

An Erosion Aggressiveness Index (EAI) Based on Pressure Load Estimation Due to Bubble Collapse in Cavitating Flows Within the RANS Solvers

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ABSTRACT

Despite numerous research efforts, there is no reliable and widely accepted tool for the prediction of erosion prone material surfaces due to collapse of cavitation bubbles. In the present paper an Erosion Aggressiveness Index (EAI) is proposed, based on the pressure loads which develop on the material surface and the material yield stress. EAI depends on parameters of the liquid quality and includes the fourth power of the maximum bubble radius and the bubble size number density distribution. Both the newly proposed EAI and the Cavitation Aggressiveness Index (CAI), which has been previously proposed by the authors based on the total derivative of pressure at locations of bubble collapse ($DP/Dt > 0$, $Da/Dt < 0$), are computed for a cavitating flow orifice, for which experimental and numerical results on material erosion have been published. The predicted surface area prone to cavitation damage, as shown by the CAI and EAI indexes, is correlated with the experiments. EAI predictions indicate the minimum bubble size above which erosion starts as also its location along the injector wall. The proposed methodology is also tested in an actual Diesel injector, operating under realistic injection cycles and pressure levels for which erosion data are available.

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INTRODUCTION

Erosion of material surfaces in flow machines and hydraulic equipment due to cavitation is a fluid mechanics phenomenon which has been the focus of investigation in recent years. Hydrodynamic cavitation is the appearance of vapour within a liquid flow due to the decrease of the local pressure below the vapour saturation pressure at the local temperature [1]. Cavitation appears in various forms and it is represented by the agglomeration of bubbles containing inert gas and vapour. These bubbles are entrained to the flow and transported to regions of higher pressure where they collapse; during the collapsing phase pressure pulses are emitted to the environment and impinge either on material surfaces or on other surrounding vapour bubbles. In the first case and depending on the amplitude of the pressure waves, erosion can take place on material surfaces [2]; in the latter case the pressure waves can initiate the collapse of the surrounding bubbles resulting in a cumulative increase of the pressure wave effect [2]. Erosion damage in marine propellers and rudders due to cavitation has been identified very early in the shipping industry; cavitation with its adverse effects is finally proven to be present in a variety of fluid mechanics processes, as fuel injectors [2], hydraulic pumps and turbines [2] and even mechanical heart valves [2]. Apart from the erosion of material surfaces the bubble collapse causes

among others, machinery vibrations, noise and loss of machinery efficiency. The ability to quantify erosion prone surfaces due to hydrodynamic cavitation is an open scientific problem today.

The collapse of a bubble (containing inert gas and vapour) due to the sudden increase of the environmental pressure can be calculated from the solution of the Rayleigh-Plesset equation, and the emitted acoustic pressure pulse during the phase of bubble collapse is given by equation (1), Brennen [1]:

$$P_{ac} = \frac{\rho_l}{4\pi r} \frac{d^2V}{dt^2} \quad (1)$$

where V is the bubble volume and r the bubble distance from the wall and ρ_l the liquid density; the acoustic pressure pulse P_{ac} exhibits a $1/r$ law of pressure pulse attenuation, which renders the distance from the wall of the collapsing bubble of primary importance. According to flow analysis of a collapsing bubble based on the solution of the Rayleigh-Plesset equation, the amplitude of the emitted pressure waves, as calculated using equation (1), can be as high as 100 MPa. Franc and Michel [2]. Such pressures despite the short duration of

