

Brake Squeal Simulation by Energy Flow Analysis

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ABSTRACT

This paper presents a new FEA approach for brake squeal simulation, the energy flow analysis. It demonstrates that under certain conditions, two system normal modes with equal or close frequencies may start injecting vibration energy into each other, from dynamic friction mechanism. The vibrations, therefore, can “self-grow” (limited cycle), resulting in system instability, such as brake squeal and brake moan. The theoretical conditions for such positive energy flows to occur are discussed. Test examples and application case studies are presented, along with comparison with, and linkage to, complex eigenvalue analysis.

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INTRODUCTION

Complex eigenvalue analysis has become one of the main analytical tools for brake squeal simulation throughout the industry. The theoretical foundation of complex eigenvalue analysis is “friction induced instabilities”. The friction has two main effects: 1) geometric coupling and 2) negative μ -V relationship (negative damping). Both effects would lead to asymmetrical system matrices, which results in some complex modes. Complex mode always occurs in pairs. The eigenvalue of one of the conjugate pair has positive real part, meaning the vibration will diverge (self-grow), hence potential squeal. The divergence of vibration will not grow to infinite; rather, it will stop at so-called “limit-cycle” due to nonlinearities. In theory, the bigger the positive real part of a complex eigenvalue, the faster the mode will diverge. In other words, the squeal is more likely to occur. Therefore, the positive real part of a complex eigenvalue is sometimes referred to as “Squeal Index” or “Squeal Propensity”, either directly or through some arithmetic manipulations [1, 2, 3, 4, 5, 6, 7].

Complex eigenvalue analysis has been widely used in brake NVH design and development. However, it is not easy to explain why some unstable modes would “self-grow” into actual squeal, while others would not, even though some of the unstable modes have higher propensity (sometimes much higher).

Gregory and Li [8] attempted to explain the generation of system instability through analyzing power flow at the frictional interface. The model was extremely simplified. Chung and Donley [9] presented a numerical perturbation analysis to explain how the friction changes system eigenvalues. However, it does not address how to relate frequency changes with system instability.

In this paper, a systematic approach of energy flow analysis is presented [11] (also see ¹ Authors' Notes about this paper). A general FEA based computer program was developed for disk brake

squeal simulation applications¹. This paper demonstrates that two close modes can potentially cause simultaneous positive energy flows, due to dynamic friction mechanism. In-plane tangential and out-of-plane vibrations are both important for positive energy flow. The linkage between energy flow analysis and complex eigenvalue analysis is also discussed.

ENERGY FLOW: THE THEORY

Basic Equations

Consider a disk brake system consisting of two parts, caliper part (C), which includes pads, support bracket, caliper body and piston, and rotor part (R).

At the friction interface between pads and rotor, subscripts **s** and **n** refer to tangential (sliding) direction DOFs and the normal (axial) direction DOFs, respectively.

Consider a friction-free brake system with a normal mode **i**. Equations (1), (2), (3) describe the displacement, velocity and interface normal force related to mode *i*,

$$\left\{ \begin{matrix} x_s^i(t) \\ x_{s:R}^i(t) \end{matrix} \right\} = \left\{ \begin{matrix} x_{s:C}^i(t) \\ x_{s:R}^i(t) \end{matrix} \right\} = \alpha^i(t) \left\{ \begin{matrix} X_{s:C}^i \\ X_{s:R}^i \end{matrix} \right\} \sin(\omega^i t) \quad (1)$$

$$\left\{ \begin{matrix} v_s^i(t) \\ v_{s:R}^i(t) \end{matrix} \right\} = \left\{ \begin{matrix} \dot{x}_{s:C}^i(t) \\ \dot{x}_{s:R}^i(t) \end{matrix} \right\} = \alpha^i(t) \omega_i \left\{ \begin{matrix} X_{s:C}^i \\ X_{s:R}^i \end{matrix} \right\} \cos(\omega^i t) \quad (2)$$

$$\{f_n^i(t)\} = \begin{Bmatrix} f_{n:C}^i(t) \\ f_{n:R}^i(t) \end{Bmatrix} = \begin{Bmatrix} f_{n:C}^i(t) \\ -f_{n:C}^i(t) \end{Bmatrix} = \alpha^i(t) \begin{Bmatrix} F_{n:C}^i \\ -F_{n:C}^i \end{Bmatrix} \sin(\omega^i t) \quad (3)$$

