

A New Semi-Empirical Method for Estimating Tire Combined Slip Forces and Moments during Handling Maneuvers

Shahyar Taheri
Virginia Tech

Terence Wei
Bridgestone Americas Inc

ABSTRACT

Modeling the tire forces and moments (F&M) generation, during combined slip maneuvers, which involves cornering and braking/driving at the same time, is essential for the predictive vehicle performance analysis. In this study, a new semi-empirical method is introduced to estimate the tire combined slip F&M characteristics based on flat belt testing machine measurement data. This model is intended to be used in the virtual tire design optimization process. Therefore, it should include high accuracy, ease of parameterization, and fast computational time.

Regression is used to convert measured F&M into pure slip multi-dimensional interpolant functions modified by weighting functions. Accurate combined slip F&M predictions are created by modifying pure slip F&M with empirically determined shape functions. Transient effects are reproduced using standard relaxation length equations. The model calculates F&M at the center of the contact patch.

The developed methodology is implemented as an external tire module for the vehicle simulation software CarSim. The validation studies are conducted using the data sets from a flat belt testing machine and outdoor handling tests. The model demonstrated a high correlation with experimental results, while featuring an improved performance in parameterization and F&M estimation routines, compared to the commercially available Magic Formula tire model.

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INTRODUCTION

Studying the interaction between the tire and the road for vehicle dynamic analysis, requires an accurate characterization of the tire behavior. Resulting handling simulations can be used to evaluate the tire and vehicle performance in handling maneuvers to ultimately improve vehicle stability and safety. For instance, paper tire studies have been performed to identify which tire characteristics improve handling performance [1]. Simulations have also been used to improve safety of testing of potentially dangerous handling maneuvers [2]. The tire response quantities of interest for the handling studies are forces and moments (F&M) at the contact patch. Therefore, tire internal states such as stresses and strains as well vibrational characteristics are not investigated for handling simulations.

During a vehicle cornering maneuver, if a braking/driving torque is applied to the spindle, the tire contact patch will slip relative to the ground, in longitudinal and lateral directions. This condition, which is called combined slip, happens during critical driving conditions such as a sudden change of path while simultaneously reducing speed in

order to avoid an obstacle. Therefore, it's essential for a tire model used in handling simulations to produce accurate predictions in these sorts of behaviors.

This study presents a new tire model for reproducing tire behavior in handling simulations. The developed model is envisioned to:

1. Describe tire F&M in both pure slip and combined slip conditions, at different normal loads and inclination angles
2. Directly become parameterized from F&M test data with the least number of correction shape factors
3. Handle asymmetric F&M data curves with proper longitudinal and lateral shifts
4. Include the effect of ply-steer and conicity
5. Have fast computational time in parameterization and calculation steps in order to be used in virtual tire development process
6. Be able to communicate with vehicle simulation software, and function as an integrated or standalone tire module.

There are several primary advantages of this model over commercially available models, like MF-Tyre. Those advantages include higher F&M accuracy in combined slip conditions, lower computational effort in parameterization steps, and less manual tuning in order to achieve an optimum set of model parameters.

In the remaining part of this section, the main tire modeling strategies that are used in vehicle handling simulations are described in more detail. Furthermore, the process of virtual tire development and its requirements are explained. In the following chapters, first the procedures for processing the F&M data and parameterization of the tire model are discussed. Second, the developed methods are elaborated and formulated. Then the implemented tire models are validated against flat belt testing machine and outdoor handling test data, and finally the conclusions are made.

Models for Tire F&M Studies

The main tire models utilized for vehicle dynamic investigations can be grouped into two main categories: analytical tire models and semi-empirical models [3]. Analytical tire models take a theoretical approach to describe the mechanics of the tire structure and tire/terrain interaction, using necessary physical parameters to parameterize the model. Additionally, empirical adjustments are applied to increase the model predictive accuracy. Due to the complex and nonlinear nature of the tire/terrain system, these models need to be detailed. The degree of their complexity range from simple models such as brush-based models [4, 5] and string-based tire models [6, 7] to more advanced, discrete tire models such as F-Tire [8, 9, 10, 11] and finite element models [12, 13, 14].

