

Directional Dynamics of Steering the Third Axle

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ABSTRACT

With the expectation that means of redundant steering will be necessary for highly autonomous vehicles, different methods of providing redundant steering can be considered. One potential for redundancy is to steer the rear axle for directional control of the vehicle in the event of a failure in the primary steered front axle. This paper will characterize the dynamics of directional control of a three-axle vehicle when steered at the rear, and compare it to a conventionally steered three-axle vehicle. Several compensators are suggested that allow similar vehicle dynamic behavior when steering the rear axle as a driver would expect when steering the front, giving hope that a steerable rear axle can provide acceptable redundancy for a failed primary steering system on the front axle.

CITATION: Williams, D. and Nhila, A., "Directional Dynamics of Steering the Third Axle," *SAE Int. J. Commer. Veh.* 8(2):2015, doi:10.4271/2015-01-2747.

INTRODUCTION

As more sophisticated vehicle guidance strategies are contemplated, managing failures in sensors, actuators, and other components required for the supporting control systems becomes increasingly important. Steer-by-wire systems and autonomously controlled vehicles both must be designed so that they are inherently safe in the event of a "single-point" failure. The overall goal of this functional safety effort is for these electronically controlled systems to be generally as safe as the mechanical systems they replace.

Much effort has been expended on developing these electronically controlled steering systems for passenger cars. This development requires significant resources that are more easily justified by potential passenger car volumes. Such thinking focuses on only one part (volume) of the potential revenue calculation, however. Commercial vehicles, while not having the potential volumes of passenger cars, offer a more tangible business case for adoption of this technology. Through appealing to lower fuel costs and better utilization of drivers' hours of service, the benefits of this advanced technology fall to the bottom line more quickly than a passenger car based appeal to comfort and safety.

Vehicle dynamics were largely developed for passenger cars. The "yaw-plane" or "bicycle" model captures significant directional control properties of a vehicle. Historically commercial vehicle dynamics has been treated as an extension of the passenger car analysis, along with certain assumptions and conventions that are appropriate for passenger cars. More recently, with slightly different assumptions and conventions, a yaw-plane model has been generated that can immediately characterize the directional control properties of a vehicle with any number of axles, any of which can be steered [1]. (The present work will use this model with a brief introduction, the interested reader can find a more thorough development in the references.)

Although less common than the typical two-axle passenger car configuration, many commercial vehicles contain three axles. The third axle of these commercial vehicles is occasionally steered, providing low-speed handling [2] and tire wear [3] benefits. Currently electronically controlled rear axle steering systems are offered on European production commercial vehicles that fully counter-steer at low speeds, and remain at their nominal center positions at higher vehicle speeds [4]. It has also been shown that steering the rear axle of a vehicle can improve high-speed lanekeeping [5], although for a three-axle vehicle this high speed benefit would come at the expense of increased tire wear. Qu and Zu describe theoretical opportunities to improve the handling of three-axle vehicles [6]. Bayar and Unlusoy likewise describe improvements in three-axle vehicle handling with rear axle steer [7]. Occasionally one of the three axles is "self-steered" or caster steered [8]. If the third axle of a three-axle vehicle were steered, it would be possible to move the second axle near the midpoint of the overall distance between the forward and rear axle, further increasing maneuverability and decreasing tire wear. Existing literature has addressed how steering the rear axle coordinated with the front can improve normal functionality of a three-axle vehicle.

This work describes how steering the third axle can substitute for a failed front axle steering system. Safety critical electronically controlled primary axle steering systems must have redundancy to protect against failure. Examples of such systems are steer-by-wire systems which lack mechanical connection between the handwheel and the roadwheel, and highly automated vehicles that either completely eliminate the driving task, or allow a detached driver some significant time (perhaps 20 sec) to reintegrate into the vehicle control task upon an event the automation cannot handle, including potentially a failure in the primary steering system [9].

Typically needed redundancy in a safety critical steering system is a second actuator placed in parallel with the primary actuator. That is to say, upon failure of the primary actuator the redundant actuator would function identically to the failed unit. This paper explores the possibility of an alternate source of redundancy. If the primary steering system on a three-axle vehicle fails, the vehicle could be directionally controlled by steering the rear axle. Lanekeeping vehicle dynamics will be developed for steering the rear axle only, and compared with conventional front axle steered lane keeping dynamics. It will be shown that steering the third axle can closely approximate the vehicle response expected by the driver when steering the front axle.

