

Design of the Linear Quadratic Control Strategy and the Closed-Loop System for the Active Four-Wheel-Steering Vehicle

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ABSTRACT

In the field of active safety, the active four-wheel-steering (4WS) system seems to be an attractive alternative and an effective tool to improve the vehicles' handling stability in lane-keeping control performance. Under normal using condition, the vehicle's lateral acceleration is comparatively small, and the mathematic relationship between the small side force excitation and the small slip angle of the tire is in the linear region. Furthermore, the effects of roll, heave, and pitch motions are neglected as well as the dynamic characteristics of the tires and suspension system in this work. Therefore, the linear quadratic control (LQC) theory is used to ensure that the output of the 4WS control system can keep track of the desired yaw rate and zero-sideslip-angle response can also be realized at the same time.

Due to the development of Steer-by-Wire (SBW) technology, the driver's steering input can be easily converted into digital signals and calculated by the SBW conversion mechatronic control system, which utilize these digital signals to regulate the front and rear wheel steering angle to achieve the ideal steering responses. And the attention is paid to the more fundamental aspects of the vehicle's horizontal motions. Based on these statements, the differential equations of 2DOF linear model for the 4WS vehicle is derived to describe the vehicle's horizontal motions with relatively high precision in essence. In addition, a numerical method is presented to evaluate the steering characteristics and handling stability. Output results show that the 4WS system with LQC algorithms has higher accuracy in path tracking and better performance in improving the active safety.

CITATION: Zhang, Z., Huang, M., Ji, M., and Zhu, S., "Design of the Linear Quadratic Control Strategy and the Closed-Loop System for the Active Four-Wheel-Steering Vehicle," *SAE Int. J. Passeng. Cars - Mech. Syst.* 8(1):2015, doi:10.4271/2015-01-9107.

INTRODUCTION

With the development of control algorithms and even further rules and observances on safety, the active chassis control technologies, such as active suspension, ABS, ASR, and DYC etc. have already got application on vehicles [1, 2, 3]. The SBW 4WS conversion mechatronic control system is computer-controlled and automatically adjusts the rear steering angle according to the steering wheel's position, vehicle velocity, and other variables. What's more, the active four-wheel-steering control system can enhance the vehicle's steering responses with the suitable optimization algorithm by improving the maneuverability at low speed and the handling stability at high speed.

The optimal control theory has made great progress due to the rapid development of computer industry and high-quality requirements in controlling vehicle system variables. The optimal control goal is to achieve the desired responses of the closed-loop system on the premise of the given limiting conditions and the evaluation function. The given limiting conditions refer to the physical constraints and the evaluation function is the objective function used to evaluate the performance of the system.

A few papers have taken the control algorithm into consideration to enhance the vehicle's comfort and performance. The study of L. Pascali, P. Gabrielli and G. Caviasso is to show that 4WS system is able not only to compensate handling performance decay due to suspension layout replacement, but even to maximize handling performance and increase longitudinal comfort performance. So the identification of the rear suspension module for maximizing longitudinal comfort is the initial step in order to build vehicle model [4]. As stated by [5], the active four steering (4WS) control system is designed to enhance vehicle lateral dynamics through controlling rear steer angle. The optimal and fuzzy logic control theories are used to design the system controllers. The integrated model connects the two systems via the Magic Formula Tire Model to represent the tire non-linearity during augmented longitudinal and lateral dynamic attitudes. The Kalman filter algorithm is successfully applied to derive a practical control strategy of 4WS system in [6], and the practical system uses only feedback signals of lateral acceleration and front steering angle to obtain the control law. Measurement noise is taken into account and results are generated to obtain the step response of the outputs of interest.

Methods of qualitative description have constantly been applied to the research and evaluation. However, none of these papers have proposed a numerical method to evaluate the steering characteristics and handling stability for quantitative analysis. In addition, a closed-loop system including the driver, the vehicle and the road is seldom taken into account to design control algorithms. In this paper, we propose a numerical method and build a system of 'driver-vehicle-road' to show the superiority of the LQC algorithm by quantitative and qualitative analysis.

In this paper, the following items are finished:

1. The model for 'driver-vehicle-road' closed-loop handling system is built to conduct the simulation and evaluation.
2. The numerical method is presented to evaluate the active safety and handling stability.
3. The comparison between the output responses of proportional control strategy and that of the LQC (linear quadratic control) strategy is made to verify the superiority of the LQC strategy for four-wheel-steering control.