To simplify the modeling process, less complex tire models assume some simplifications to the tire/terrain system properties, such as elliptical or parabolic normal pressure distribution at the contact patch [15], steady-state condition, string representation for the tire, and/or discretizing the tire tread as elastic bristles [16, 17, 18, 19, 20]. These assumptions limit the tire model to certain tire properties and operational conditions. The more complicated analytical models are computationally expensive; but at the same time they remain powerful in terms of accuracy and the wider domain of application [21, 22]. On the other hand, the extensive number of the model parameters needed for a given tire and high computational cost make most of the analytical models too inefficient for the iterative tire design optimization processes.

The semi-empirical tire models describe the tire F&M corresponding to its inputs using shape functions, and find the related shape coefficients using curve-fitting methods. Therefore, they mostly characterize the tire F&M instead of the actual mechanical system. These models are the most accurate tire models within the domain of the test data that are used for parameterization, but they perform poorly when extrapolating beyond that range. The semi-empirical tire models are widely used by the industry due to their fast speed, flexible parameterization for different tire types, and modifiability to accommodate new applications.

Virtual Tire Development

Semi-empirical F&M tire models are constructed using actual tire measurement data, which requires testing the physical tire. To simplify the tire design process, the virtual tire prototypes are constructed using FEM tools, and tested within a simulation environment at the test conditions similar to F&M measurements [1, 23, 24, 25]. The estimated tire responses are further processed to characterize the parameters for the tire model used in handling studies. These models help with determining and optimizing the tire design parameters that can ultimately improve handling performance.

Due to the high computational workload of the FEM simulations, it is not practical to estimate the tire behavior across the entire range needed for parameterizing the F&M tire models. Instead, virtual F&M test results are obtained at a reduced number of test configurations, and employed to modify the F&M model parameters. As mentioned before, the developed tire model in this study is designed to fit seamlessly in the described virtual tire development process.

Tire Model Benchmarking

Initially, some prominent commercially available tire models were studied to find the best solution that can characterize the tire combined slip F&M, in a virtual tire development process. These models are expected to be able to fully integrate with CarSim in order to function in a full vehicle simulation. The two main tire models that are used for benchmarking are Magic Formula and CarSim internal tire model.

The Magic Formula [26] model is one of the most adopted empirical tire model, for the simplicity and accuracy in handling simulations. The model parameters are calculated using the MF-Tool toolbox, which accept the measurement data in a pre-processed TYDEX format, and curve-fit the model parameters. Using the assumed shape functions and calculated shape coefficients, this model calculates the F&M at the center of the contact patch.

The Magic Formula model is not a viable option for the iterative virtual tire development process due to the following facts: The process of parameter identification is computationally intensive, it involves knowledge of parameter searching space, presence of a local minima, and/or other mathematical searching algorithm requirements. Furthermore, the model parameters do not have a clear physical interpretation, and consequently cannot be easily tuned to achieve specific model response characteristics [5, 27, 28].

The CarSim internal tire model is based on a study done by Pacejka [29]. The parameterization for this model is done by the F&M two dimensional tabular data, which include the F&M dependencies to slip variables (slip angle or slip ratio) and normal load. These tables are generated from the F&M measurements at the pure slip condition, and are required to only have the positive side of the slip values. CarSim automatically generates the negative side by mirroring the data, and adds a row for zero slip, and a column for zero load to constrain the curves to pass through the origin. This symmetric F&M

behavior does not reproduce F&M shape characteristics that are inherently different in the positive and negative slip regions, and neglects plysteer and conicity effects.