In above equations, $\begin{Bmatrix} X_{s:C}^i \\ X_{s:R}^i \end{Bmatrix}$ and $\begin{Bmatrix} F_{s:C}^i \\ -F_{s:C}^i \end{Bmatrix}$ are the mass-normalized modal shape vector of mode (i) , and the corresponding modal force vector, respectively. $\alpha^i(t)$ is the modal coordinate response of mode i , and $\{f_n^i(t)\}$ is the dynamic normal contact force of mode I (see [Appendix](#): Definitions and Formulations for more information).

The dynamic portion of the friction force of mode i is therefore,

$$\{f_s^i(t)\} = \begin{Bmatrix} f_{s:C}^i(t) \\ f_{s:R}^i(t) \end{Bmatrix} = \mu \alpha^i(t) \begin{Bmatrix} F_{n:C}^i \\ -F_{n:C}^i \end{Bmatrix} \sin(\omega^i t) \quad (4)$$

Similar groups of equations as [Equations \(1\), \(2\), \(3\), \(4\)](#) can be written for any other system normal mode j , by replacing superscript i by j , and replacing $\omega^i t$ by $\omega^j t + \phi_{ij}$, where ϕ_{ij} represents the phase difference between mode j and mode i at time zero.

Energy Flow to Mode I and Mode J

When an object moves at a speed of $\vec{v}(t)$ and a force $\vec{f}(t)$ is applied on it, the energy going into this object by the applied force is,

$$E(t) = \int_0^t \vec{v}(t) \cdot \vec{f}(t) dt \quad (5)$$

In a brake system, friction at the pad/rotor interface is the only mechanism that may inject energy into the system, and hence potentially leading to unstable vibrations.

A brake assembly has many system normal modes. Each of these modes can potentially exert friction force on mode i , therefore influencing the vibration of mode i .

Assuming constant sliding, and the sliding motion is superimposed with the above mentioned vibration related to mode i . Therefore, within a full cycle of mode i ($T^i = 2\pi/\omega^i$), the total energy going into mode i , $E_{\rightarrow i}(T^i)$, is,

$$E_{\rightarrow i}(T^i) = \int_0^{T^i} \{f_s^{all}(t)\}^T \{v_s^i(t)\} dt \quad (6)$$

where,

$$\{f_s^{all}(t)\} = \{f_s^{st}(t)\} + \{f_s^i(t)\} + \{f_s^j(t)\} + \{f_s^{rest}(t)\} \quad (7)$$

In [Eq. \(7\)](#), $\{f_s^{st}(t)\}$ is static friction force, and considered constant. Its net energy contribution to mode i over one cycle period of T^i is averaged out to zero; $\{f_s^i(t)\}$ is dynamic friction force from mode i itself. $\{f_s^j(t)\}$ is dynamic friction force from mode j , whose frequency ω^j is close to that of mode i , ω^i . As illustrated later in this paper, under certain conditions, $\{f_s^j(t)\}$ can become the driving force to input energy into mode i and to make it grow; $\{f_s^{rest}(t)\}$ is dynamic friction force from all other modes. Depending on the frequencies of these modes, over one cycle (T^i) of this mode, the net energy to mode i by these forces may not be zero. However, because of frequency differences between these modes and mode i , the net energy to mode i , integrated over multiple cycles, will be much smaller, hence negligible.