THREE-AXLE VEHICLE MODEL

The characterization of vehicle handling was first performed on two-axle passenger cars. It has been long appreciated that for vehicles travelling at a relatively constant forward speed with lateral accelerations so limited to correspond to the linear range of the tires, a simple model including yaw rate and lateral velocity degrees of freedom can be used to predict vehicle performance. A simple and yet illuminating vehicle model can be constructed by assuming that side forces generated by either tire on a given axle combine to act equivalently at a point where the axle intersects the centerline of the vehicle. The “axle” can be a convenient abstraction to describe a pair of independently suspended tires located the same longitudinal displacement from the center of mass. Geometry errors due to steering linkage and dual tires [10] are considered negligible. Longitudinal dynamics are likewise neglected so a constant forward speed is assumed. This assumption neglects suspension effects, therefore roll dynamics are not considered. Because such a model neglects sprung mass roll, and the action of tires on a common axle are combined, the result is referred to for obvious reasons as the “bicycle model,” and its parsimonious combination of simplicity and ability to characterize well-known vehicle properties has resulted in wide use [11, 12, 13, 14].

The development of the bicycle model is briefly repeated in this study of multi-axle vehicle dynamics for three reasons: 1.) The reader is likely most familiar with two-axle vehicles, 2.) The simple two-axle bicycle model results in well-known vehicle dynamic terminology that will be applied to more complex vehicles, and most importantly 3.) Slightly different conventions used in this work result in minor differences in this familiar formulation of a simple vehicle model, but these changes will result in a significant reduction in complexity when generalized. This revised yaw plane model has been developed in detail for use in multi-axle vehicles and is briefly summarized here [1].

In general the lateral side force developed by the i th tire can be expressed as a function of the angular difference between the direction the tire is pointing and its instantaneous velocity vector as shown in Figure 1.

The side force F_i of a displaced tire i can be expressed as:

$$F_i = C_i \left(\delta_i - \frac{v + x_i r}{u} \right) \quad (1)$$

where C_i is the linear cornering coefficient of the i th tire, δ_i is the angular steering input at the tire, x_i is the location of the tire relative to the vehicle center of gravity, v is the lateral velocity relative to a reference frame at the vehicle center of mass and rotating with the vehicle, r the yaw rate, and u the constant longitudinal velocity. For a positively defined δ_i the result is a positively defined side force.

Knowing side force generated by a given tire, a vehicle model can be constructed by summing the forces and moments produced by each tire. The front tire is steered with a displacement δ_1 as shown in Figure 1. The generalized side force expressed in (1) can be used to produce the classic bicycle model, by a steered axle 1 producing a side force F_1 a distance x_1 ahead of the vehicle center of gravity, and a fixed axle 2 producing a side force F_2 a distance x_2 behind the center of gravity of the vehicle. Since the positively defined x-axis is projected forward from the center of gravity on the centerline of the vehicle, the value of x_2 is negatively signed as it is behind the center of gravity. As shown in Figure 2 the second axle can be possibly steered with an input δ_2 .

A third axle can be located a negative distance x_3 behind the center of mass as shown in Figure 3. In this work this third axle can be either steered or rigid. Lateral forces can be summed and equated to lateral acceleration in a reference frame moving with the vehicle. As discussed, (1) can be used to express the side force of an individual tire, or with the assumptions of the yaw plane model tires can be combined on an axle to form a composite axle side force coefficient. Moving forward in this work it is assumed that a side force coefficient C_i is the combined cornering coefficient of all tires on the i th axle.

The translational equation of lateral motion with respect to a coordinate frame fixed to the vehicle is

$$m(\dot{v} + ur) = F_1 + F_2 + F_3 \quad (2)$$

where m is the total mass of the vehicle. These lateral forces also produce yawing moments about the vehicle center of gravity that can be summed and equated with the yaw acceleration, giving the yaw or rotational equation of motion,

$$I\dot{r} = x_1 F_1 + x_2 F_2 + x_3 F_3 \quad (3)$$

where I is the moment of rotational inertia of the vehicle. Together, (2) and (3) describe the dynamic behavior of a vehicle with a constant longitudinal velocity u . A vehicle drift angle β can be formed by scaling the time-varying lateral velocity v by the constant longitudinal velocity u . This scaling is occasionally found in the literature, [14] and β can be physically interpreted as the opposite of the “angle of attack” of the vehicle. If the vehicle drifts it has a negative angle of attack, and vice versa.