In the extended version of the model, the camber effect is added through assigning tables that list the residual F&M, as functions of the inclination angle and normal load. By using this approach, the same value of the residual F&M are added to the F&M calculations across all slip angle values at a given inclination angle and normal load. A different approach was used for including the inclination effects in the proposed model after verifying the benchmarked model against the test data.

TIRE F&M TESTS

In order to identify the tire model parameters, the test data from F&M tests are post-processed to find the F&M dependencies to tire/road system inputs. The test data are acquired from a flat belt testing machine, which rolls the tire on a moving belt and controls the tire system input parameters, including slip angle α , inclination angle γ , slip ratio κ , normal load F_z , inflation pressure p , and belt velocity V_x . Some studies tried to identify the tire F&M characteristics only based on the measurements carried out on board vehicle during outdoor experimental tests [30]. This approach reduces the range of model inputs due to the limitations of the test data used for parametrization.

The F&M output signals are measured at the spindle center using a force transducer, that are then transformed to the W-axis coordinate system located at the center of the contact patch (CTC). This coordinate system, which is employed to define the sign conventions, is shown in Figure 1. The conventions used for the coordinate systems and developed communication interface for tire-vehicle models are in accordance with the TYDEX-Format and the standard tire interface [31].

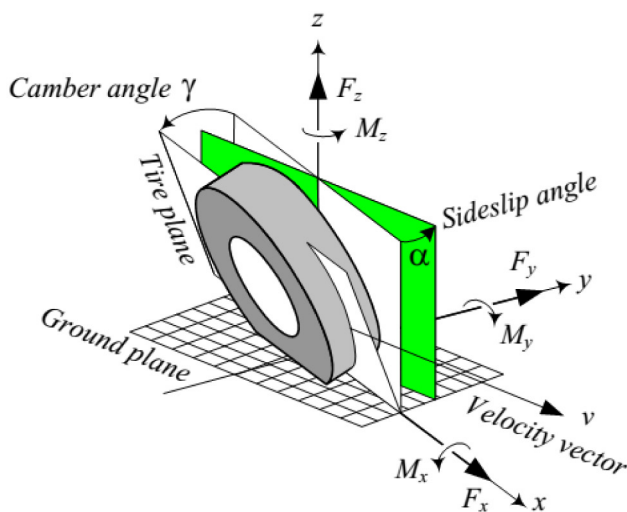


Figure 1. Tire coordinate system [32]

The experimental tests required for characterizing tire's different longitudinal, lateral, and combined slip behaviors are identified and properties of each test run are characterized. During each test

segment, one of the model inputs is varied and other inputs are held constant. Three types of tests are performed for model fitting purposes: pure braking/driving, pure cornering and combined slip.

The pure braking test is conducted by controlling the braking/driving torque at the spindle, in order to vary the longitudinal slip ratio at 0° slip angle. Other control parameters including vertical load, inclination angle and inflation pressure are kept constant. Based on the defined coordinate system, the longitudinal slip ratio κ is considered negative for braking and positive for driving, and is defined as:

$$\kappa = -\frac{V_{sx}}{V_x} = -\frac{V_x - \Omega R_e}{V_x} \quad (1)$$

Where V_{sx} is the longitudinal slip velocity, V_x is the forward velocity, Ω is the wheel angular velocity, and R_e is the effective rolling radius. It should be mentioned that the flat belt testing machine used for conducting these tests was only able to perform braking tests at normal loads below 5000 N. Pure braking tests were performed at 65 km/hr and the slip ratio was swept at 10 %/sec.

As for the pure cornering test, the spindle torque motor is removed so that free-rolling conditions are produced without active control and the tire slip angle is varied at a constant normal load, inclination angle, and inflation pressure. Slip angle sweeps are performed at 4 deg/sec. Pure cornering tests are performed at 50 km/hr. Pure cornering tests are also performed in order to characterize transient characteristics and are described in the Tire Transient Dynamics section found later in this paper.