Assume that within one cycle of mode i , the changes of modal responses of mode i and mode j are small ($\alpha^i(t) \approx \bar{\alpha}^i$ and $\alpha^j(t) \approx \bar{\alpha}^j$). By substituting [Equations \(2\)](#) and [\(4\)](#) into [\(6\)](#), the energy flow to mode i by its own force, $E_{i \rightarrow i}(T^i)$, is,

$$\begin{aligned} E_{i \rightarrow i}(T^i) &= \{F_s^i\}^T \{V_s^i\} \int_0^{T^i} \alpha^i(t) \alpha^i(t) \sin(\omega^i t) \cos(\omega^i t) dt \\ &\approx \bar{\alpha}^i \bar{\alpha}^i \{F_s^i\}^T \{V_s^i\} \int_0^{T^i} \sin(\omega^i t) \cos(\omega^i t) dt = 0 \end{aligned} \quad (8)$$

From [Eq. \(8\)](#) we can see that the energy flow to mode i from its own friction force is always 0. This is because, for any mode i , its own friction force and velocity in the frictional direction are always perpendicular (90 degree in phase) to each other. In other words, a normal mode cannot energize itself on its own by absorbing vibration energy from friction.

[Eq. \(6\)](#) finally becomes

$$\begin{aligned} E_{\rightarrow i}(T^i) &= E_{j \rightarrow i}(T^i) = \int_0^{T^i} \{f_s^j(t)\}^T \{v_s^i(t)\} dt \\ &\approx \bar{\alpha}^j \bar{\alpha}^i \{F_s^j\}^T \{V_s^i\} \int_0^{T^i} \sin(\omega^j t) \cos(\omega^i t) dt \end{aligned} \quad (9)$$

Similarly, the energy flow to mode j , $E_{\rightarrow j}(T^j)$ is,

$$\begin{aligned} E_{\rightarrow j}(T^j) &= E_{i \rightarrow j}(T^j) \\ &\approx \bar{\alpha}^i \bar{\alpha}^j \cdot \{F_s^i\}^T \{V_s^j\} \int_0^{T^j} \sin(\omega^i t) \cos(\omega^j t + \phi_{ij}) dt \end{aligned} \quad (10)$$

Idealized Energy Flow

For a conservative system with mode i and j , the vibration can never “self-grow”. If mode i gained energy from mode j , then mode j would lose same amount of energy. In reality, the vibration cannot even sustain, due to structural damping and frictional heat dissipation.

However, friction force is not conservative. From Equations (9) and (10), we can see that it is possible for mode i and j to both gain energy simultaneously due to friction. When the energy gained is enough to overcome damping, the vibration can sustain or even “self-grow”.

Below is an idealized case to help conceptually explain the energy flow theory. Assuming: (1) modes i and j have same frequency ($\omega^i = \omega^j$); (2) mode j is lagging mode i by 90 degrees ($\phi_{ij} = -90^\circ$); (3) for mode i , the dynamic friction forces $\{f_s^i(t)\}$ are all in-phase with the corresponding displacements in frictional direction, $\{x_s^i(t)\}$; (4) for mode j , the dynamic friction forces $\{f_s^j(t)\}$ are all out-of-phase (180°) with the corresponding displacements in frictional direction, $\{x_s^j(t)\}$. (Assumptions (3) and (4) are possible because the phase of dynamic friction force of each mode is associated with the corresponding normal force, which can be in-phase or out-of-phase with displacement at friction interface.)

The assumptions (3) and (4) are made only to simplify the explanation. They are not a necessary condition for the energy flow theory.

Fig. 1 shows the possible relationship of friction direction displacements, velocities and friction forces of mode i and j .

It can be seen from Fig. 1 that the friction forces of mode i and mode j , $\{f_s^i(t)\}$ and $\{f_s^j(t)\}$, are always in-phase with the velocities of mode j and mode i , $\{v_s^j(t)\}$ and $\{v_s^i(t)\}$, respectively. Thus, as long as the above phase relationships are maintained, there are always positive energy flows into mode i from the friction forces of mode j , and similarly, positive energy flows into mode j from the friction forces of mode i .

