

Introducing Functional Data Analysis to Coast Down Modeling for Rolling Resistance Estimation

Lasse G. Andersen and Jesper K. Larsen
Roskilde University Center

ABSTRACT

Coast-down modeling has been widely used to assess vehicle aerodynamic drag and rolling resistance by fitting a vehicle resistance model to speed measurements and thereby get an estimate on model parameters. Here a coast-down model is used for assessing how road surface characteristics influence rolling resistance. Parameter estimation as well as an extensive perturbation analysis of the parameter fit with respect to data noise has been performed. Functional Data Analysis (FDA) is introduced and discussed as a tool for this. It is concluded that FDA is a powerful tool for 1) approximating derivatives, 2) assessing the degree of smoothing of the data 3) handling noise sources in the perturbation analysis and 4) enabled numerical solutions of the coast-down Ordinary Differential Equation (ODE) model. Investigations showed that MPD was the most important parameter compared to IRI although MPD data required smoothing for optimal model fit. Furthermore, it is concluded that the model responds nicely to the statistical tests in the perturbation analysis. However, certain parameters associated with surface related rolling resistance were unstable in the sensitivity tests.

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INTRODUCTION

Environmental concerns related to energy consumption has increased rapidly in the last decades. Since transportation by car accounts for about 20% of the worlds energy consumption [21], reducing energy consumption in this area is highly desirable. One way of lowering energy consumption in car transportation is to decrease rolling resistance, primarily by developing low rolling resistance tires and pavements. Thus a deeper understanding of the relationship between rolling resistance and the tire/pavement interaction is an important step in reducing energy consumption (as well as noise) and thereby reducing CO₂ emissions.

Four standard methods exist for measuring or estimating rolling resistance [19]: Drum (laboratory) measurements, measurements by trailer [4, 18, 23, 24], coast-down procedures [6, 11, 20, 2, 12, 7, 8, 9] and fuel consumption measurements [17, 1]. These methods can be ranked by how idealized the test setup is: The drum only measures the tire, the trailer method includes road properties and some transmission losses, coast-down includes even more features such as whole-car transmission losses, various road properties etc. Finally, the fuel consumption measurement procedure is the most natural setting compared to real-world car transportation. However, as the measuring methods come closer and closer to actual driving, the number of error sources increases as well, since more and more factors influence the measurements and have to be taken into account. Thus, these four methods represents different trade-offs between disturbing factors and affinity with actual driving on real roads. Each of these methods have their advantages and short-comings so they are suited for different purposes. In this paper the coast-down method is

used since it includes transmission losses that can be considered as part of rolling resistance, and which is not included in trailer measurements. The downside of the coast-down is the many potential disturbing factors that can influence measurements such as, wind, traffic, and irregularities in transmission losses etc.

Large data sets with a comprehensive amount of supplement data, obtained by the Swedish National Road and Transport Research Institute (VTI) [7], is analyzed. The widely used coast-down procedure can be performed to various degrees of detail both with respect to data and regression model. In this coast-down experiment different measurements accounting for road properties and meteorological conditions have been collected in connection with speed data. A suitable model that utilizes these data has been formulated.

The mathematical and statistical framework of 'Functional Data Analysis' (FDA) is proposed as a resourceful tool in handling these large data sets and the primary goal of this paper is to show that FDA is a useful choice in handling these amounts of high resolution data. In addition, Monte Carlo and bootstrapping methods are applied in a sensitivity analysis of the coast-down parameter estimation results.

COAST-DOWN DATA AND MODEL

The coast-down measurements used here have been performed by VTI and the procedure will be briefly described here. For a detailed account of the coast-down measurement procedure, see [7]. A suitable set of road strips were selected which was subject to certain requirements. Among these are

- The longitudinal gradient and road curvature must be low.
- The set of road strips must have high variation in Mean Profile Depth (MPD) and International Roughness Index (IRI) in order to get the optimal experimental setting.
- Different speed limits such that the coast-downs could be performed with varying initial velocities.

Furthermore, meteorological variations had been kept at a minimum, e.g., no rain, low wind and steady temperature. Fourteen different road strips have been selected and by including direction this yields 28 different road strips. The average length of the strips ranged between 400 and 1000 meters. Beginning and end of the coast-down sections were marked by reflective tape which could be detected by a photo sensor mounted on the rear bumper of the car. On each of the 28 strips various quantities were measured with a 1 meter resolution before the actual coast-down procedures were performed. The measured quantities of interest here are

- Roughness: IRI [#]
- Road texture: MPD [mm]
- Longitudinal gradient [%]
- Road crossfall [%]
- Road curvature radius [m]

