

On Optimum Choice of Forchheimer Equation Coefficients for Computational Fluid Dynamics Simulation of Heat Exchangers

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ABSTRACT

The simulation of heat exchanger air flow characteristics using Computational Fluid Dynamics requires knowledge of the experimental pressure drop across the element concerned. This is normally achieved through wind tunnel testing of either full scale heat exchangers or by using laminations of various porous materials and honeycombs to represent these. The current paper both continues and compliments prior work published by the lead author, which entailed a series of measurements of the pressure drop in both the near and far field, across screens with porosity (β) in the range $0.41 < \beta < 0.76$. This experimental investigation established a relationship between the porosity and the pressure drop characteristics of a given material at various angles of inclination to the free-stream flow. In addition, the effect of screen depth was investigated using honeycombs. The present paper investigates the use of a weighted least squares regression model to correct the previously obtained Forchheimer Equation coefficients for residual errors associated with the data being a function of the underlying experimental measurements. Following a review of the available relations for pressure drop through woven screens, it was concluded that the best method to use in obtaining the Forchheimer Equation coefficient is when each screen is treated individually rather than using a general correlation that relates the permeability and inertial coefficient to the porosity of the screen. Porous jump conditions in the computational procedure were used to model thin membranes with known velocity vs. pressure-drop characteristics. The CFD results matched closely the experimental data, with only a slight variation when modelled using coefficients obtained through data fitting. In contrast, the simulation results varied from the experimental data when only the material thickness value was modified, and not the corresponding porous jump coefficients. A best practice recommendation was thus established.

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INTRODUCTION

The flow of fluid through porous screens is of engineering interest due to the wide range of potential applications including heat exchanger simulations [1], insect entry into greenhouses [2], and flow control in wind tunnels [3].

Previous investigations of the flow through such materials have been performed by wind tunnel testing as well as by numerical simulation. These have mainly involved the measurement of the pressure distribution and flow visualization around porous screens and gauzes. The first of such was carried out by Taylor [4] with the study of the theoretical relationship between gauze porosity and drag. Subsequently, Taylor and Davies [5] extended their study to the air resistance of perforated sheets, gauze and fabrics.

Schubauer et al. [3] conducted a series of experiments dealing with the pressure drop and streamline deflection of damping screens for angles of inclination up to 45 deg. They showed that the pressure drop is only a function of the solidity of the screen and the velocity component normal to it.

Baines et al. [6] conducted analytical and experimental studies on the relative pressure drop as a function of screen form and porosity, capacity of screens to modify velocity distribution and the characteristics of turbulence that the screens produce. Carrothers et al. [7] summarized all the available correlations for the pressure drop coefficients through wire mesh screens and textile nettings.

Further studies of flow in the wakes of perforated plates and gauze screens were carried out by Reynolds [8], who set up a mathematical model to predict the deflection of the flow passing through gauzes.

Brundrett [9] formulated a new general correlation equation for pressure drop through woven wire and cloth screens for incompressible flows using a pressure loss coefficient (K_θ).

$$\Delta P = 0.5K_\theta \rho u^2$$

$$K_\theta = \cos^2 \theta \left(\frac{1 - \beta^2}{\beta^2} \right) \left[\frac{\sigma_m}{\sigma_k} \frac{7.125}{Re \cos \theta} + \frac{0.88}{\log(Re \cos \theta + 1.25)} \right]$$

$$+ 0.055 \log(Re \cos \theta)$$



Miguel [10] investigated airflow characteristics of a number of screening materials. Based on the obtained results he concluded that the shape of the yard and mesh geometry have a negligible influence on the airflow characteristics of screening materials. Additionally, he proposed equations that relate the screen permeability (K) and inertial factor (Y) to the screen porosity (β):

$$K = 3.44 \times 10^{-9} \beta^{1.6} \quad Y = 4.3 \times 10^{-2} \beta^{-2.13} \quad (2)$$

Valera et al. [11] evaluated the geometrical characteristics and airflow resistance of upright insect-proof screens. Based on obtained pressure drop coefficients, they proposed an overall governing equation for the relationship between the porosity of the screen and Reynolds number of the airflow. Different correlations were suggested for the screen permeability and inertial factors as a function of the porosity:

$$K = 5.68 \times 10^{-8} \beta^{3.68} \quad Y = 5.67 \times 10^{-2} \beta^{-1.1604} \quad (3)$$