DESIGN OF THE OPTIMAL CONTROLLER

Physical and Mathematical Model of the 4WS Vehicle

The simplest possible model of the vehicle that runs at a constant speed over the even horizontal road is taken and considerable insight into the vehicle's horizontal motions is given. We neglect the influence of roll, heave, and pitch motions as well as the dynamic characteristics of the tires and suspension system [7]. The slip angles of front and rear tires are in the region where the linear tire physical model is valid (lateral acceleration $a_y < 0.4g$) [8]. Derived from these statements, the simple physical model used to exemplify the vehicle dynamics is shown in Figure 1. This is an equivalent bicycle physical model, only taking into account lateral and yaw motions of the vehicle body. We place the moving rectangular coordinate system's origin at the vehicle's center of gravity and set the vehicle body's longitudinal direction as coordinate axis X and lateral direction as coordinate axis Y, respectively. In the steady-state condition, the rotation of the vehicle can be modeled in the moment and force equation as being about the center of gravity.

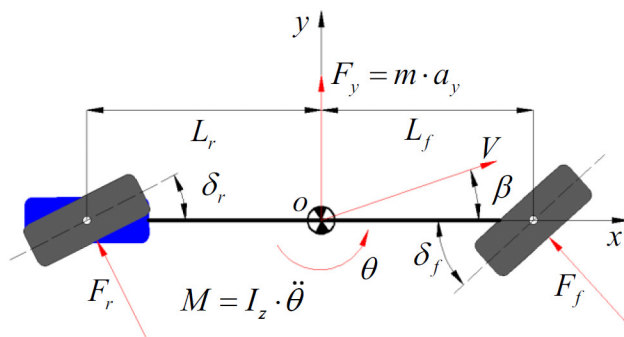


Figure 1. Equivalent bicycle physical model with lateral force characteristics for front and rear tire.

The equivalent bicycle mathematical model corresponding to the Euler-Lagrange equations of motions may be described as:

$$\begin{cases} F_y = F_f \cos \delta_f + F_r \cos \delta_r \\ M = L_f F_f \cos \delta_f - L_r F_r \cos \delta_r \end{cases} \quad (1)$$

Under normal circumstances, the steering and slip angles of tires may be limited to relatively small values. So the variation of the geometry is supposed to remain linear, which is described as:

$$\begin{cases} \lim_{x \rightarrow 0} \sin x = x \\ \lim_{x \rightarrow 0} \cos x = 1 \end{cases} \quad (2)$$

And this is similar for the steering angle. Then, the driving force required to keep the vehicle's speed constant can be assumed to remain small with respect to the lateral tire force and the influence of the driving force's lateral component is neglected. With the use of Equation (1) and Equation (2), the following equations of motion for the bicycle model are established:

$$\begin{cases} F_y = F_f + F_r \\ M = L_f F_f - L_r F_r \end{cases} \quad (3)$$

The schematic diagram for the bicycle model's kinematics analysis in the case of a curved trajectory, as depicted in Figure 2, shows the vehicle's horizontal motion in a given time interval Δt in the steady-state condition. The reference coordinate system (O, X, Y) is fixed on the ground and the moving rectangular coordinate system (o, x_p, y_p) are attached to the center of gravity of the vehicle.

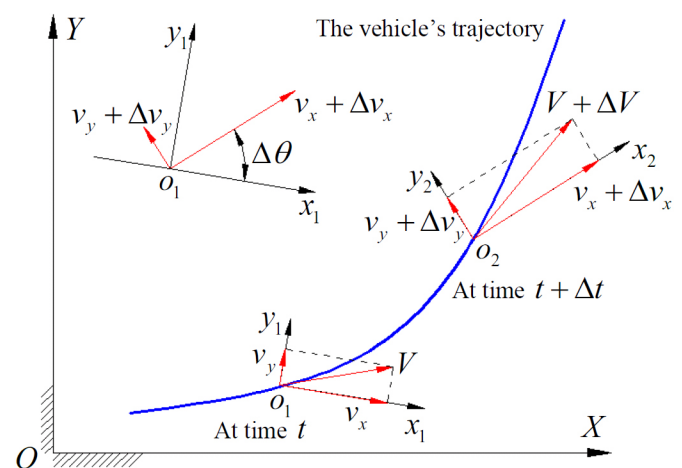


Figure 2. Schematic diagram for the simple vehicle model's kinematics analysis.