During the combined slip tests, the tire slip ratio is varied at a non-zero slip angle while normal load, inclination angle, and inflation pressure, are kept constant. Performance limitations of the slip ratio controller made it challenging to sweep the slip ratio while keeping the slip angle at different constant values. Combined slip tests are performed at 65 km/hr and the slip ratio was swept at 10 %/sec.

It should be mentioned that three tires of the same specification are used for F&M testing. One is used for free-rolling (pure cornering) tests, one for pure braking tests, and the last for combined slip tests. This is due to the fact that the tests cause severe tread wear and consequently change the initial force characteristics of the tire. Differences between tires of the same nominal construction caused by manufacturing tolerances are a source of uncertainty for later validation steps.

Model Fitting: Pure Slip Data

The pure slip F&M are treated as functions of slip value (α or κ), inclination angle and normal load. Initially the effects of the slip value and normal load changes are modeled and then enhanced using the effects of inclination angle. Using the pure slip test data, F&M can be modeled as functions $\Psi(\alpha \text{ or } \kappa, F_z)$ of the sweep parameter (α or κ) as well as normal load, where each function represents an asymmetrical surface in the 3D space. This approach is more accurate

than averaging the data points at each normal load level to construct individual F&M curves, and use them to interpolate at the intermediate normal loads.

The method that is used for fitting a surface to the data is called locally weighted scatterplot smoothing (LOESS). This method fits individual quadratic polynomials to small subsets of data using a weighted least square approach. LOESS does not assume that the data have a particular overall shape, and consequently is very flexible in modeling the complex F&M behaviors. This is an advantage over MF-Tyre model which considers a set of predefined empirical equations to fit the trend of F&M data. Therefore, in the proposed model there is no need for the macro-coefficients (C, D, E, B) of MF-Tyre model in order to regulate the shape of the force-slip curves.

On the other hand, using the LOESS method during the parametrization step requires more computation effort compare to simpler tire models such as CarSim internal tire model. Given the fact that the F&M pure slip data model fitting occurs only a few times during the tire virtual development process, this step does not impose a major delay on the entire process. This method requires many data points, which is possible by using high resolution F&M data. Finally, LOESS is sensitive to data outliers, which can be attenuated by incorporating a robust version of this method as described in the following paragraphs.

The regression procedure starts with calculating the regression weights for each of the data points in a span. This span sweeps the domain of data similar to a moving average function. The data points inside the span have the largest weights and consequently the highest influence on the fit. On the other hand the data points outside the span have zero weight and no effect on the fitting process. Having the calculated weight values, a weighted least-squares regression is performed using a second degree polynomial.

To minimize the effect of data outliers, the robust algorithm computes the residuals from the initial regression fit. A new set of robust weights are calculated based on the fit residuals. Using these weight values, a new weighted smoothing regression fit is constructed. The process of calculating the fit residuals, computing the robust weights, and fitting the data is performed iteratively until a desirable degree of accuracy is achieved.

Braking/Driving Tests

The F&M data are divided into smaller sub-segments, each containing a slip ratio sweep from 0 % to -30 % and back to 0%. In each sub-segment, the slip angle, the inclination angle, and normal load are held constant. As discussed earlier, a regression surface $F_x(\kappa, F_z)$ can be fitted to the data that shows the longitudinal force dependency to the slip ratio and normal load input variables. The $F_x(\kappa, F_z)$ surface which is fitted using the LOESS method is shown in Figure 2. The coefficient of determination R^2 displayed in this figure indicates very good agreement between the data and the smoothing surface.

