

Reduced Order Model Approach for Efficient Aircraft Loads Prediction

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ABSTRACT

Flight loads calculations play a fundamental role in the development and certification of an aircraft and have an impact on the structural sizing and weight. The number of load cases required by the airworthiness regulations is in the order of tens of thousands and the analysis must be repeated for each design iteration. On large aircraft, CS-25 explicitly requires taking into account for loads prediction, airframe flexibility, unsteady aerodynamics and interaction of systems and structure, leading to computationally expensive numerical models. Thus there is a clear benefit in speeding-up this calculation process. This paper presents a methodology aiming to significantly reduce the computational time to predict loads due to gust and maneuvers. The procedure is based on Model Order Reduction, whose goal is the generation of a Reduced Order Model (ROM) able to limit the computational cost compared to a full analysis whilst retaining accuracy. The method is applied to a commercial transport aircraft modeled with beam elements, unsteady aerodynamics based on Doublet Lattice Method and servo-hydraulic actuators for the control surfaces. The aeroelastic equations of motion are formulated in the time-domain, through the Rational Function Approximation and application of the Balanced Truncation method. The results obtained with the reduced model shows a very good accuracy with respect to the full model and a significant saving in computational time. The impact of flexibility on the gust load factor is also highlighted, comparing it with the quasi-static analysis by Pratt's formula, current standard for Part 23 aircraft.

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INTRODUCTION

Flight loads calculation is one of the first and most important processes in the design and certification of an aircraft. The certification specifications for large aircraft, EASA CS-25 and FAA FAR-25 [1,2], mandate for rational analysis taking into account all the relevant structural degrees of freedom, unsteady aerodynamics and interaction of structure and systems, which dictate the use of complex aeroelastic models [3]. Besides, the number of load cases to be considered to assure structural integrity can easily reach the order of hundreds of thousands. For all these reasons, the turnaround time of loads calculation is high and impacts on the downstream activities; therefore there is a clear benefit in speeding it up.

The common industrial approach is to generate a numerical model based on a Finite Element discretization of the airframe and unsteady aerodynamics. Whereas classical aeroelastic analyses have been performed routinely in frequency domain [4], when actuators driving control surfaces and control laws are considered it is easier to cast the equations in time domain [5] to obtain a Linear Time Invariant (LTI) state-space model. This also allows reducing the size of the model

applying Model Order Reduction (MOR) techniques [6]. These techniques produce Reduced Order Models (ROM) which are capable of retaining the accuracy of the original model, but with a significant reduction of computational time.

In this paper a reduced order aeroservoelastic model of a generic transport aircraft is developed for gust and flight maneuvers responses and a MOR technique applied to reduce its size and show the advantage in terms of calculation time and loads prediction accuracy.

The paper is organized as follows: in the first section the building blocks of the aeroservoelastic model are presented, i.e. structure, aerodynamics and control surfaces actuators. A time domain state-space model is generated and its size reduced to obtain the ROM. Discrete gust responses and pitching maneuver simulations according to the certification specifications are then performed to compute airframe loads and assess the validity of the approach and the saving in computational time. A brief comparison of the gust load factor and wing loads obtained through a dynamic response

analysis and through the application of a quasi-static methodology such as Pratt's formula, currently the standard for Part 23 aircraft, is also presented.

AEROSERVOELASTIC MODEL

Flight loads calculations are typically performed using an aeroservoelastic model of the aircraft capable of representing, with a different degree of fidelity and uncertainty according to the phase of the development cycle, the vibration behavior over a frequency range of interest, the aerodynamics characteristics, including control surfaces, and the interaction of systems and structure (as stated in CS 25.302), mainly the Flight Control System.

The procedure proposed in this work is applied to a generic commercial transport aircraft, depicted in Figure 1; its main geometric and mass properties are summarized in Table 1. This model was developed as a part of the EU funded FFAST FP7 project. The building blocks of the aeroservoelastic model are now presented.

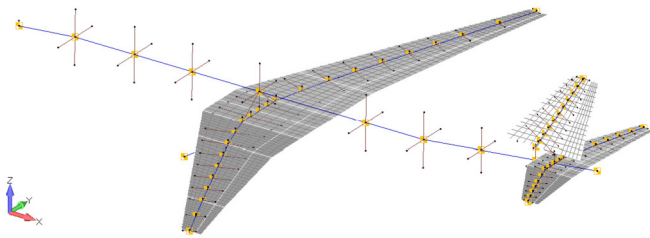


Figure 1. Aeroelastic model of the generic commercial transport aircraft

Table 1. Main properties of the generic commercial transport aircraft

Property	Value
Length	67m
Wingspan	65m
Height	17m
Wing area	445m ²
Mean Aerodynamic Chord	6.07m
MTOW	268tons
Empty weight	192tons
Cruise Mach	0.85
Max operating Mach	0.89
Max operating altitude	43000ft
M _A	0.82
M _C	0.89
M _D	0.95
n _{z,max}	2.5g
n _{z,min}	-1g

Structural Model

The structural model of the aircraft, shown in Figure 1, is a beam-stick finite element model built in the commercial solver Nastran. The fuselage, wing, engine pylons and tailplanes are represented by bar elements (CBAR) and rigid elements (RBE2) are used to connect the different subcomponents. The mass distribution is represented both

by distributed masses on the bar elements, for the structural mass, and by lumped masses (CONM2), these accounting for the engines, systems, furniture, payload and fuel.

The structural dynamics equations are formulated via the modal approach and selecting the first 30 normal modes of the free-free structure, including the 6 rigid body modes, to obtain the modal mass, damping and stiffness matrices, namely $\mathbf{M}_{hh}$, $\mathbf{C}_{hh}$ and $\mathbf{K}_{hh}$. This subset of lower frequencies modes is sufficient to obtain converged airframe loads.

Aerodynamic Model

The aerodynamic model is based on the Doublet Lattice Method (DLM) available in Nastran. The DLM is a 3D subsonic unsteady panel code formulated in the frequency domain and has long established itself as the standard methodology employed in the industry for aeroelastic calculations. The wing and tailplanes are modelled by a flat plate mesh, whereas the aerodynamic effect of the fuselage is neglected. The model contains also control surfaces (elevator, rudder and ailerons). The interpolation between the structural and aerodynamic meshes is based upon the Infinite Plane Spline method [7]. The DLM provides the Generalized Aerodynamic Forces (GAF) due to structural motion, gust disturbance and control surfaces deflections as complex matrices tabulated as a function of Mach number and a finite set of n_k reduced frequencies, namely $\mathbf{Q}_{hh}(M, k)$, $\mathbf{Q}_{hg}(M, k)$, and $\mathbf{Q}_{hc}(M, k)$.

Flight Control System

The control surfaces of the aircraft are driven by servo-hydraulic actuators. The modelling of the actuator follows [8]. Its dynamics is represented as a 3rd order transfer function between the pilot input u_c and the actual control surface deflection δ . The open-loop transfer function of the actuator relates the control surface deflection to the displacement of the servo-valve spool u_v

$$G(s) = \frac{\delta}{u_v} = \frac{g}{s \left(\frac{s^2}{\omega_h^2} + \frac{2\zeta}{\omega_h} s + 1 \right)} \quad (1)$$

where ω_h is the hydraulic angular frequency, ζ the hydraulic damping and g the gain. Since the equations governing the actuator dynamics are non-linear, to obtain the transfer function these are linearized in the neighborhood of an operating point, assumed to be the neutral position of the valve spool. A feedback between the valve spool displacement and the desired input u_c is introduced and the closed-loop transfer function becomes

$$H_c(s) = \frac{\delta}{u_c} = \frac{k_c G(s)}{1 + k_c G(s)} \quad (2)$$

with k_c being the proportional gain.

Under this modelling assumption, the actuator is seen as a shaping filter of the pilot input command and in series with the aeroelastic plant.

STATE-SPACE MODEL

The classical approach used in aeroelasticity for loads and flutter analyses solves the equations of motion in the frequency domain. To apply the Model Order Reduction method presented in this paper, the equations of motion must be translated into the time domain and casted in state-space form, leading to the Linear Time Invariant system

$$\begin{aligned}\dot{\mathbf{x}}_{ae} &= \mathbf{A}_{ae}\mathbf{x}_{ae} + \mathbf{B}_{ae}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}_{ae}\mathbf{x}_{ae} + \mathbf{D}_{ae}\mathbf{u}\end{aligned}\quad (3)$$

where $\mathbf{x}_{ae}$ are the states of the aeroservoelastic system, including the structural, aerodynamic and actuator states, $\mathbf{u}$ is the input exciting the aircraft, a gust or a pilot command, and $\mathbf{y}$ the output, loads on the airframe or other Interesting Quantities (IQ).

Since the aerodynamic forces are available in the frequency domain in tabulated form, to obtain Eq. (3) a Rational Function Approximation (RFA) of the matrices $\mathbf{Q}_{hh}(M, k)$, and $\mathbf{Q}_{hg}(M, k)$ is performed. This consists of approximating the aforementioned matrices by a rational polynomial in the Laplace variable s such that

$$\begin{aligned}\widetilde{\mathbf{Q}}_{hh}(s, M) &= \mathbf{D}_0 + \frac{l_a}{V}\mathbf{D}_1s + \left(\frac{l_a}{V}\right)^2\mathbf{D}_2s^2 + \mathbf{C}_a\left(s\mathbf{I} - \frac{V}{l_a}\mathbf{A}_a\right)\mathbf{B}_as \\ \widetilde{\mathbf{Q}}_{hg}(s, M) &= \mathbf{D}_{0g} + \frac{l_a}{V}\mathbf{D}_{1g}s + \left(\frac{l_a}{V}\right)^2\mathbf{D}_{2g}s^2 + \mathbf{C}_g\left(s\mathbf{I} - \frac{V}{l_a}\mathbf{A}_g\right)\mathbf{B}_gs\end{aligned}\quad (4)$$

Many approaches have been developed to perform this approximation; in the following Roger's method [9] is employed. It assumes that Eq. (4) are rewritten as

$$\begin{aligned}\widetilde{\mathbf{Q}}_{hh}(s, M) &= \mathbf{D}_0 + \frac{l_a}{V}\mathbf{D}_1s + \left(\frac{l_a}{V}\right)^2\mathbf{D}_2s^2 + \frac{V}{l_a}\sum_{i=1}^{n_a}\frac{s}{s + V/l_a\beta_{li}}\mathbf{A}_i \\ \widetilde{\mathbf{Q}}_{hg}(s, M) &= \mathbf{D}_{0g} + \frac{l_a}{V}\mathbf{D}_{1g}s + \frac{V}{l_a}\sum_{i=1}^{n_g}\frac{s}{s + V/l_a\beta_{gi}}\mathbf{A}_{gi}\end{aligned}\quad (5)$$

In the original formulation the unknowns are the coefficients of $\mathbf{D}_0$, $\mathbf{D}_1$, $\mathbf{D}_2$ and $\mathbf{A}_i$, while the number and values of the aerodynamic poles β_i are selected a-priori and $\beta_i < 0$ to ensure asymptotic stability. The coefficients of the polynomial are then identified by a linear least-square procedure applied term by term to the matrix $\mathbf{Q}_{hh}$, i.e. a curve fitting is performed. In this work, Roger's formulation is extended considering the aerodynamic poles as free design variables of an optimization process whose objective function is the minimization of the squared error between the approximated and tabulated GAF. Several studies have been published on the nonlinear optimization of the aerodynamic poles [10,11,12]. In the present work, nonlinear non-gradient constrained optimizations are performed to select the aerodynamic poles minimizing the following objective function

$$\begin{aligned}\mathcal{F} &= \sum_{j=1}^{n_h} \left(\sum_{i=1}^{n_h} \sum_{m=1}^{n_k} w_{ij} \varepsilon_{ijm} \right)^{1/2} \\ \varepsilon_{ij} &= \frac{|\widetilde{Q}_{ij} - Q_{ij}|^2}{\max_m \{1, |Q_{ij}|^2\}}\end{aligned}\quad (6)$$

Where w_{ij} are weighting factors that can be chosen if some specific elements of the GAF matrices are deemed more important to be approximated accurately. The whole RFA procedure consists thus of a two-level optimization: an inner linear least-square curve fitting for the coefficients matrices at the numerator of Eq. (5) and an outer nonlinear optimization for the aerodynamic poles β_i .

The GAF of the control surfaces are instead cast into time domain through a quasi-steady approximation so that

$$\begin{aligned}\widetilde{\mathbf{Q}}_{hc}(s, M) &= \mathbf{D}_{0c} + \frac{l_a}{V}\mathbf{D}_{1c}s + \left(\frac{l_a}{V}\right)^2\mathbf{D}_{2c}s^2 = \\ &= \mathbf{Q}_{hc}(0, M) + \frac{l_a}{V}\mathbf{Q}'_{hc}(0, M)s + \frac{1}{2}\left(\frac{l_a}{V}\right)^2\mathbf{Q}''_{hc}(0, M)s^2\end{aligned}\quad (7)$$

Optimized Rational Function Approximation

The RFA is performed employing and comparing three non-gradient optimization algorithms: a Nelder-Mead simplex scheme, both in its unconstrained and constrained bounded version [13], a genetic algorithm [14] and simulated annealing [15]. As indicators of the goodness of the fit, the total root mean square error and the Frequency Response Assurance Criterion (FRAC) are calculated as

$$\begin{aligned}ERR &= \frac{1}{\sqrt{n_k}} \sum_{j=1}^{n_h} \left(\sum_{i=1}^{n_h} \sum_{m=1}^{n_k} w_{ij} \varepsilon_{ijm} \right)^{1/2} \\ FRAC_{ij} &= \frac{|\widetilde{\mathbf{Q}}_i(ik)^T \mathbf{Q}_j(ik)|^2}{|\widetilde{\mathbf{Q}}_i(ik)^T \widetilde{\mathbf{Q}}_i(ik)| \cdot |\mathbf{Q}_j(ik)^T \mathbf{Q}_j(ik)|}\end{aligned}\quad (8)$$

The RFA of $\mathbf{Q}_{hh}$ is performed assuming 5 aerodynamic poles at 14 reduced frequencies. The total root mean square error for the unoptimized and the three optimization algorithms is reported in Table 2. For the same number of aerodynamic poles, there is an improvement of the curve fitting error for the optimized RFAs. It is interesting to note that the three algorithms deliver different poles, but the total error is comparable.

Table 2. Total root mean square error for unoptimized and optimized $\mathbf{Q}_{hh}$ RFAs

Method	ERR	Aerodynamic poles
RFA standard	1.290E-03	0.057, 0.227, 0.510, 0.907, 1.417
RFA Nelder-Mead	6.422E-04	0.520, 0.689, 0.741, 0.999, 1.001
RFA Genetic Algorithm	5.309E-04	0.633, 0.784, 0.869, 1.041, 1.106
RFA Simulated Annealing	5.135E-04	0.608, 0.899, 0.932, 0.973, 1.189