Table 1. Overview of road sections

Road ID	Length [m]	Speed Limit [km/h]	# of runs	Avg. IRI [mm/m]	Avg. MPD [mm]
1	1000	90	19	0.83	1.10
2	1000	90	19	1.05	1.12
3	300	70	21	2.04	2.54
4	300	70	19	1.79	2.49
5	600	70	13	1.18	1.19
6	600	70	15	1.68	1.21
7	1000	90	14	1.40	0.82
8	1000	90	14	1.08	0.76
9	500	70	15	2.64	2.42
10	500	70	14	3.23	2.40
11	900	90	15	3.30	1.00
12	900	90	14	3.55	0.93
13	500	70	14	2.18	1.07
14	500	70	14	2.15	0.90
15	400	70	14	2.57	0.74
16	400	70	14	2.84	0.59
17	580	70	14	2.82	0.86
18	580	70	14	2.48	0.86
19	550	90	15	1.11	0.39
20	550	90	14	1.02	0.44
21	570	70	14	2.23	1.76
22	570	70	15	2.40	1.72
23	560	90	15	3.66	0.64
24	560	90	14	3.05	0.75
25	620	70	14	1.77	0.59
26	620	70	14	1.50	0.58
27	630	90	16	0.79	0.66
28	630	90	14	0.83	0.58

Other measured quantities of interest are wind speed, air temperature and air pressure which are measured at the beginning of the coast-down. An overview of measured quantities and extreme values are shown in table 1. The test car used is a Volvo 940 that had been warmed up at least 30 minutes before every coast-down series to ensure equilibrium tire temperature and pressure. The speed measurements were obtained by a wheel pulse instrument that measured position and time at approximately every 77 cm. In order to have a data set of varying speed, coast-down measurements were carried out in sequences of varying speed. Depending on the speed limit for a particular road strip, the measurements were carried out in sequences of 70-65-60-65-70 km/h or 90-80-70-80-90 km/h. The initial velocities are target values, since a specific initial speed is difficult to achieve. To avoid additional error sources, the driver was able to press an abort button indicating that measurements henceforth are invalid in subsequent analysis. E.g., if oncoming traffic occurred during a coast-down procedure, the abort button was activated and the measurements discarded. An average of 15 coast-downs were performed for each road strip and direction yielding approximately 420 coast-downs.

The measurements were used to estimate the coefficients of a coast down model [7], which can be written as a sum of forces acting on the vehicle during coast-down

$$F = F_{roll} + F_{air} + F_{side} + F_g \quad (1)$$

where F is the total force acting on the vehicle, F_{roll} is the rolling resistance force, F_{air} is aerodynamic drag, F_{side} is the side force resistance and F_g is gravitational pull. It is assumed that the forces acting on the vehicle is additive. Note that F_{roll} used here is empirical and tentative and is given by [7, p. 24]

$$F_{roll} = m(\eta_0 + \eta_1 T + \eta_2 \text{IRI} + \eta_3 \text{IRI}(v - 20) + \eta_4 \text{MPD} + \eta_5 \text{MPD}(v - 20)) \quad (2)$$

with v being speed, T temperature and the η_i 's ($i = 1..5$) are rolling resistance regression coefficients to be estimated. F_{roll} is defined in a fairly general way such that η_0 also accounts for transmission losses from, e.g., the gearbox [7, p.15], in addition to surface independent tire losses. Since the latter is known to be dependent on air temperature through the pressure-temperature relation of the tire at equilibrium, an additional surface independent and temperature dependent term $\eta_1 T$ is added. The surface texture and roughness dependent part of the rolling resistance are assumed to be linearly dependent on speed, as shown in the latter four terms. F_{air} is a simplified version of the drag equation

$$F_{air} = 0.5\rho(T, p)A_{yz}C_L[v - \cos(\alpha)w]^2 \quad (3)$$

where A_{yz} is projected frontal area of the vehicle, C_L the air dynamic drag coefficient, w wind speed, α is the wind angle, and $\rho(T, p)$ is the air density function given by

$$\rho(T, p) = K_\rho \frac{p}{T + T_0} \quad (4)$$

with p being atmospheric pressure, $K_\rho = 0.3847 \text{ }^\circ\text{K s}^2/\text{m}^2$ and $T_0 = 273.2 \text{ }^\circ\text{K}$ are conversion constants. Since w is low, eq. 3 can be simplified by expanding the quadratic term and only retaining the linear term (assuming $w^2 \approx 0$) such that

$$F_{air} = 0.5\rho(T, p)A_{yz}C_L[v - \cos(\alpha)w]^2 \approx \eta_6\rho(T, p)v^2 + \eta_7\rho(T, p)\cos(\alpha)vw \quad (5)$$

where η_6 and η_7 are regression parameters. Simplifying the expression by substituting $0.5\rho(T, p)A_{yz}C_L$ with two regression coefficients η_i seems reasonable since C_L and w are not very precise (i.e. the latter is measured only in the beginning of the coast-down and the former is hard to determine experimentally), thus viewing them as unknowns. The fact that eq. 5 is reduced by $w^2 = 0$ supports this decision as well. More elaborate expressions of F_{air} can be formulated [7, 9], but since the term is being calibrated by regression, it has been omitted here. By the same argument, tire stiffness C_A in

$$F_{side} = -\frac{F_y^2}{C_A} = \eta_8 F_y^2 \quad (6)$$

is replaced by a regression parameter η_8 . F_y is given by

$$F_y = m[\cos(\gamma)v^2/R - g\sin(\gamma)\cos(\beta)] \quad (7)$$

with γ being crossfall angle, m vehicle mass, g gravitational constant, β longitudinal gradient and R radius of road curvature. F_g is straightforwardly formulated as

$$F_g = -mg\sin(\beta). \quad (8)$$

Finally, the inertial resistance is

$$F = (m + m_{rot})\frac{dv}{dt} = (m + n_{wh}K_J J/r_{wh}^2)\frac{dv}{dt} \quad (9)$$

where m_{rot} is the inertial mass of a wheel plus additional rotating transmission parts, n_{wh} is the number of wheels, r_{wh} the radius of the wheels, J the inertial moment per wheel and K_J a correction factor of J to include rotating transmission parts. Inserting eq. 2, 5, 6, 8 and 9 into eq. 1, and rearranging, yields

Table 2. List of parameters in coast-down model

Parameter	unit	description
v	[m/s]	Velocity of vehicle
IRI	[mm/m]	Roughness measure
MPD	[mm]	Macrottexture measure
$\rho(T, p)$	[kg/m ³]	Air density
p	mbar	Pressure
A_{yz}	[m ²]	Projected frontal area of the vehicle
C_L	[-]	Air dynamic coefficient
α	[rad]	Angle between wind and driving direction
w	[m/s]	Wind speed
T	[°C]	Temperature
T_0	[°C]	Temperature
C_A	[N/radians]	Tire stiffness
m	[kg]	Vehicle mass
g	[m/s ²]	Earth's gravitational acceleration
β	[radians]	Longitudinal slope
γ	[radians]	Road crossfall
K_J	[-]	Correction factor
K_ρ	[kg°K m ⁻³ mbar ⁻¹]	Conversion constant
n_{wh}	[-]	Number of wheels
J	[kg m ²]	Inertial moment per wheel
r_{wh}	[m]	Wheel radius
m_{rot}	[kg]	Inertial mass of rotating parts



$$\begin{aligned} \frac{dv}{dt} = & \mu_0 + \mu_1 T + \mu_2 \text{IRI} + \mu_3 \text{IRI}(v - 20) \\ & + \mu_4 \text{MPD} + \mu_5 \text{MPD}(v - 20) + \mu_6 \frac{\rho(T, p)v^2}{m + m_{rot}} \\ & + \mu_7 \frac{\rho(T, p) \cos(\alpha) wv}{m + m_{rot}} + \mu_8 \frac{F_y^2}{m + m_{rot}} - \kappa g \sin(\beta) \end{aligned} \quad (10)$$

where $\kappa = m/(m + m_{rot})$ and $\mu_i = \kappa \eta_i$ for $i \in \{0, \dots, 5\}$ and $\mu_i = \eta_i$ for $i \in \{6, 7, 8\}$. $\mu = [\mu_0, \dots, \mu_8]'$ are the regression coefficients to be estimated from the coast-down data above. See [table 2](#) for at complete list of the parameters presented here. Since the road data is measured per meter the final regression model must have distance as the independent variable. Therefore [eq. 10](#) is converted to

$$\begin{aligned} \frac{dv}{ds} = & \frac{1}{v} [\mu_0 + \mu_1 T + \mu_2 \text{IRI} + \mu_3 \text{IRI}(v - 20) \\ & + \mu_4 \text{MPD} + \mu_5 \text{MPD}(v - 20) + \mu_6 \frac{\rho(T, p)v^2}{m + m_{rot}} \\ & + \mu_7 \frac{\rho(T, p) \cos(\alpha) wv}{m + m_{rot}} + \mu_8 \frac{F_y^2}{m + m_{rot}} - \kappa g \sin(\beta)] \end{aligned} \quad (11)$$

making an assessment of rolling resistance and its various components achievable. Normally the acceleration data have to be calculated by finite differencing of the speed measurements, which usually yields noisy fluctuating results. For this reason, speed measurements have been sought to be avoided entirely [12]. Below the derivative of the speed data is extracted from the speed data in functional form. Furthermore, the supplement data will be converted to functions as well, such that it is possible to study the model by solving [eq. 11](#) numerically by, e.g., a Runge-Kutta algorithm.

FUNCTIONAL DATA ANALYSIS

This is a very brief introduction to functional data analysis tailored to the present needs. For a general and thorough exposition to the computational, mathematical and statistical aspects, see [15] and [16]. The basic idea in FDA is to convert discrete data into functions for further study, e.g., statistical investigations. Data in functional form can be treated by different methods that utilizes properties pertaining solely to functions, such as smoothness. Usually a functional data function is a linear combination of several basis functions. This can be a truncated Fourier series expansion, sum of wavelets or a sum of B-spline polynomials, the latter being used here. We have

$$\tilde{y}(t) = \sum_{i=0}^m c_i \phi_i(t) = \mathbf{c}' \boldsymbol{\phi}.$$

where $\boldsymbol{\phi} = [\phi_1(t), \dots, \phi_m(t)]'$ is the vector of B-spline basis polynomials and $\mathbf{c} = [c_1, \dots, c_m]'$ is the vector of coefficients ('denotes the transpose operation). Since B-splines are closed with respect to linear combinations $\tilde{y}(t)$ is a B-spline as well. The degree of smoothness of $\tilde{y}(t)$ depends on the degree of the $\phi_i(t)$'s such that N degree

polynomials $\phi_i(t)$ yields a C^{N-1} function $\tilde{y}(t)$. For a detailed account of B-splines, see [3]. Normally $N = 3$ which is also the case here, so the functional data used below have continuous second order derivatives.

Fitting the B-spline basis system is done by an extension of the ordinary least squares (OLS) algorithm that includes a *smoothness penalty*. Assuming a set of data points $y = [y_1, \dots, y_n]$ measured at times $t = [t_1, \dots, t_n]'$ we have

$$\begin{aligned} L(\mathbf{c}) = & \sum_{i=1}^n [\tilde{y}(t_i) - y_i]^2 + \lambda \int [D^2 \tilde{y}(t)]^2 dt \\ = & (\mathbf{y} - \boldsymbol{\Phi} \mathbf{c})' (\mathbf{y} - \boldsymbol{\Phi} \mathbf{c}) + \lambda \mathbf{c}' \mathbf{R} \mathbf{c} \end{aligned} \quad (12)$$

where the first term is classical OLS and the second term is the aforementioned smoothness penalty. Φ is the $n \times m$ matrix of basis function evaluated at times t and R is the $m \times m$ matrix of inner products where the (i, j) entry is given by $\int D^2 \phi_i(s) D^2 \phi_j(s) ds$. The specific choice of differential operator $D^2 f(t) = \frac{d^2 f}{dt^2}$ is based on the assumption that a smooth curve means a curve with low curvature [16, p.84]. Since the integral equals zero if $\tilde{y}(t) = at + b$ the fit based on [eq. 12](#) will converge to a straight line as $\lambda \rightarrow \infty$. Thus, the smoothness parameter λ represents a balance between fitting the data and obtaining a smooth curve with zero curvature. Estimating λ for some of the data is one of the objectives here. For fixed λ a unique $\hat{\mathbf{c}}$ exists which minimizes [eq. 12](#). Since no numerical optimization is involved, finding $\hat{\mathbf{c}}$ is quick and computationally simple.

When the speed and additional data have been brought in a functional form it is straightforward to estimate the regression parameters μ by minimizing

$$\begin{aligned} L(\boldsymbol{\mu}) = & \int \left\{ \frac{d\tilde{v}(s)}{ds} - F[\tilde{v}(s), \tilde{R}(s), \tilde{\beta}(s), \tilde{\gamma}(s), \tilde{\text{MPD}}(s), \tilde{\text{IRI}}(s)] \right\}^2 ds \end{aligned} \quad (13)$$

where F is the right hand side of [eq. 11](#) and $\tilde{v}(s)$, $\tilde{R}(s)$, $\tilde{\beta}(s)$, $\tilde{\gamma}(s)$, $\tilde{\text{MPD}}(s)$, $\tilde{\text{IRI}}(s)$ are the additional data in functional form. In the optimization process the integral must be approximated by a quadrature rule which is Simpsons rule in this case. Using a quadrature rule reduces the optimization problem to a squared residual one, for which several specific algorithms exist, e.g., the Gauss-Newton optimization routine. More advanced FDA-based parameter estimation techniques exist, which takes, e.g., c into the optimization process [14, 22]. However, this case is linear in the parameters so a simple method have been used.

Several advantages exist in the simple methodology sketched above. Firstly all the functional approach assumes that the measurements represent samples from a continuous process which is certainly true here. Secondly, estimating μ requires the derivative of the speed which is easily obtained by analytically calculating the first derivative