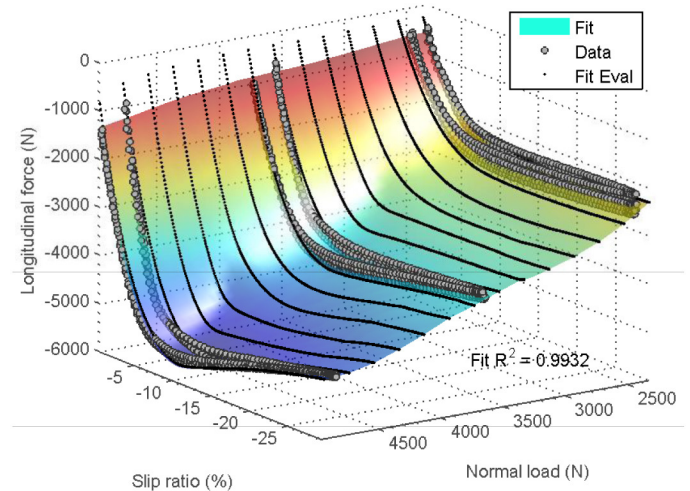


Figure 2. A smoothing surface is fitted to the longitudinal force data using LOESS method to construct the $F_x(\alpha, F_z)$ function

In order to construct the driving range segment of the surface, the pure braking data are mirrored from the negative slip ratio region to the positive. Once the entire range of $F_x(\kappa)$ curves are shaped at all specified normal load levels, an interpolating surface is fitted to the resulting curves as shown in Figure 3.

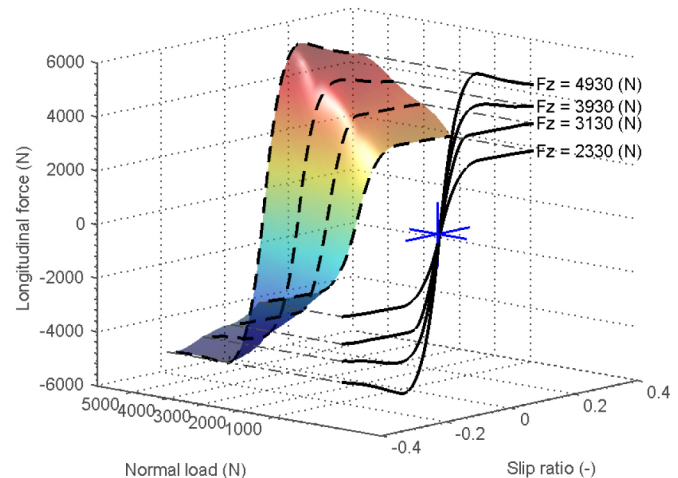


Figure 3. Constructing the full range of longitudinal force from surface fitted data

In order to consider the effect of inclination angle, additional asymmetric $F_x(\kappa, F_z)$ surfaces are constructed based on the test data at different inclination angle values. These set of surfaces are used as a 3D look-up table $F_x(\kappa, F_z, \gamma)$ to interpolate the longitudinal force at given input values, using a trilinear interpolation method. This procedure is discussed in detail later on.

Cornering Tests

The cornering data were measured at three inclination angle values, during which the slip angle is swept between -20° and $+20^\circ$, and the normal load is held constant at approximately four different amplitudes. The data points at each inclination angle are grouped together and a surface is fitted to them using the LOESS method. The fitted surfaces for the lateral force $F_y(\alpha, F_z)$ and the aligning moment $M_z(\alpha, F_z)$ at 0° inclination, are shown in Figure 4 and Figure 5.

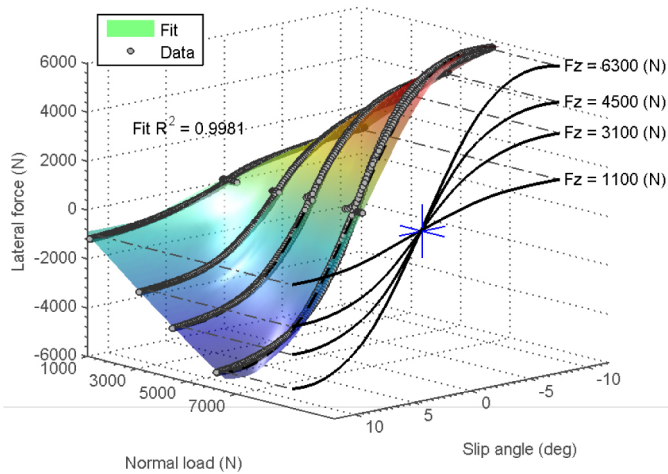


Figure 4. A smoothing surface is fitted to the lateral force data using LOESS method to construct the $F_y(\alpha, F_z)$ function