time in which they develop (typical time scale is of $1\mu\text{s}$) are high enough to create erosion damage on the material surface if there is a cumulative effect due to synchronous bubble collapse. Iwai et al. [3] conducted experiments on the erosion characteristics of various materials due to cavitation using a vibratory facility; they measured the impact loads attributed to the collapse of vapour bubbles and were able to correlate erosion material loss with impact loads. In particular they confirmed that there exists an impact threshold value after which the bubble collapse contributes to the fatigue fracture of the material surface; the cavitation erosion loss of mass is proportional to the energy acting on the surface to a power experimentally defined. Fortes- Patella et al. [4] concluded that the damage of the material due to cavitation phenomena can be related to the characteristics of pressure waves emitted by the collapse of vapour structures; they were able to show experimentally and computationally that the pressure wave power density (which is proportional to the square of the maximum pressure pulse upon bubble collapse) is well correlated to the pit volume created on the material surface due to its elastic-plastic deformation; the previous point was proved by coupling the pressure waves load on the material surface (an output of the numerical solution of a Rayleigh-Plesset equation) with a numerical fluid-structure interaction analysis of the elastic-plastic deformation of the material surface, demonstrating that the pressure wave power density and the generated pit volume are linearly correlated. Dular et al. [5] investigated experimentally a cavitating flow around a hydrofoil in a water tunnel for various operating conditions and cavitation numbers; they visually observed the strong correlation of cavitation and erosion damage. These experiments suggested that erosion damage is well correlated with the cavitation unsteadiness and vapour cloud separation and collapse; the cloud collapse initiates a shock wave that upon impinging on the bubbles near the solid surface force them into oscillation and due to the presence of the solid wall they start an asynchronous bubble collapse. According to Plesset and Chapman [6], as a result of the asynchronous bubble collapse, a micro jet is formed towards the wall surface with an impulse which, if above a threshold value, is capable of creating plastic deformation on the material surface. They proposed an erosion model based on the correlation of the standard deviation of the deformation effects the micro jet has on the material surface, which was measured experimentally, with the emitted pressure wave. Dular and Coutier-Delgosha [7] investigated the possibility of predicting cavitation erosion in hydrofoil flows using Computational Fluid Dynamics (CFD); the CFD model was based on the numerical solution of the unsteady RANS equations coupled with a homogeneous equilibrium barotropic model. As erosion model they employed the model of Dular et al. [5]. They assumed that the micro jet is responsible for the plastic deformation of the material and the creation of pits on the solid surface, as Plesset and Chapman [6]. Li Ziru [8] investigated various erosion criteria within the concept of RANS equation on a post processing basis; she examined the predictability of the instantaneous static pressure, time derivative of pressure, time derivative of vapour volume fraction and time integrals of the previous indicators and others; she concluded that among the tested erosion indexes the time derivative of pressure seems to perform best, whilst the time derivative of the vapour volume fraction seems not to be correlated to the erosion regions. Bergeles et al. [9] proposed a Cavitation Aggressiveness Index (CAI) based on the

concept of the total derivative of pressure in the flow regions of vapour collapse within the framework of RANS methodology; the regions of bubble collapse were identified as those regions of negative total derivative of the vapour volume fraction (α) (in a single fluid concept-homogeneous mixture of liquid and vapour), [equation \(2\)](#);

$$\frac{DP}{Dt} > 0, \frac{D\alpha}{Dt} < 0 \quad (2)$$

The cavitation aggressiveness index (CAI) was defined as a non dimensional expression of the total derivative of pressure comparing its value to the emitted pressure pulses either due to bubble collapse or to cavity oscillations using typical corresponding time scales. The proposed index identified successfully the regions prone to erosion as reported in the measurements of Franc et al. [10]; the proposed model was also tested in another industrial case by Koukouvinis et al. [11] with success. Kato et al. [12] were among the first to propose an analytic model to compute material erosion due to cavitation; their model was based in estimating the impact force due to bubble collapse and the frequency of bubble collapses on the material surface. The number density distribution of the bubble size was taken into account in calculating the cumulative impact force due to synchronous bubble collapse; the authors pointed out the importance of the number density distribution on predicting erosion. The method successfully predicted erosion rates in cavitating hydrofoils, however it requires information not readily available, unless 'reasonable' estimations are made. Perkovic et al. [13] applied a commercially available software in predicting the flow development and the erosion rates of an injector type flow for which experimental data were available. The mathematical model was based on an incompressible, turbulent two phase flow simulation; they were able to predict the erosion velocity due to cavitation under various experimental operating conditions in the fuel injector and various fuels; the erosion model used was that employed by the commercial CFD code. Hattori et al. [14] conducted erosion tests on material surfaces due to cavitation of impacting liquid jets on the surface. The collapsing bubbles on the surface were creating impact loads which were measured with piezoelectric sensors. They found that the impact load raised to a power (which is material dependent with values from 1.1 to 1.79) multiplied with the frequency of occurrence of pressure impacts is well correlated to erosion rate of the material surface. Recently, more sophisticated models have been employed for the prediction of cavitation erosion. For example, Skoda et al [16] investigated a cavitating flow in square sectioned throttle geometry, similar to the one of the present investigation; a 2D density-based compressible Euler CFD code was used which could capture the pressure peaks created due to shock waves from the imploding bubbles. They proposed an Erosion Probability Indicator related to the maximum pressure peak on the wall and thus they were able to predict qualitatively the surface region prone to erosion. Edelbauer et al [17] investigated a similar to [16] cavitating flow and employed an LES approach for the numerical solution of the flow; they applied an erosion model which qualitatively predicted the material surface region to be eroded; the highest erosion damage was found around the tail of the vapour cloud.

In the following the proposed concept of the Erosion Aggressiveness Index (EAI) is derived and its reliability is tested in predicting two cavitating flows of engineering importance for which some data on the flow development and on erosion rates have been published.