Flores-Velazquez et al. [12] suggested yet another relationship between Y and K , based on previously published data:

$$K = 2.0 \times 10^{-7} \beta^{3.3531} \quad Y = 0.342 \times 10^{-2} \beta^{-2.5917} \quad (4)$$

Teital et al. [2, 13, 14] investigated the effect of screen inclination on the flow parameters behind porous screens using Computational Fluid Dynamics (CFD). Furthermore, they compared the simulation of the flow through a realistic woven screen and that through the porous media approach.

Gerova et al [1] measured the pressure drop through screens with porosity in the range 0.41- 0.76. They established a relationship between the porosity and the pressure drop characteristics of a given material at various angles of inclination to the free-stream flow. Furthermore, the effect of screen depth was investigated using honeycombs.

In the present study, the simulation of porous materials and honeycombs was computationally investigated. The aim being to establish the most appropriate choice of Forchheimer coefficients.

METHODOLOGY

The purpose of the study is to discuss the optimum choice of Forchheimer Equation coefficients for CFD simulation of heat exchangers. The experimental set-up used to obtain the data discussed in this paper was published by Gerova et al. [1]. Due to this only a brief overview of the fore mentioned set-up and fluid flow physics will be presented in this Methodology.

In previous work Gerova et al. obtained pressure drop data across screens of various porosities and at different angles of inclination. This was achieved using a blower wind tunnel with the porous screen fixed to the working section so as to cover its entire cross section.

The pressure tubes from the static, Pitot-static and reference ring pressure tapping were connected to OMEGA PX 13 pressure transducers. The relative error of the pressure transducers specified by the manufacturer was $\pm 0.3\%$ of the full Range of 0.3 psi (2.068 kPa), and thus giving an error of ± 0.0009 psi (± 6.2 Pa).

The wind tunnel measurements were conducted at 0, 15, 30 and 45 degrees angles of inclination. The wind tunnel velocity was measured with a centre line Pitot-static tube at 113.5 mm from the working section floor. The wind tunnel speed varied between 3 and 35 m/s.

Porosity (β) was defined as the ratio of the open screen area to complete screen area, thus, values can be between 100% (theoretically) and 0% (solid flat plate). The stated porosities had a measurement tolerance of approximately ± 0.03 .

For the purpose of nondimensionalising the obtained data, a pressure drop coefficient $\frac{\Delta P}{q}$ was introduced. It was defined as the ratio of the pressure drop across the screen to the dynamic pressure in the working section.

Assuming an average thickness of 1 mm for the tested meshes in the current experiments, the Reynolds number range was between approx. 400 (7 m/s) and 2000 (35 m/s).

For a thin porous medium with finite thickness, the pressure change can be defined by Darcy's Law:

$$u = \frac{K}{\mu} \frac{\partial P}{\partial x} \quad (5)$$

where μ is the dynamic viscosity and, K the permeability, P the static pressure and x the direction of flow. It is valid for isothermal laminar fluid flow with Reynolds number less than 1. However, for Reynolds numbers larger than 10 a breakdown in linearity is observed [15]. This type of flow regime can be expressed with Forchheimer's Equation [16]:

$$\frac{\partial P}{\partial x} = \frac{\mu}{K} u + \rho \left(\frac{Y}{K^2} \right) u^2 \quad (6)$$

Where ρ is the fluid density, μ is the dynamic viscosity, K the permeability and Y the inertial factor, u is the velocity normal to the porous face.

The permeability and inertial factor can be obtained by equating a second order polynomial ($\Delta P = Au^2 + Bu$) to Forchheimer's Equation [2,10,11,17,18]:

$$K = \frac{\mu \Delta x}{B} \quad Y = \frac{A \sqrt{K}}{\Delta x \rho} \quad (8)$$

The porous media approach is widely used in simulating screens in Computational Fluid Dynamics. When the Navier-Stokes equations are solved numerically the source term in the momentum equation is presented by Forchheimer's Equation.