The R^2 indices presented in Figure 4 and Figure 5 show that a close fit to the test data is achieved. Given the fact that the test data includes both negative and positive slip angle regions, the mirroring procedure used in the longitudinal force fitting process is not required. These sets of surfaces are used to create 3D look-up tables (equivalent to 3D volumes) for $F_y(\alpha, F_z, \gamma)$ and $M_z(\alpha, F_z, \gamma)$ respectively.

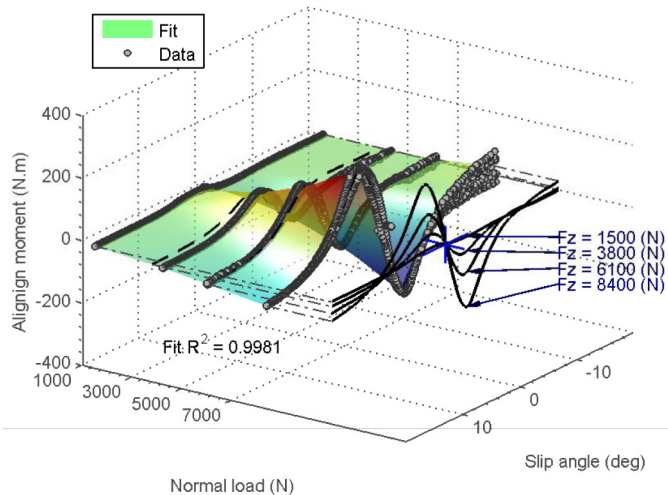


Figure 5. A smoothing surface is fitted to the aligning moment data using LOESS method to construct the $M_z(\alpha, F_z)$ function

Combined Slip Tests

The combined slip F&M tests are conducted at 3 inclination angles, 2 non-zero slip angles, and 2 levels of normal load. Throughout each test segment, the tire slip ratio is swept in a complete cycle between -10% and +10%, while the other input signals are held constant. The effect of the slip ratio changes on the lateral force is shown in Figure 6. At a constant inclination angle, slip angle and normal load, the peak lateral force occurs at approximately zero slip ratio and decreases with either increasing or decreasing the slip ratio. To model this behavior, regression curves are fitted to the F_y data points, using

a smoothing spline at constant (α, F_z, γ) input variables. The processed curves are used later on to mathematically model the effect of the slip ratio on lateral force.

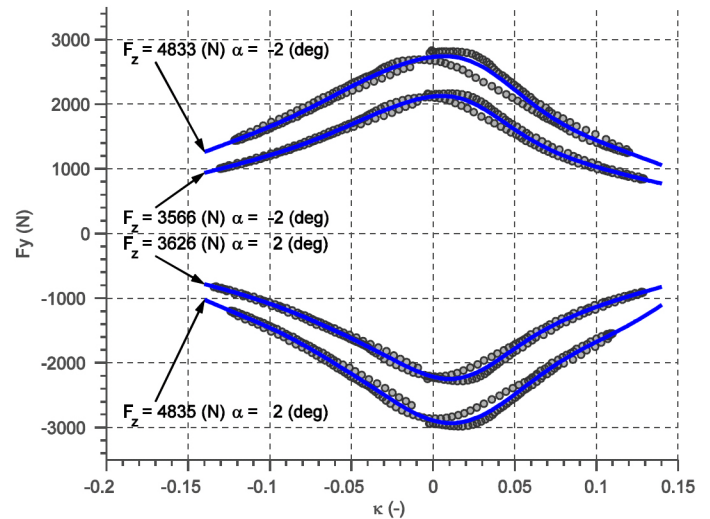


Figure 6. Curve-fitting the lateral force changes due to slip ratio variations using a smoothing spline method (test data at zero inclination angle: gray points, curve-fitted: blue lines)

Using the curve-fitted data for parameterizing the final mathematical function is a robust procedure that allows averaging of data points from portions where the slip ratio is increasing with the portions where the slip ratio is decreasing. This process also can help attenuate the effect of the data outliers. Since slip angle sweeps were not conducted in combined slip, it is not possible to study the effect of the slip angle changes on the longitudinal force generation. The solution to this problem is discussed in the combined slip data estimation section.

