

An Application of Shape Optimization to Brake Squeal Phenomena

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ABSTRACT

The present paper describes an application of non-parametric shape optimization to disc brake squeal phenomena. A main problem is defined as complex eigenvalue problem in which the real part of the complex eigenvalue causing the brake squeal is chosen as an objective cost function. The Fre'chet derivative of the objective cost function with respect to the domain variation, named as the shape derivative of the objective cost function, is evaluated using the solution of the main problem and the adjoint problem. A selection criterion of the adoptive mode number in component mode synthesis (CMS), which is used in the main problem, is presented in order to reduce the computational error in complex eigenvalue pairs. A scheme to solve the shape optimization problem is presented using an iterative algorithm based on the H^1 gradient method for reshaping. For an application of the optimization method, a numerical example of a practical disc brake model is presented. The numerical result illustrates that the real part of the target complex eigenvalue monotonously decreases until it reach zero.

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INTRODUCTION

Brake squeal is a vibration phenomenon in the frequency range of between 1kHz and 15 kHz caused by friction between the disc rotor and the pads. Since brake squeal causes customer dissatisfaction, a method for preventing brake squeal during the design stage is strongly desired. In the design stage, the target value of brake squeal performances in assembly level are detailed into the target values in each components level such as eigenvalues. Then, the shape of each component is designed in order to satisfy the detailed target value. In this design manner, it is necessary for engineer to execute reiterative study between estimate of the performances in assembly level and design of shapes in component level. In order to complete this reiterative study by engineer, an effective solution is to utilize optimization method in assembly level based on mathematical model of brake squeal phenomena.

A number of studies to mathematical model of the disc brake squeal were conducted. Mills [1] explained brake squeal using the stick-slip vibration phenomenon caused by the friction force. North [2] introduced a simple model of a rotor and a pad between which Coulomb friction occurs and considered the brake squeal to be a self-excited vibration induced by the friction force. Based on these findings, Millner [3] revealed that the stiffness matrix becomes

asymmetric in the rotor and pad model with Coulomb friction and by that the natural vibrations are determined by solutions of a complex eigenvalue problem. He also reported that if the real part of a complex eigenvalue is positive, since the amplitude of the natural vibration increases with respect to time, a dynamic instability occurs. The comprehensive review of mathematical model of the disc brake squeal is summarized by Kinkaid et al. [4]. Based on the mathematical model, a number of studies were produced to analyze the dynamic instability of brake squeal by using the finite element method [5,6].

Moreover, since the 2000s, researches in order to determine the optimum shape have been started. Lee and Kikuchi [7] and Guan et al. [8] presented formulations of the parametric optimization problem by choosing eigenvalues of the components in the brake model and the real part of the complex eigenvalue that generates the brake squeal as the design variables and the objective cost function respectively. Nelagadde and Smith [10] presented a method to obtain optimum shapes of the components of the brake model by using commercial software, in which the frequency separation between the critical modes are increased under constraining conditions of the frequency separation between other selected modes. Soh and Yoo [11] analyzed the optimum shape of the caliper housing using the real part of the complex eigenvalue as the objective function. However,

since these studies are based on parametric optimization method, it seems that it is difficult to apply these methods to the shape optimization problem which has large number of design variables.

On the other hand, there were some researches based on non-parametric shape optimization method which can deal with large number of design variables. Goto et al. [9] presented a method to find the shapes of the brake components as a solution of the non-parametric shape optimization problem using the differences of the eigenvalues from the target values as the objective cost function. However, since this method is only based on component level, it is not sufficient for engineers to execute reiterative study between estimation of the performances in assembly level and shapes design in component level. Shintani et al. [12] presented a solution of the non-parametric shape optimization problem to brake squeal phenomena in assembly level based on the H^1 gradient method [13, 14, 15] for reshaping. In this study, the positive real part of the complex eigenvalue generating the brake squeal, which is regarded as performance in assembly level, is chosen as an objective cost function. By using this solution, reiterative study between estimation of the performances in assembly level and shape design in component level is automatically executable. However, no practical application by using this solution has been presented yet. In order to apply this solution to practical brake model using finite element analysis with a large number of DOFs, it is necessary to solve main and adjoint problem with high accuracy due to the optimization result based on H^1 gradient method depends on the accuracy of shape derivative which is calculated from numerical results of main and adjoint problems.