The velocity increment in Y direction of the moving rectangular coordinate system (o_1, x_1, y_1) is obtained as:

$$\Delta v_{o_1 y_1} = (v_x + \Delta v_x) \sin \Delta \theta + (v_y + \Delta v_y) \cos \Delta \theta - v_y \quad (4)$$

With all the terms of the second order of magnitude (products of variables) are neglected and the use of Equation (2), Equation (4) becomes:

$$\begin{cases} v_x = V \cos \beta \approx V \\ v_y = V \sin \beta \approx V \beta \\ \Delta v_{o_1 y_1} = v_x \Delta \theta + \Delta v_y \end{cases} \quad (5)$$

Then, the following useful formulae for the expression of the lateral acceleration in Y direction of the moving rectangular coordinate system (o_1, x_1, y_1) can be found:

$$a_y = \lim_{\Delta t \rightarrow 0} \frac{v_x \Delta \theta + \Delta v_y}{\Delta t} = V \frac{d\theta}{dt} + V \frac{d\beta}{dt} = V(\omega + \dot{\beta}) \quad (6)$$

Therefore the mathematical model of the equivalent bicycle can be expressed in terms of the relative parameters as follows:

$$\begin{cases} m a_y = m V(\omega + \dot{\beta}) = F_f + F_r \\ I_z \dot{\omega} = L_f F_f - L_r F_r \end{cases} \quad (7)$$

The slip angles of the front and rear tires can be obtained as [6]:

$$\begin{cases} F_f = k_f \left(\beta + \frac{L_f \omega}{V} - \delta_f \right) \\ F_r = k_r \left(\beta - \frac{L_r \omega}{V} - \delta_r \right) \end{cases} \quad (8)$$

Our goal of the control algorithm is to regulate the lateral deviation and yaw deviation at the center of gravity, which ensures the output of the vehicle system can keep track of the ideal steering response as we expected. By deriving Equation (7) and Equation (8), the control law of the front and rear steering angle can be obtained as the following transfer function:

$$\begin{cases} \frac{\delta_f(s)}{\delta(s)} = \frac{1}{e_f} [(a_f s + b_f) \frac{\beta(s)}{\delta(s)} + (c_f s + d_f) \frac{\omega(s)}{\delta(s)}] \\ \frac{\delta_r(s)}{\delta(s)} = \frac{1}{e_r} [(a_r s + b_r) \frac{\beta(s)}{\delta(s)} + (c_r s + d_r) \frac{\omega(s)}{\delta(s)}] \end{cases} \quad (9)$$

$$\begin{cases} a_f = -m L_r V; a_r = -m L_f V \\ b_f = (L_f + L_r) k_f; b_r = (L_f + L_r) k_r \\ c_f = -I_z; c_r = I_z \\ d_f = -m L_r V + \frac{L_f (L_f + L_r) k_f}{V} \\ d_r = -m L_f V - \frac{L_r (L_f + L_r) k_r}{V} \\ e_f = (L_f + L_r) k_f; e_r = (L_f + L_r) k_r \end{cases} \quad (10)$$

Converting the differential equations of motion (Equation (7)) into the state space representation is always effective when properties of the linear control system are the subject of investigation. We choose two state variables (the body sideslip angle β and the yaw rate ω) for the linear first order vehicle system. The dynamic behavior of the vehicle is studied by three quantities: the lateral acceleration a_y , the yaw rate ω and the sideslip angle β defined at the center of gravity. The equation in matrix notation is expressed as:

$$\dot{X} = AX + BU \quad (11)$$

$$\text{with } X = \begin{bmatrix} \beta \\ \omega \end{bmatrix}; \quad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}; \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$U = \begin{bmatrix} \delta_f \\ \delta_r \end{bmatrix};$$

$$\begin{cases} a_{11} = -\frac{k_f + k_r}{mV}; a_{12} = \frac{L_r k_r - L_f k_f}{mV^2} - 1 \\ a_{21} = \frac{L_r k_r - L_f k_f}{I_z}; a_{22} = -\frac{L_f^2 k_f + L_r^2 k_r}{I_z V} \\ b_{11} = \frac{k_f}{mV}; b_{12} = \frac{k_r}{mV} \\ b_{21} = \frac{L_f k_f}{I_z}; b_{22} = -\frac{L_r k_r}{I_z} \end{cases} \quad (12)$$

Design of the Ideal 4WS Vehicle Model

For the conventional 4WS vehicles, the steady-state gain of the yaw rate is obviously different compared with the FWS (front-wheel-steering) vehicle. This difference makes it more difficult in driving and bring more passive fatigue to the driver, especially at high speed. Most drivers do not always respond correctly to the yaw rate and the lateral acceleration, but they are acutely sensitive to them. So it is reasonable to regulate the front and rear steering angles to achieve the same steady-state gain as that of the FWS vehicle and to get the zero-sideslip-angle response as expected. To get better driving experiences and road feels, there should be no phase lag in lateral