

Probabilistic Life and Damage Assessment of Components under Fatigue Loading

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ABSTRACT

This study presents a probabilistic life (failure) and damage assessment approach for components under general fatigue loadings, including constant amplitude loading, step-stress loading, and variable amplitude loading. The approach consists of two parts: (1) an empirical probabilistic distribution obtained by fitting the fatigue failure data at various stress range levels, and (2) an inverse technique, which transforms the probabilistic life distribution to the probabilistic damage distribution at any applied cycle. With this approach, closed-form solutions of damage as function of the applied cycle can be obtained for constant amplitude loading. Under step-stress and variable amplitude loadings, the damage distribution at any cycle can be calculated based on the accumulative damage model in a cycle-by-cycle manner. For Gaussian-type random loading, a cycle-by-cycle equivalent, but a much simpler closed-form solution can be derived. Several case studies are provided to demonstrate the effectiveness of the approach.

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1. INTRODUCTION

Vehicle components and systems are subjected to mechanical, thermal or combined fatigue loading during operation, and their durability and reliability performance is a significant consideration in product design and validation [1, 2, 3]. Durability describes the ability of a product to sustain required performance over a period of time or cycles without undesirable failure. Reliability describes the ability and probability of a component or system to function under stated conditions.

Repeated fatigue failure data often exhibit large scatter for a given testing condition and the scatter comes from many sources, such as material uncertainty and loading uncertainty, which cannot be appropriately described using the traditional deterministic approach. In many cases, especially for components subjected to constant amplitude loading, continuous probability density functions (PDFs) and cumulative distribution functions (CDFs) of lognormal or Weibull distribution functions can be effectively used for characterizing the scattered fatigue test data [3], especially for components showing a single failure mechanism and mode. For failure with two or more failure mechanisms and modes, the fatigue data can be described using bi-modal or multiple-modal distributions [4, 5, 6]. Even though the underlying driving force of the probabilistic distributions and the reasons why the fatigue data follow certain distributions are still not fully understood, the role of probabilistic analysis in the analysis of durability/reliability and product validation is becoming increasingly more important.

During operation, vehicle components and systems are subjected to variable amplitude loading, and the probabilistic life assessment under such conditions is a significant issue. However, the nature of the uncertainty in the fatigue life under variable amplitude loading is not commonly considered in a majority of these applications. A particular application is the durability and reliability analysis of vehicle exhaust systems under random cyclic loading through road load data acquisition (RLDA) [2]. Additionally, the probabilistic damage assessment of a component and system at any applied cycle is also important for applications such as RLDA. Again, the current analysis methods use the deterministic approach and the linear damage accumulation rule, and therefore, cannot solve this problem. In engineering design and validation, understanding the lower bound and upper bound of damage will be beneficial in risk assessment and decision making. Recently, an inverse technique has been developed to calculate the probabilistic damage distribution at any cycle for components subjected to constant amplitude loading. The fatigue failure data and the linear or nonlinear damage accumulation law are required for the damage analysis [7].

In this paper the probabilistic life and damage assessment approach is extended to more complex loading conditions: step-stress loading and variable amplitude loading. For step-stress loading the probabilistic life and damage distribution is calculated in a cycle-by-cycle manner with the accumulative damage model. For Gaussian-type random loading, a cycle-by-cycle equivalent, but a much simpler closed-form solution is derived. Several examples are provided to demonstrate the effectiveness of the new probabilistic life and damage assessment approach and provide new insights to its engineering applications including RLDA.

2. PROBABILISTIC FATIGUE LIFE ASSESSMENT BASED ON CUMULATIVE DAMAGE ASSUMPTION

Probabilistic fatigue life assessment based on cumulative damage model [8], is provided below for three commonly used loading scenarios, i.e. constant loading, step-stress loading, and variable amplitude loading.

Under constant amplitude stress/load loading, the data scatter, variation and distribution of the fatigue cycles can be properly characterized using continuous probability density functions (PDFs) $f(N)$ and cumulative distribution functions (CDFs) $F = \int f(N)dN$. The variability in fatigue life at different stress range levels is often characterized as probabilistic distributions, which are illustrated schematically in Fig. 1 [9]. The degrees of the scatter or spread of the data are usually different at different stress range levels indicating different degrees in uncertainty of test data. Typically, smaller scatter appears at higher stresses and larger scatter at lower stresses because crack initiation, which often shows higher uncertainty than crack growth, dominates the life at the lower stress range levels [9]. The full probabilistic distributions of the fatigue failure data at all stress levels can often be properly characterized using a single continuous probability density function (PDF) $f[N(S)]$ and a cumulative distribution function (CDF) $F = \int f[N(S)]$, indicating both PDF and CDF are the functions of stress range S [8].

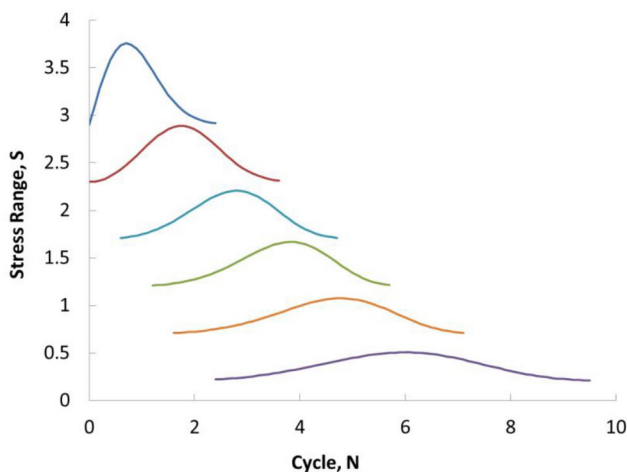


Fig. 1. Schematic illustration of the fatigue variability behavior with respect to stress level.

For step-stress loading, Nelson provides a cumulative damage model for estimating damage using the cycle-by-cycle approach [8]. A simple example is presented below to describe the procedure. A specimen is first subjected to a specified constant stress range S_1 for a specified number of cycles N_1 , followed by stress range S_2 for $N_2 - N_1$ cycles, and then by stress range S_3 for $N_3 - N_2$ cycles. The schematic step-stress loading profile for the three stress ranges is shown in Fig. 2 (a).

Fig. 2 (b) depicts the cumulative distribution functions (CDFs) of failure as functions of cycles N at the three constant stress ranges (S_1 , S_2 , and S_3). The specimen first follows the cumulative distribution for S_1 up to the first number of applied cycles N_1 . When the stress range increases from S_1 to S_2 , the specimen follows the cumulative

distribution for S_2 , starting at the cumulated damage fraction. Similarly, when the stress range increases from S_2 to S_3 , the specimen follows the next cumulative distribution, starting at the accumulated damage fraction. The cumulative distribution for life under the step-stress pattern is shown in Fig. 2 (c). It consists of the segments of the cumulative distributions for the constant stresses. In this simple way, this cycle-by-cycle model takes into account the previous damage history.

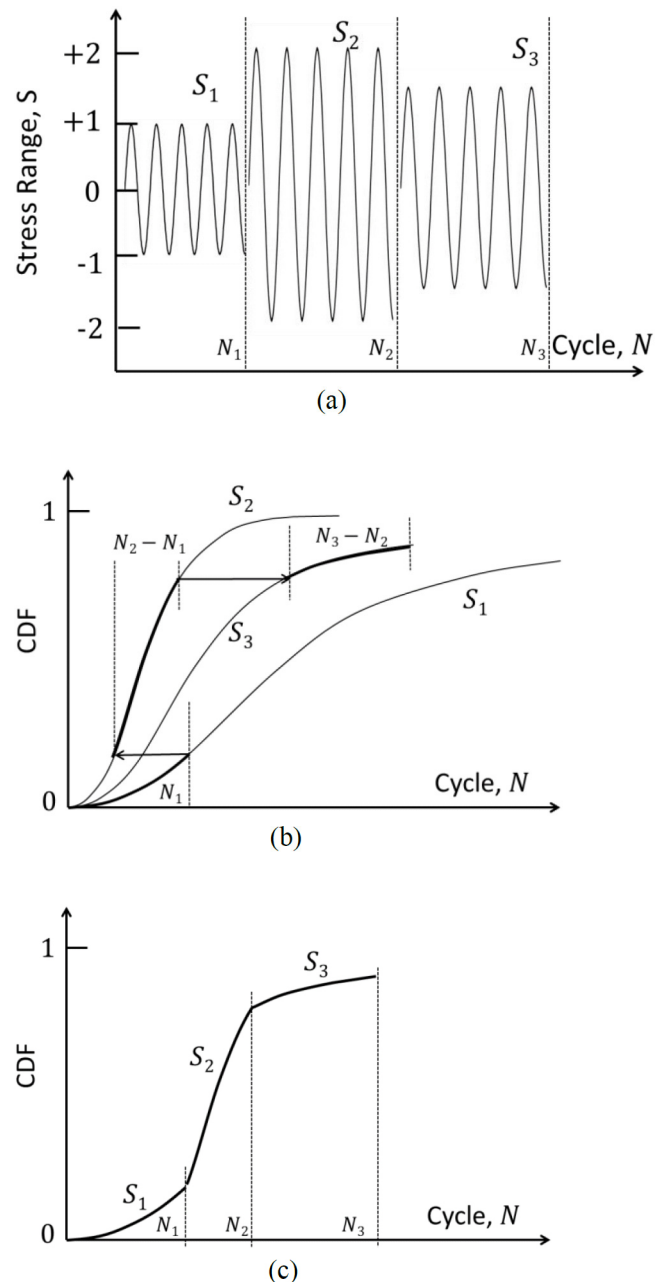


Fig. 2. Schematic of (a) step-stress loading, (b) cumulative damage estimation using the cycle-by-cycle approach, and (c) the CDF of failure under the step-stress loading.

For variable amplitude loading, the cycle-by-cycle damage assessment procedure is obviously tedious and difficult to implement, and oftentimes unnecessary, for engineering applications. A more effective way is strongly preferred. Nelson [8] suggests an equivalent

but much simpler way to handle it: a sequence independent approach. The approach is applicable to some specific categories of probabilistic distribution functions such as simple linearlognormal, linear-Weibull, and linear-exponential [8]. For a two-parameter Weibull cumulative distribution function (CDF) $F(N) = 1 - \exp[-(N/\eta)^\beta]$, it can be shown that the population fraction failed at

$$N = \sum_{i=1}^I n_i \text{ after } I \text{ steps is [8]}$$

$$F(N) = 1 - \exp \left[- \left(\sum_{i=1}^I (n_i/\eta_i) \right)^\beta \right] \quad (1)$$

where n_i is the total cycle number at stress range S_i , and n_i as functions of S_i can be obtained using cycle counting methods, such as rain flow counting method. In this model, the characteristic life is assumed to be a function of stress range S and is expressed in a power law form: $\eta(S) = (S_0/S)^\beta$, where S_0 is a reference stress range. The shape parameter β is assumed to be a constant at all stress levels for simplicity purpose [8]. For a random loading with Gaussian type, which indicates that the distributions of load range or amplitude are statistically similar for any given time (long enough to be representative) period, Eq. (1) can be simplified as

$$F(N) = 1 - \exp \left\{ - \left[N \left(\sum_{i=1}^I (\gamma_i/\eta_i) \right) \right]^\beta \right\} \quad (2)$$

where $\gamma_i = n_i/N$ are nondimensional values measuring the relative percentage of the cycles at step i against the total number of cycles N .

Clearly, $\sum_{i=1}^I \gamma_i = \sum_{i=1}^I n_i/N = 1$. Eq. (2) can also be simply expressed in an equivalent form:

$$F(N) = 1 - \exp \left[- (N/\eta_e)^\beta \right] \quad (3)$$

$$\text{with } \eta_e = \sum_{i=1}^I (\eta_i/\gamma_i).$$

Eq. (1), Eq. (2), Eq. (3) have no sequence effect, and they are a probabilistic analog of Miner's rule [2]. It is noted that for some failure modes, products, and materials, the failure behavior depends on the order of the steps, or the sequence effect [8]. Finally, it should be noted that the cumulative damage models are very convenient for engineering applications; however, the models are empirical in nature and have not been adequately verified by experience [8]. Therefore, caution must be exercised when use the models and interpret the results obtained from the models.

3. PROBABILISTIC FATIGUE DAMAGE DISTRIBUTION AT ANY APPLIED CYCLE BASED ON CUMULATIVE DAMAGE ASSUMPTION

The damage calculation is important in the durability and reliability assessment of vehicle components and systems under road loading or rig testing with RLDA data. Currently, the life or damage assessment procedure is essentially based on the deterministic approach, which does not account for the uncertainty and the probabilistic characteristics as observed in real test data. In order to overcome the shortcomings of the current approaches, a new approach has been developed recently [7]. This approach is based on the fatigue life data collected and the inverse transformation technique [10].

The procedure can be schematically illustrated with the damage-cycle diagram as shown in Fig. 3. When $N = 0$, the damage $D = 0$. $N = N_F$ indicates the failure status of the component and $D = 1$. With the PDF $f[N_F(D = 1)]$, and CDF $F[N_F(D = 1)]$, the probabilistic damage distributions at any cycle N (or time t) $f[D(N)]$ and $F[D(N)]$ can be derived provided the damage evolution process is known [7]. Both linear and nonlinear damage evolutions can be used, however, the most commonly used damage law is the linear damage rule or Miner's rule, with which damage vs cycle plot is simply a cluster of straight lines [7]. For constant amplitude loading, $D = N/N_F$ and the damages made at each cycle are the same: $D_j = 1/N_F$, j here represents the j th cycle. Mathematically, the problem is equivalent to seeking a target distribution function $F_Y(y)$ for a given initial distribution function $F_X(x)$ and transformation functions: $y = \varphi(x)$ and $x = \psi(y)$. Closed-form solutions can be obtained as follows [10].

$$f_Y(x) = \frac{dF_Y(y)}{dy} = f_X[\psi(y)] \left| \frac{d\psi(y)}{dy} \right| \quad (4)$$

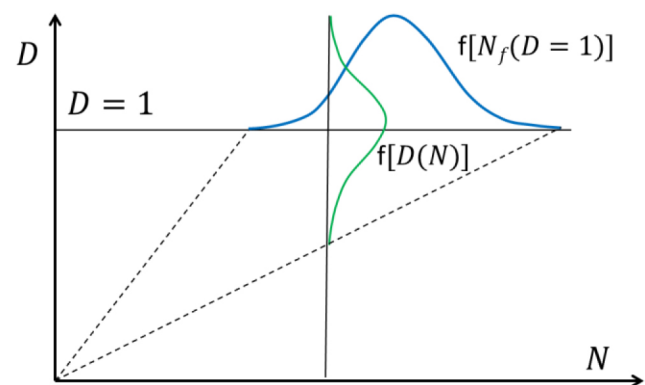


Fig. 3. The schematic of a continuous probabilistic life (failure) distribution $f[N_F(D = 1)]$ and probabilistic damage distribution $f[D(N)]$ under constant amplitude loading for a population.

For step-stress loading shown in Fig. 2, the probabilistic damage assessment procedure can be schematically illustrated in the damage-cycle diagram shown in Fig. 4. However, the cycle-by-cycle damage calculation process becomes tedious when the number of stress steps is high. For variable amplitude loading with the Gaussian