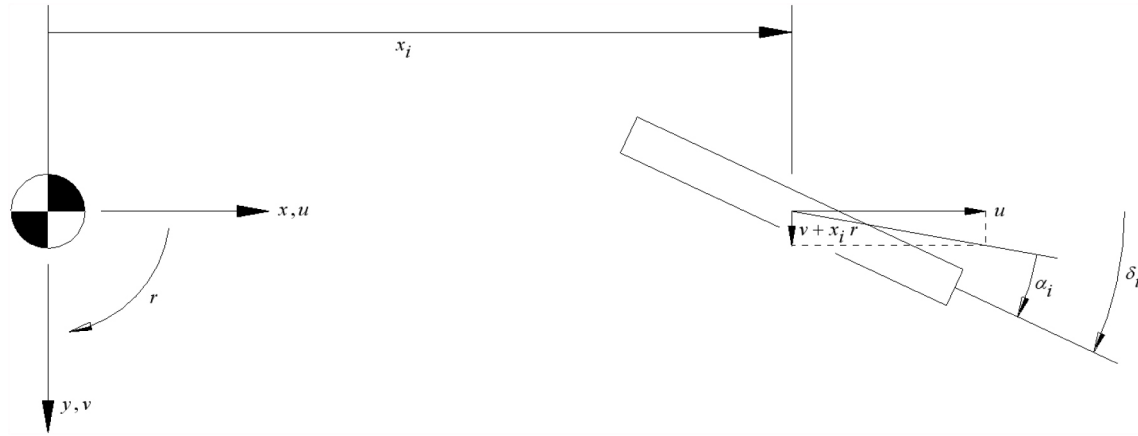


Figure 1. Generalized Tire Force Geometry

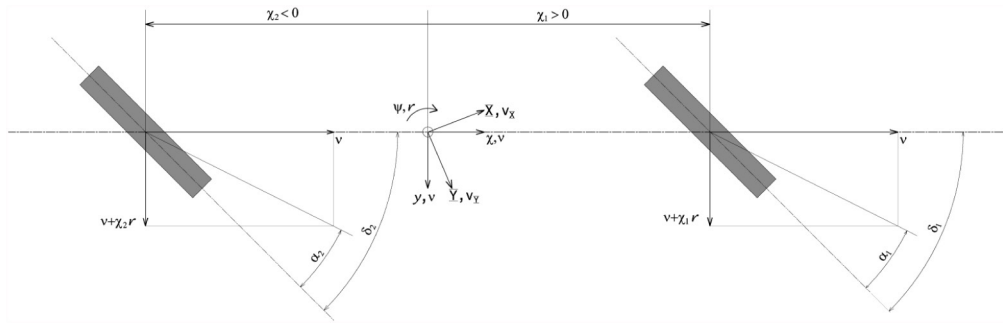


Figure 2. Geometry of Front and Rear Steering

Relative to the inertial frame, the body fixed coordinate system is rotated an angle ψ which is the integral of the yaw rate as shown in Figure 2. Using ψ the body fixed vehicle dynamic velocities can be transformed to velocities relative to the inertial frame where lane position is referenced.

Using the definition of the vehicle side-slip angle β , and if β and ψ are both small, a condition consistent with lane-keeping, velocities in the inertial frame can be written as linear functions of the state variables.

$$V_x = u$$

(4)

$$V_y = u(\psi + \beta)$$

(5)

In the lane maintenance mode the longitudinal velocity is the same constant value in both the body fixed and inertially fixed reference frames as seen in (4). Lateral velocity in the inertially fixed frame is a function of the constant forward velocity and state variables, and is the derivative of lateral displacement Y with respect to time.

$$\dot{Y} = u\beta + u\psi$$

(6)

Another inertially referenced dynamic state is apparent by noting that the time derivative of yaw angle is simply yaw rate.

$$\dot{\psi} = r$$

(7)

Equations (2), (3), (6) and (7) can be written in state-space form using (1), recalling that C_1 and C_2 are the combined side force coefficients for the front and rear axles respectively.

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \\ \dot{\psi} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} \frac{-C_1 - C_2 - C_3}{mu} & \left(\frac{-x_1 C_1 - x_2 C_2 - x_3 C_3}{mu^2} - 1 \right) & 0 & 0 \\ -x_1 C_1 - x_2 C_2 - x_3 C_3 & \frac{-x_1^2 C_1 - x_2^2 C_2 - x_3^2 C_3}{lu} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ u & 0 & u & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ \psi \\ Y \end{bmatrix} + \begin{bmatrix} \frac{C_1}{x_1 C_1} & \frac{C_2}{x_3 C_3} \\ \frac{mu}{x_1 C_1} & \frac{mu}{x_3 C_3} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}$$

(8)

Where (8) is of the form $\dot{x} = Ax = Bu$ when x is the state vector and u is the control input vector.

For brevity some convenient notation is defined.

$$C_a = C_1 + C_2 + C_3$$

$$C_b = x_1 C_1 + x_2 C_2 + x_3 C_3$$

$$C_c = x_1^2 C_1 + x_2^2 C_2 + x_3^2 C_3$$

(9 a, b, c)

The signed axle convention allows uniformity of sign in these equations.