RESULTS AND DISCUSSION

Use of Weighted Least Squares for Coefficient Generation

The standard linear regression model used to obtain the A and B coefficients makes strong assumptions about the behavior of the errors, more specifically, that the errors are normally and independently distributed with zero mean and constant variance (σ^2). Experimental measurements, however, have inherently non constant variance of the error term. In this experiment the measurements were obtained using strain gauge pressure transducers. The accuracy of the transducers is related to the inverse of the measured pressure. The errors of the underlying (pressure measurement) were therefore non constant, with a larger variance for lower velocities. Examination of the residuals using the Breusch-Pagan test found that it was significant at a level of 5% and therefore the residuals were found to be heteroskedastic.

In order to obtain the original A and B coefficients by Gerova et al. [1] the Curve Fitting Toolbox in Matlab was used. Matlab implements the method of least fitted squares. To obtain the coefficient estimates, the least-squares method minimizes the summed square of the residuals. The residual for the i^{th} data point (r_i) is defined as the difference between the observed response value (y_i) and the fitted response value ($\hat{y}_i$):

$$r_i = y_i - \hat{y}_i \quad (9)$$

The summed square of residuals is given by:

$$S = \sum_{i=1}^n r_i^2 = \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (10)$$

Where n is the number of data points included in the fit and S is the sum of the squared error estimates.

To remove the heteroskedasticity present within the residuals, weighted least squares regression was used. This was done through the addition of scale factor or weight (w_i) to the fitting process. The weights define how much each value influences the final parameter estimates.

$$S = \sum_{i=1}^n w_i (y_i - \hat{y}_i)^2 \quad (11)$$

$$w_i = \frac{1}{\sigma_i^2} \quad (12)$$

The weighting matrix used was White's heteroskedasticity-consistent estimator [19]. A summary of the unweighted and weighted A and B coefficients is presented in Table 1 below.

Table 1. Summary of calculated weighted and unweighted A and B coefficients for tested screens with a variable porosity (β) and angle of inclination between 0 and 45 deg.

0 deg	β	A	B	A weighted	B weighted
	0.76	0.37	1.52	0.31	0.26
	0.70	0.27	1.07	0.35	0.88
	0.64	0.38	0.23	0.38	0.20
	0.50	0.81	1.23	0.83	0.82
	0.41	1.30	7.13	1.41	4.95
15 deg	β	A	B	A weighted	B weighted
	0.76	0.30	0.43	0.27	1.11
	0.70	0.29	0.97	0.30	0.76
	0.64	0.35	0.30	0.35	0.47
	0.50	0.78	1.24	0.79	0.17
	0.41	1.39	5.68	1.37	3.9
30 deg	β	A	B	A weighted	B weighted
	0.76	0.26	0.19	0.26	0.15
	0.70	0.27	0.61	0.27	0.56
	0.64	0.34	0.42	0.33	0.52
	0.50	0.75	0.40	0.75	0.40
	0.41	1.26	4.42	1.27	3.96
45 deg	β	A	B	A weighted	B weighted
	0.76	0.22	0.04	0.21	0.24
	0.70	0.26	0.36	0.25	0.49
	0.64	0.32	0.17	0.31	0.41
	0.50	0.59	0.48	0.59	0.58
	0.41	1.05	3.48	1.06	3.13

Estimation of Forchheimer Equation Coefficients

Few studies exist that relate screen porosity and its permeability and inertial factor. Nonetheless, the studies predict vastly varying values for the A , B coefficients. Such a study into the discrepancies in published literature was conducted by Teitel et al. [13] for $Re < 60$. Due to the limited range of Re investigated by Teitel et al., a secondary study was needed to be conducted for flow regimes applicable to simulation of heat exchangers ($Re > 500$, $V \sim 8\text{ m/s}$).

Teitel et al. [13] investigates the accuracy of the proposed global relationship between permeability and inertial factor to porosity by Miguel [10]. They concluded that the proposed relations given in Equation 2 lead to a large overestimation of the pressure drop values. They predicted approximately 5 times higher K and 1.5 times higher Y values. Similar discrepancy was observed when comparing the obtained pressure drop by Miguel for higher Reynolds numbers (see Figure 1).