THE EROSION AGGRESSIVENESS INDEX (EAI)

Erosion damage due to cavitation is a time evolving process of the material damage due to pressure waves, impinging on the material surface. Various theories have been proposed for explaining material erosion and which correlate the amplitude of the pressure signature on the wall with the material properties in the elasticity and fracture regions. Vapour bubbles increase in size when moving in low pressure regions and obtain their maximum radius R_{max} at the region of lowest pressure, which usually is taken as equal to the vapour saturation pressure at the liquid temperature; as they are convected by the flow at pressure recovery regions and under the pressure difference ΔP between the surrounding pressure and the bubble pressure ($P_l - P_v$) the collapse phase is initiated. The emitted pressure pulse at a distance r from the bubble centre, following [equation \(1\)](#), is given by [equation \(3\)](#):

$$P_{ac} = \frac{\rho_l}{r} \frac{d}{dt} (R^2 \dot{R}) \quad (3)$$

where R is the bubble radius and $\dot{R}$ is the velocity of the bubble wall. A mean velocity of the bubble wall during collapse, which is valid at the last stages of bubble collapse with small amount of non-condensable gas in the bubble, is given by [equation \(4\)](#), [1] and [2],

$$\dot{R} = \sqrt{\frac{2 \Delta p}{3 \rho_l}} \left(\frac{R_{max}}{R} \right)^{3/2} \quad (4)$$

R_{max} is the maximum bubble radius at the region of minimum pressure, which is given by [equation \(5\)](#), assuming an isothermal expansion (Brennen [1]):

$$R_{max} = \left(P_{g0} \frac{R_0^3}{2\sigma} \right)^{1/2} \quad (5)$$

Expression (4) introduced into [equation \(3\)](#) and after some algebra leads to an approximate expression for the emitted pulse pressure in shock waves during the collapse phase and at a distance r from the bubble centre, [equation \(6\)](#)

$$P_{ac} = \frac{\Delta P}{3r} R_{max} \left(\frac{R_{max}}{R} \right)^2 \quad (6)$$

This expression is in agreement with the estimation of the magnitude of pressure pulses emitted from a collapsing bubble, Brennen (1995, equation 3.2) [1] for a value of $\frac{R_{max}}{R} = 10$. The emitted pressure impulse due to bubble collapse is evaluated from [equation \(7\)](#),

$$P_{w,imp} = P_{ac} t_b \quad (7)$$

where t_b is the Rayleigh bubble collapsing time.

The mean impact load on the wall surface due to synchronous bubble collapse inside a volume V_{vol} can be evaluated from [equation \(8\)](#):

$$P_w = \iiint_{V_{vol}} P_{ac} t_b \cos \varphi \left[-n_0 \frac{D\alpha}{Dt} \right] dV \quad (8)$$

where the quantity in brackets expresses in an approximate way the number of bubbles which collapse per unit time and volume; φ is the surface inclination of the wall to the radius (rad) connecting the bubble centre and the centre of the wall surface; the integration volume is a cylinder sitting on the surface with radius and height equal to L_v . Substituting expression (6) for P_{ac} and the expression of t_b ([equation \(11\)](#)) in [equation \(8\)](#) the following expression for the impact pressure (using the middle value theorem) is obtained,

$$P_w = 0.4 \sqrt{\rho_l \Delta P} R_{max}^2 L_v^2 \left(\frac{R_{max}}{R} \right)^2 \left[-n_0 \frac{D\alpha}{Dt} \right] \quad (9)$$

where n_0 is the number of bubbles per unit volume and R is the local bubble radius obtained from the volume fraction. The constant 0.4 in [equation \(9\)](#) is an approximate value considering also the wave attenuation due to local absorption. In the results presented below $L_v = R_{max}$ and the mean value in brackets is calculated at a distance $R_{max}/2$ from the wall. [Equation \(9\)](#) predicts that the impact pressure on the material surface is proportional to the square of the flow velocity ($\sqrt{\Delta P} \cdot D\alpha/Dt$); as the erosion rate is proportional to the square of the impact pressure, therefore it is predicted that the erosion rate would be proportional to the fourth power of the flow velocity; this finding qualitatively agrees with the experimental evidence that the erosion rate is proportional at least to the flow velocity to the fourth power; [equation \(9\)](#) takes into account also the effect of non-condensable gas content in the liquid by indirectly affecting the maximum bubble radius. [Equation \(9\)](#) also indicates that the erosion rate depends on the initial bubble size as it affects R_{max} and also on the initial bubble number density distribution as reported by Kato et al. [12].

The calculated impact pressure should be compared to the yield stress of the material considering however the cycling loading, which leads to material fatigue. Therefore an Erosion Aggressiveness Index is defined (EAI):

$$(EAI) = \frac{P_w}{\sigma_y} \log \left(\frac{L}{t_b} \right)$$