For solving the solution of complex eigenvalue problem, namely the main and the adjoint problem in brake squeal phenomena, a direct solver using the Hessenberg method and an iterative solver using the Lanczos method are widely used. By applying a direct solver, all eigenvalue pairs can be calculated with high accuracy, but the computational cost is large. As a result, its application is limited to small problems of approximately 10,000 DOF. On the other hand, calculating the eigenvalue pairs of large DOF problems by using an iterative solver become more possible but it has its own disadvantages, such as missing actual modes and accepting quasi-modes. To overcome these disadvantages, a component mode synthesis (CMS) that uses both a direct solver and an iterative solver is used. Using CMS make it possible to avoid the disadvantages of missing actual modes and accepting quasi-modes while using fewer computer resources.

Due to the calculated eigenvalue pairs contain computational error caused by CMS and the optimization result depends on the computational error [16], in this paper, we also present the appropriate number of adaptive modes in CMS in order to estimate shape derivative with accuracy.

BRAKE MODEL

Let us define the initial domains for a brake model as depicted in Fig. 1. Let $\Omega_0 \subset \mathbf{R}^3$ be an initial domains of linear elastic continua with Lipschitz boundary, which is denoted by $\partial\Omega_0$. $\Gamma_D \subset \partial\Omega_0$ be the

homogeneous Dirichlet boundary On $\partial\Omega_0$. Let Ω_{R0} , Ω_0 and Ω_{P0} , Ω_0 be a disc rotor domain and a pad domain, respectively. On the boundary of disc rotor $\partial\Omega_{R0}$ and the boundary of pad $\partial\Omega_{P0}$, $\Gamma_{R0} \subset \partial\Omega_{R0}$ and $\Gamma_{P0} \subset \partial\Omega_{P0}$ denote contact boundaries respectively. Let ν_P and ν_R be the normal vector, and let τ_R and τ_P be the tangents vector on Γ_{R0} and Γ_{P0} , respectively. Moreover, let α denote the stiffness on the relation between the rotor and the pad, and let μ denote the coefficient of the Coulomb friction.

We define design variable in shape optimization problem by domain variation ϕ , in which the domain changing is created by continuous one-to-one mapping $i + \phi: \Omega_0 \rightarrow \mathbf{R}^3$ by

$$\Omega(\phi) = \{(i + \phi)(x) | x \in \Omega_0\}. \quad (1)$$

The symbol i is used as the identity mapping in the present paper. To keep the property of continuous one-to-one mapping, we define the admissible set of ϕ as

$$D = \{\phi \in Y | \|\phi\|_Y < \sigma, \phi = 0 \text{ on } \Gamma_C\}, \quad (2)$$

where Y is defined by $W^{1,\infty}(\mathbf{R}^3, \mathbf{R}^3)$, and $\sigma > 0$ is chosen such that ϕ becomes bijection. Γ_C denote the shape restrict boundary during shape optimization process. In this paper, $X = H^{1,\infty}(\mathbf{R}^3, \mathbf{R}^3) \supset Y$ is used as the Banach space for the perturbation φ of ϕ in order to define the Fréchet derivatives of cost functions with respect to the domain variation as shown later.

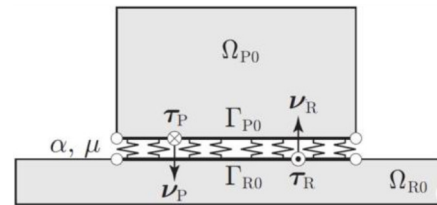


Fig. 1. Contact boundary between disc rotor domain and pad domain. The contact stiffness and Coulomb friction are defined on the contact boundary.

MAIN PROBLEM

Using the domains for the brake model described above, let us define the main problem for disc brake squeal. First, let us consider the natural vibration of the brake model. Let $\mathbf{u} \in \mathbf{R}^3$ be the displacement expressing natural vibration. In the present paper, let $\mathbf{u}$ denote the displacement in $\Omega_0(\phi)$ and let $\mathbf{u}_R$ and $\mathbf{u}_P$ denote the displacements in $\Omega_{R0}(\phi)$ and $\Omega_{P0}(\phi)$, respectively. Let

$$\mathbf{E}(\mathbf{u}) = \frac{1}{2} \left(\nabla \mathbf{u}^T + (\nabla \mathbf{u}^T)^T \right), \quad (3)$$

$$\mathbf{S}(\mathbf{u}) = \mathbf{C} : \mathbf{E}(\mathbf{u}), \quad (4)$$

denote the strain tensor and the Cauchy stress tensor, respectively, where $\mathbf{C} \in \mathcal{W}^{1,\infty}(\mathbf{R}^3; \mathbf{R}^{3 \times 3 \times 3 \times 3})$ is the stiffness of material. In the present paper, let $(\cdot)^T$ denote the transpose. Let ρ denote the density. α , μ and ρ are given as positive constants for simplicity and the notation $(\cdot)$ denote the time derivative. Based on these definitions, let us define the equations of motion for the disc brake phenomena.