TIRE F&M ESTIMATION

Once the regression functions, which express the F&M as functions of test independent input variables α , κ , F_z , and γ were developed, the estimation process can be initiated. At this step, the pure slip F&M are estimated from the regression functions derived based on the pure cornering and pure braking data sets. This process is done using a trilinear interpolation algorithm which is described in the following section. For combined slip cases, a new weighting factor is introduced that when multiplied by the corresponding pure slip estimations, it can characterize the effects of slip angle on longitudinal force and slip ratio on lateral force. This process will be described in detail later on.

Pure Slip Data Estimation

Consider the case that the pure lateral force value at a given set of input parameters $(\alpha_d, F_{z_d}, \gamma_d)$ is desired. The inclination angle is required to be in the range of $\gamma_1 \leq \gamma_d \leq \gamma_2$, where γ_1 and γ_2 are the inclination angles at which the test data are available. As a result, the interpolation point is located between the two regression surfaces of $F_y(\alpha, F_z, \gamma_1)$ and $F_y(\alpha, F_z, \gamma_2)$. Regarding the fact that both surfaces are

meshed similarly, the eight points surrounding the interpolation point will form a cube with $\alpha_1 \leq \alpha_d \leq \alpha_2$ and $Fz_1 \leq Fz_d \leq Fz_2$. Initially, the normalized input variables are derived as:

$$\alpha^* = \frac{\alpha_d - \alpha_1}{\alpha_2 - \alpha_1} \quad (2)$$

$$Fz^* = \frac{Fz_d - Fz_1}{Fz_2 - Fz_1} \quad (3)$$

$$\gamma^* = \frac{\gamma_d - \gamma_1}{\gamma_2 - \gamma_1} \quad (4)$$

Using the normalized input variables and adjacent data point values, the following formulation is used to interpolate along the slip angle direction:

$$f_{y00} = Fy(\alpha_1, Fz_1, \gamma_1)(1 - \alpha^*) + Fy(\alpha_2, Fz_1, \gamma_1)\alpha^* \quad (5)$$

$$f_{y10} = Fy(\alpha_1, Fz_2, \gamma_1)(1 - \alpha^*) + Fy(\alpha_2, Fz_2, \gamma_1)\alpha^* \quad (6)$$

$$f_{y01} = Fy(\alpha_1, Fz_1, \gamma_2)(1 - \alpha^*) + Fy(\alpha_2, Fz_1, \gamma_2)\alpha^* \quad (7)$$

$$f_{y11} = Fy(\alpha_1, Fz_2, \gamma_2)(1 - \alpha^*) + Fy(\alpha_2, Fz_2, \gamma_2)\alpha^* \quad (8)$$

Following, interpolate along the normal load direction provides:

$$f_{y0} = f_{y00}(1 - Fz^*) + f_{y10}Fz^* \quad (9)$$

$$f_{y1} = f_{y01}(1 - Fz^*) + f_{y11}Fz^* \quad (10)$$

Finally, by interpolating between the two intermediate points, located on the adjacent surfaces of $Fy(\alpha, Fz, \gamma_1)$ and $Fy(\alpha, Fz, \gamma_2)$, the lateral force at the desired point is calculated as:

$$f_y = f_{y0}(1 - \gamma^*) + f_{y1}\gamma^* \quad (11)$$

The same procedure is established to find the longitudinal force $F_x(\kappa_d, Fz_d, \gamma_d)$, the aligning moment $M_z(\kappa_d, Fz_d, \gamma_d)$, and the overturning moment $M_x(\kappa_d, Fz_d, \gamma_d)$ at the given input pure slip conditions, using the neighbor data points.