This means that when mode i gains energy with the help of mode j , the energy level of mode j does not necessarily go down. Instead, both modes i and j may help each other to gain vibration energy simultaneously - through friction mechanism, not from each other. This is the first conclusion of energy flow theory.

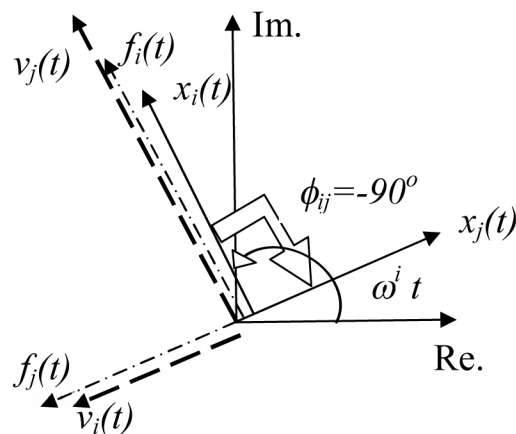


Figure 1. Ideal phase relationship for energy flow between mode i and mode j

Practical Energy Flow

Some simplifications and idealizations made in previous section may not always hold in reality. Therefore, more practical considerations are discussed in this section.

Phase Condition of Energy Flow

This refers to the phase relationships between the force and displacement (and velocity) vectors at the interface within a normal mode. In the previous section, idealized conditions (3) and (4) were assumed.

Under the condition of $\omega_i = \omega_j (= \omega)$, hence $T = T (= T)$, Equations (9) and (10) become (details see Appendix),

$$E_{\rightarrow i}(T) = \beta_{ij} \cdot \{F_{n:C}^j\}^T \cdot (\{X_{s:C}^i\} - \{X_{s:R}^i\}) = \beta_{ij} \cdot EFI_{\rightarrow i} \quad (11)$$

$$E_{\rightarrow j}(T) = -\beta_{ij} \cdot \{F_{n:C}^i\}^T \cdot (\{X_{s:C}^j\} - \{X_{s:R}^j\}) = -\beta_{ij} \cdot EFI_{\rightarrow j} \quad (12)$$

where $EFI_{\rightarrow i}$ and $EFI_{\rightarrow j}$ are Energy Flow Intensity (EFI) to mode i and j , respectively; and β_{ij} is defined as

$$\beta_{ij} = \mu \pi \bar{\alpha}^i \bar{\alpha}^j \cdot \sin(\phi_{ij}) \quad (13)$$

If modes i and j are not in-phase nor out-of-phase, then $\beta_{ij} \neq 0$. We can see that it is still possible to have positive energy flows into both modes i and j , simultaneously, if and when Eq. (14) is satisfied:

$$\begin{cases} \text{sign}(\beta_{ij} \cdot EFI_{\rightarrow i}) > 0 \\ -\text{sign}(\beta_{ij} \cdot EFI_{\rightarrow j}) > 0 \end{cases} \quad (14)$$

where $\text{sign}(Y) = Y/|Y|$ ($Y \neq 0$). Some discussions:

- If Eq. (14) is satisfied, the bigger $|EFI_{\rightarrow i}|$ (or $|EFI_{\rightarrow j}|$) is, the more energy mode i (or mode j) will gain, hence the vibration of mode i (or mode j) more likely to grow.
- If $|EFI_{\rightarrow i}|$ (or $|EFI_{\rightarrow j}|$) is too small to overcome damping, the energy flow may attenuate very quickly, and the system is likely to remain stable.
- Therefore, Eq. (14) is necessary but not sufficient condition for squeal to occur.
- Eq. (14) does not have to hold all the time. A system can become unstable during the time period when Eq. (14) holds. This might explain squeals' often intermittent behavior.
- The energy flow approach establishes the relationship between system instability and in-plane & out-of-plane vibrations at pad/rotor interface, and quantifies its impact on squeal generation.

However, in actual implementation of energy flow approach, it is not known. For practical application, we modify Eq. (14) as below,