For $\phi \in D$, initial displacement $\bar{\mathbf{u}}_0$ and initial velocity $\bar{\mathbf{v}}_0$, find $\mathbf{u}$ such that

$$\begin{aligned} \rho \ddot{\mathbf{u}} - (\nabla \cdot \mathbf{S}(\mathbf{u}))^T &= \mathbf{0} \quad \text{in } \Omega(\phi) \times \mathbf{R}, \\ \mathbf{S}(\mathbf{u})\nu &= \mathbf{0} \quad \text{on } \partial\Omega(\phi)/(\Gamma_R \cup \Gamma_P) \times \mathbf{R}, \\ \mathbf{S}(\mathbf{u}_R)\nu_R &= \alpha \{(\mathbf{u}_R - \mathbf{u}_P) \cdot \nu_R\} \quad \text{on } \Gamma_R \times \mathbf{R}, \\ \mathbf{S}(\mathbf{u}_P)\nu_P &= \alpha \{(\mathbf{u}_P - \mathbf{u}_R) \cdot \nu_P\} \quad \text{on } \Gamma_P \times \mathbf{R}, \\ \mathbf{S}(\mathbf{u}_R)\nu_R &= \mu \alpha \{(\mathbf{u}_R - \mathbf{u}_P) \cdot \tau_R\} \quad \text{on } \Gamma_R \times \mathbf{R}, \\ \mathbf{S}(\mathbf{u}_P)\nu_P &= -\mu \alpha \{(\mathbf{u}_P - \mathbf{u}_R) \cdot \tau_P\} \quad \text{on } \Gamma_P \times \mathbf{R}, \\ \mathbf{u}_R &= \mathbf{u}_P \quad \text{on } (\Gamma_R \cup \Gamma_P) \times \mathbf{R}, \\ \mathbf{u} &= \bar{\mathbf{u}}_0 \quad \text{on } \Gamma_D \times \{0\}, \\ \dot{\mathbf{u}} &= \bar{\mathbf{v}}_0 \quad \text{on } \Gamma_D \times \{0\}, \\ \mathbf{u} &= 0 \quad \text{on } \Gamma_D \times \mathbf{R}. \end{aligned}$$

Here, the negative sign in equation on $\Gamma_P \times \mathbf{R}$ makes the strain energy term in the weak form of equation of motion asymmetric with respect to $\mathbf{u}$.

The eigenvalue problem for the natural vibrations of the brake model becomes a complex eigenvalue problem. Since this brake model is a linear system with respect to $\mathbf{u}$, the form of separation of variables is given for some $s \in \mathbf{C}$ as

$$\mathbf{u}(\mathbf{x}, t) = e^{st} \hat{\mathbf{u}}(\mathbf{x}) + e^{s^*t} \hat{\mathbf{u}}^*(\mathbf{x}). \quad (5)$$

By substituting equation (5) into equation of motion above, we have a complex eigenvalue problem for natural vibrations as

$$\begin{aligned} \rho s_k^2 \hat{\mathbf{u}}_k - (\nabla \cdot \mathbf{S}(\hat{\mathbf{u}}_k))^T &= \mathbf{0} \quad \text{in } \Omega(\phi) \\ \mathbf{S}(\hat{\mathbf{u}}_k)\nu &= \mathbf{0} \quad \text{on } \partial\Omega(\phi)/(\Gamma_R \cup \Gamma_P) \\ \mathbf{S}(\hat{\mathbf{u}}_{Rk})\nu_R &= \alpha \{(\hat{\mathbf{u}}_{Rk} - \hat{\mathbf{u}}_{Pk}) \cdot \nu_R\} \quad \text{on } \Gamma_R \\ \mathbf{S}(\hat{\mathbf{u}}_{Pk})\nu_P &= \alpha \{(\hat{\mathbf{u}}_{Pk} - \hat{\mathbf{u}}_{Rk}) \cdot \nu_P\} \quad \text{on } \Gamma_P \\ \mathbf{S}(\hat{\mathbf{u}}_{Rk})\nu_R &= \mu \alpha \{(\hat{\mathbf{u}}_{Rk} - \hat{\mathbf{u}}_{Pk}) \cdot \tau_R\} \quad \text{on } \Gamma_R \\ \mathbf{S}(\hat{\mathbf{u}}_{Pk})\nu_P &= -\mu \alpha \{(\hat{\mathbf{u}}_{Pk} - \hat{\mathbf{u}}_{Rk}) \cdot \tau_P\} \quad \text{on } \Gamma_P \\ \hat{\mathbf{u}}_k &= 0 \quad \text{on } \Gamma_D \end{aligned}$$