F&M Extrapolation

The described estimation scheme provides highly accurate results within the test data boundaries. A linear extrapolation method is incorporated for finding the desired values beyond the data limits. To accomplish this, the following first-order variation formulation is used:

$$\begin{aligned} \Psi(\alpha_d, \kappa_d, Fz_d, \gamma_d) = \\ \Psi(\alpha_n, \kappa_n, Fz_n, \gamma_n) + \frac{\partial \Psi}{\partial \alpha} \Delta \alpha + \frac{\partial \Psi}{\partial \kappa} \Delta \kappa + \frac{\partial \Psi}{\partial Fz} \Delta Fz + \frac{\partial \Psi}{\partial \gamma} \Delta \gamma \end{aligned} \quad (12)$$

Where Ψ corresponds to any of the F&M system response quantities (SRQ), $\Psi(\alpha_d, \kappa_d, Fz_d, \gamma_d)$ is the SRQ at the desired point of interest, $\Psi(\alpha_n, \kappa_n, Fz_n, \gamma_n)$ is the SRQ at the nearest data point, $\Delta \alpha = \alpha_d - \alpha_n$, $\Delta \kappa = \kappa_d - \kappa_n$, $\Delta Fz = Fz_d - Fz_n$, and $\Delta \gamma = \gamma_d - \gamma_n$. In order to find the nearest data point, first a Delaunay triangulation is constructed at the vicinity of the search area, and then a Quickhull algorithm is used to search for the nearest data point at the desired point of interest.

Combined Slip Data Estimation

As it was discussed previously, when the F&M values are requested at a given set of independent test input variables $(\alpha_d, \kappa_d, Fz_d, \gamma_d)$, first the pure F&M variables $(F_x(\kappa_d, Fz_d, \gamma_d), F_y(\alpha_d, Fz_d, \gamma_d), M_z(\alpha_d, Fz_d, \gamma_d), \text{ or } M_x(\alpha_d, Fz_d, \gamma_d))$ are calculated using the described method. Next, the calculation of the combined slip forces is treated differently than the combined slip moments; this will be described in the remaining portion of this chapter.

Lateral Force

As shown in Figure 6, lateral force decreases from its peak value at $\kappa = 0$ by either increasing or decreasing the slip ratio. The same hill shape trend can be observed for the slip angle interaction with longitudinal force. In order to capture this behavior, a weighing function is introduced that is multiplied by the estimated pure slip data to result in the combined slip F&M values. The idea of modifying the pure slip data, using the correction factors was initially developed by Bayle [33]. In this study, a new formulation is used for defining a new weighing function that can result in a better estimation compared to the previous methods. The combined slip lateral force is calculated as following:

$$F_y^{comb} = (F_y^{pure}(\alpha, Fz, \gamma) + F_y^{res}(\alpha, \kappa, Fz, \gamma)) \cdot \frac{\Omega_y(\alpha, \kappa, Fz, \gamma)}{\Omega_y(\alpha, 0, Fz, \gamma)} \quad (13)$$

Where $F_y^{pure}(\alpha, Fz, \gamma)$ is the estimated lateral force at pure slip condition, $\Omega_y(\alpha, \kappa, Fz, \gamma)$ is the weighing function which is normalized with its value at zero slip ratio, and $F_y^{res}(\alpha, \kappa, Fz, \gamma)$ is the residual lateral force which is added to the pure lateral force to shift the peak of the F_y vs. κ curve to the proper lateral force level, based on the combined slip data. It should be noted that at zero slip ratio, the residual lateral force is zero, and the normalized weighing function is unity, so that the estimated lateral force does not change from its value at pure slip condition. The weighing function at constant normal load and inclination angle can be calculated as: