

On Practical Implementation of the Ramberg-Osgood Model for FE Simulation

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ABSTRACT

The three parameter Ramberg-Osgood (RO) method finds popular usage for extracting complete stress-strain curve from limited data which is usually available. The currently popular practice of assuming the plasticity to set in only at the Yield point provides computational advantage by separating the complete nonlinear curve, obtained from RO method, into elastic and plastic regions. It is shown, with an example problem, that serious errors are committed by using this method if one compares the obtained results with results of complete stress-strain curve.

In the present work we propose a simple Taylor series based approach based on RO method to overcome the above deficiency. This method is found to be computationally efficient. The proposed method is applicable for stress-strain curves of materials for which RO method provides a good approximation.

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INTRODUCTION

Mechanical design of a product or a component involves meeting various requirements in test as set by guidelines or standards for operating successfully in the field. Increasingly, engineering decisions are based on computed information with the expectation that the computed information will provide a reliable quantitative estimate of some attributes of a physical system or process. Finite Element Analysis (FEA) is widely used in industry to predict the stresses in components in such scenarios. The finite element method (FEM) is one of the most powerful and widely used numerical methods for finding approximate solutions to mathematical problems formulated so as to simulate the responses of physical systems to various forms of excitation. [1] [2]

It is not uncommon to observe stresses above material Yield limit in practice. For example, typically high stress near notches and fillets can lead to local yielding of material. Many physical scenarios are inherently elasto-plastic in nature. This requires a proper modeling of hardening behavior in analysis.

In practice, complete stress-strain curve is seldom available to an FE analyst and immediate mechanical testing is not typically an option. A simplified three parameter model to describe the stress-strain curve as suggested in Eq. (1) by [3] of materials for which RO method provides a good approximation.

$$\epsilon(\sigma) = \epsilon_e + \epsilon_p = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/n} \quad (1)$$

where, E is the Young's Modulus. K and n are material constants. This is a relation between True stress and True strain. The first part of the equation, σ/F , is equal to the elastic part of the strain (ϵ_e), while the second term, $(\sigma/K)^{1/n}$, accounts for the plastic part (ϵ_p). The value of n gives a measure of the material's work hardening behavior and is usually between 0 and 0.5 [4]. The equation was found to be adequate for most practical purposes, especially metals [5] [6] [7] [8] [9] [10]. The question of accuracy of Ramberg-Osgood (RO) model to model stress-strain curve for the entire stress-strain range will not be discussed any further in the present work. For this topic, the reader is referred to [11]. It is worth mentioning that for all practical purposes, no distinction is made between material Proportional limit and Yield limit.

Introducing Yield strength of the material, σ_y , and defining a new parameter, α (dimensionless parameter), related to K as

$$\alpha = \left(\sigma_y/K\right)^{1/n} \quad (2)$$

The Ramberg-Osgood equation was modified by [12]

$$\epsilon(\sigma) = \frac{\sigma}{E} + \alpha \left(\frac{\sigma}{\sigma_y}\right)^{1/n} \quad (3)$$

For a non-hardening material ($n=0$), the equation degenerates into a pair of straight lines meeting at Yield point ($\sigma=\sigma_y$). The main specification for cold formed stainless steel members in the USA is the ANSI/ASCE-8-90 [13], which adopts this modified form of RO model to describe the stress-strain behavior of material. The code defines value of the multiplication factor $\alpha=0.2\%$ which is also a common in practice. It must be pointed out that the Eq. (3) is nonlinear in σ . [14] describes a method of identification of material parameters by an inverse algorithm. [15] presents an a priori error estimates of finite element approximations of the solution for the equilibrium equation of an axially loaded RO bar.

The equation can be further simplified by bifurcating the linear/elastic and nonlinear/plastic regions of the stress strain curve as follows,

$$\varepsilon = \begin{cases} \sigma/E, & \sigma \leq \sigma_y \\ \frac{\sigma}{E} + \alpha \left(\frac{\sigma}{\sigma_y} \right)^{1/n}, & \sigma \geq \sigma_y \end{cases} \quad (4)$$

This new parameter K , is related to σ_y and α by Eq. (2). This form is particularly useful for FE simulation. In practical analysis, most of the solid domain falls under linear region and this formulation provides computational efficiency by taking advantage of the linearity of the stress-strain relation. This form finds its application in most commercial software (ANSYS®, Abaqus®, etc.). The analyst usually has material E (Young's modulus), σ_y (Yield limit), σ_{ut} (Ultimate limit) and el (uniform elongation at Ultimate limit) available. el can be used to calculate ε_u . This data along with Eq.(4), can be used to calculate the parameter n using [16],

$$n = \frac{\log(\sigma_u) - \log(\sigma_y)}{\log(\varepsilon_{pu}) - \log(\varepsilon_{py})} \quad (5)$$

where ε_{pu} and $\varepsilon_{py} = \alpha$ are plastic strain at Ultimate limit and plastic strain at Yield limit respectively.

PRESENT METHOD

This simplification does not come without a price. The resultant stress-strain curve possesses a discontinuity between the elastic and plastic region. This is due to the fact that the proportional limit and the Yield point are not coincident and the Yield point is taken at 0.2% offset. In order to remove the discontinuity, the plastic region of the curve is shifted to the left, eliminating the 0.2% offset, and connecting the plastic region of the curve to the linear elastic region at σ_y/E which has been depicted in Figure 1.

$$\varepsilon = \begin{cases} \sigma/E, & \sigma \leq \sigma_y \\ \frac{\sigma}{E} + \alpha \left(\frac{\sigma}{\sigma_y} \right)^{1/n} - \alpha, & \sigma \geq \sigma_y \end{cases} \quad (6)$$

This method will be referred henceforth as the “Shifted approach”. This shifting also changes the Yield and the Ultimate points on the stress-strain curve. This artificial shifting of curve has no physical justification and adds significantly to the modeling error, as will be shown in the Results and Discussion section.

In the present work, we propose a Taylor series based approach to overcome this deficiency and effectively capture the stress-strain curve obtained by using the RO curve fit. The methods are compared with the stress-strain curve derived from RO relations. This methodology was developed in response to an analyst's need to define the stress-strain response of the material with minimal material information.

PROPOSED METHOD

We extrapolate the second part of the Eq.(4) at the Yield point using Taylor series expansion,

$$\begin{aligned} \varepsilon = & \left(\frac{\sigma}{E} + \alpha \left(\frac{\sigma}{\sigma_y} \right)^{1/n} \right) \Bigg|_{\sigma_y} \\ & + \frac{1}{1!} \left(\frac{1}{E} + \frac{\alpha}{n\sigma_y} \left(\frac{\sigma}{\sigma_y} \right)^{1/n-1} \right) \Bigg|_{\sigma_y} (\sigma - \sigma_y) \\ & + \frac{1}{2!} \left(\frac{\alpha}{n\sigma_y^2} \right) \left(\frac{1}{n} - 1 \right) \left(\left(\frac{\sigma}{\sigma_y} \right)^{1/n-2} \right) \Bigg|_{\sigma_y} (\sigma \\ & - \sigma_y)^2 + \text{H. O. T} \end{aligned} \quad (7)$$

where H.O.T stands for higher order terms. Eq.(7) can be simplified to,

$$\begin{aligned} \varepsilon = & \left(\frac{\sigma}{E} + \alpha \right) \Bigg|_{\sigma_y} \\ & + \frac{1}{1!} \left(\frac{1}{E} + \frac{\alpha}{n\sigma_y} \right) (\sigma - \sigma_y) \\ & + \frac{1}{2!} \left(\frac{\alpha}{n\sigma_y^2} \right) \left(\frac{1}{n} - 1 \right) (\sigma - \sigma_y)^2 + \text{H. O. T} \end{aligned} \quad (8)$$

Not much error is committed if we ignore the higher order terms as will be shown in the Results and Discussion section. Considering only up to second order terms, we find the point of intersection of the elastic part of Eq.(4) with the curve obtained in Eq.(8)