for all mode number $k \in \{1, 2, \dots\}$. For compact expression of the weak form, we define the Lagrange function of the complex eigenvalue problem as

$$L_M(\phi, s_k, \hat{\mathbf{u}}_k, \hat{\mathbf{v}}_k^c) = h(s_k, \hat{\mathbf{u}}_k, \hat{\mathbf{v}}_k^c) + h(s_k^c, \hat{\mathbf{u}}_k^c, \hat{\mathbf{v}}_k^c), \quad (6)$$

where

$$\begin{aligned} h(s, \hat{\mathbf{u}}, \hat{\mathbf{v}}^c) &= a(\hat{\mathbf{u}}, \hat{\mathbf{v}}^c) + s^2 b(\hat{\mathbf{u}}, \hat{\mathbf{v}}^c) - c_R(\hat{\mathbf{u}}_R - \hat{\mathbf{u}}_P, \hat{\mathbf{v}}_R^c) \\ &\quad - c_P(\hat{\mathbf{u}}_R - \hat{\mathbf{u}}_P, \hat{\mathbf{v}}_P^c) - d_R(\hat{\mathbf{u}}_R - \hat{\mathbf{u}}_P, \hat{\mathbf{v}}_R^c) + d_P(\hat{\mathbf{u}}_R - \hat{\mathbf{u}}_P, \hat{\mathbf{v}}_P^c), \\ a(\hat{\mathbf{u}}, \hat{\mathbf{v}}) &= \int_{\Omega(\phi)} \mathbf{S}(\mathbf{u}) \cdot \mathbf{E}(\hat{\mathbf{v}}) \, d\mathbf{x}, \\ b(\hat{\mathbf{u}}, \hat{\mathbf{v}}) &= \int_{\Omega(\phi)} \rho \mathbf{u} \cdot \hat{\mathbf{v}} \, d\mathbf{x}, \\ c_{(\cdot)}(\hat{\mathbf{u}}, \hat{\mathbf{v}}) &= \int_{\Omega(\phi)} \alpha (\hat{\mathbf{u}} \cdot \nu_{(\cdot)}) (\hat{\mathbf{v}} \cdot \nu_{(\cdot)}) \, d\mathbf{x}, \\ d_{(\cdot)}(\hat{\mathbf{u}}, \hat{\mathbf{v}}) &= \int_{\Omega(\phi)} \alpha (\hat{\mathbf{u}} \cdot \nu_{(\cdot)}) (\hat{\mathbf{v}} \cdot \tau_{(\cdot)}) \, d\mathbf{x} \end{aligned}$$

Using the above definitions, we define the weak form of the eigenvalue problem for natural vibrations of the brake model and its Lagrange formulation for convenience of later description as follows.

$$L_M(\phi, s_k, \hat{\mathbf{u}}_k, \hat{\mathbf{v}}_k^c) = 0 \quad (8)$$

We call equation (8) as main problem.

SOLUTION OF COMPLEX EIGENVALUE PROBLEM

By using Galerkin method, the characteristic equation of brake squeal is given from (8) as

$$(\mathbf{K} + \mathbf{K}_\mu + s_k^2 \mathbf{M}) \hat{\mathbf{u}}_k = 0, \quad (9)$$

where, $\mathbf{K}$ is stiffness matrix obtained from the first term of the right-hand side of equation (7), $\mathbf{M}$ is mass matrix obtained from the second term of the right-hand side of equation (7) and $\mathbf{K}_\mu$ is the asymmetric stiffness matrix caused by friction between rotor and pad obtained from the third and sixth term of the right-hand side of equation (7).

To solve the complex eigenvalue problem in equation (9), the CMS is widely used due to the reason mentioned in introduction part. The procedure of CMS is as follows. Firstly, the k -th real eigen value $\lambda_k \in \mathbf{R}$ and its real eigen modes $\mathbf{p}_k \in \mathbf{R}^3$ are solved without considering $\mathbf{K}_\mu$.

$$(\mathbf{K} - \lambda_k^2 \mathbf{M}) \mathbf{p}_k = 0 \quad (10)$$

Secondly, the complex eigenmodes $\hat{\mathbf{u}} \in \mathbf{C}^3$ is assumed by using real eigenmodes as

$$\hat{\mathbf{u}} \equiv \mathbf{P} \boldsymbol{\xi}, \quad (11)$$

$$\mathbf{P} \equiv [\mathbf{p}_1 \quad \mathbf{p}_2 \quad \dots \quad \mathbf{p}_n], \quad (12)$$