R_1 are the specific heat capacities of air and water respectively, and

$$\Delta T_{ad} = T_0(R_0/c_p)(p_1 - p_0)/p_0 \quad (20)$$

The ice number N_1 at the end of the vortex phase is determined by the soot index and a survival factor based on the ice mass ratio.

$$N_1 = f_{surv} N_0 \quad (21)$$

$$N_0 = EI_{soot} m_F, \quad f_{surv} = \frac{I_1}{I_0} \quad (22)$$

Dispersion Phase

The state of each contrail segment along the flight path evolves with time and is described by the state vector X , denoted as $X(\text{Position}, \text{Ambient}, \text{Plume}, \text{Particle})$ - its position in space and time, the relevant ambient conditions, and its plume and ice particle properties. The position at time t is given as

$$X.\text{Position} = (x(t), y(t), z(t), t) \quad (23)$$

where x , y and z gives the longitude, latitude and altitude of the contrail. The advection of the contrail with time is described by

$$x(t + \Delta t) = x(t) + U(t)\Delta t \quad (24)$$

$$y(t + \Delta t) = y(t) + V(t)\Delta t \quad (25)$$

$$z(t + \Delta t) = z(t) + W(t)\Delta t \quad (26)$$

with the absolute distances converted to geodetic coordinates based on approximations provided by [15]. The ambient conditions include the parameters

$$X.\text{Ambient} = (p, T, \rho_{air}, q_a, q_s) \quad (27)$$

which are pressure, temperature, density, and ambient and saturation specific humidity respectively, obtained from the GFS, and are functions of $X.\text{Position}$. The plume parameters consist of

$$X.\text{Plume} = (\sigma, B, D, D_{eff}, A, L, M, I, N, n, M_{H2O}, D_H, D_V)$$

Respectively, these are the:

Covariance matrix σ ,

Contrail width B ,

Contrail depth D ,

Effective depth D_{eff} ,

Contrail area A ,

Contrail length L ,