

Flow-Acoustic Analysis of the Perforated-Baffle Three-Chamber Hybrid Muffler Configurations

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ABSTRACT

In this work, the noise attenuation characteristics of a three-chamber U-bend hybrid muffler have been investigated. Acoustic performance is quantified by the Transmission Loss (TL) parameter. One-dimensional Transfer Matrix based Muffler Program (TMMP) and three-dimensional Finite Element Method (FEM) have been used for the prediction of the TL of the muffler. Presence of perforated baffles necessitates use of the Integrated Transfer Matrix (ITM) approach for the one-dimensional analysis because the sound fields in the adjacent chambers would be multiply coupled with each other, and for the 3D FEM analysis LMS Virtual Lab software has been used. The mean flow distribution in each of these configurations has been evaluated by means of a lumped flow resistance network. The resulting values of the grazing flow and bias flow have been used to calculate the perforates' acoustic impedance. TL calculated with the help of the 1D analysis shows good agreement with the simulated results from the 3D FEA. Apart from the three-chamber U-bend hybrid muffler, three additional configurations have been analyzed, and the effects of the absorptive material, porosity of the baffle plates, porosity of the perforated pipes, and the length of perforation of the perforated pipes have been studied. Based on the above studies, design guidelines have been developed for these types of muffler configurations in order to improve their performance.

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INTRODUCTION

Exhaust noise is invariably the predominant source of noise in an automobile. Therefore, starting with the pioneering work of Davis et al. [1] in 1954, analysis and design of mufflers has attracted attention of researchers as well as designers [2]. A breakthrough came in late seventies with analysis of the perforated element mufflers [3, 4, 5, 6]. Transfer matrices were derived for different elements constituting cascaded element mufflers [7, 8, 9]. The problem with the cascaded element mufflers was high back pressure and rather poor acoustic performance. This was resolved by means of multiply connected mufflers which called for special analytical techniques [10, 11, 12, 13, 14]. With the advent of long strand fibers, absorptive materials could now be used in the hot exhaust system along with reactive elements. This resulted in the development of the hybrid mufflers or combination mufflers [15, 16, 17].

Analysis of the multiply-connected mufflers calls for a knowledge of the mean flow distribution because the perforate impedance depends on the grazing flow Mach number and the bias flow Mach number [12]. The mean flow distribution can be evaluated readily by means of lumped flow resistances in a flow-electric analogy network theory [12, 18]. This approach helps us to predict the overall back pressure of the muffler too. Multiply connected mufflers involving perforated baffles as well as pipes with or without absorptive elements can best be analyzed by the Vijayasree and Munjal's Integrated Transfer Matrix (ITM) approach [14].

Unlike the cascaded element mufflers, each multiply-connected muffler configuration calls for ingenious modeling or handling. Some of the researchers have found it easier to make use of numerical modeling [19 - 20]. Elnady et al. have developed a comprehensive software which combines analytical modeling with numerical modeling making use of a two port framework [12, 13, 21].

In the present investigation, an analytical procedure is outlined for flow acoustic analysis of a three-chamber combination muffler with perforated intermediate baffles, making use of the ITM approach as well as the lumped flow resistance network theory [18, 22]. The predicted values of Transmission Loss (TL) are corroborated against the finite element simulations. Four different configurations are thus investigated in order to bring out the effect of acoustically absorptive material in third chamber and a bent tube projecting into the first chamber. Having validated the 1D ITM approach, the same is used for parametric studies in order to evolve design guidelines for similar muffler configurations.

Throughout the paper; for 1D analysis as well as 3D FEA, the following assumptions have been made:

1. The medium has been considered to be inviscid (ideal) gas;
2. The end plates and the shell of the muffler are assumed to be rigid;
3. The convective effect of mean flow has been neglected.

Nevertheless, the dissipative/ absorptive effect of the mean flow has been duly considered in the perforate impedance.

DETAILS OF MUFFLER CONFIGURATION

A schematic diagram of the commercial muffler investigated here is shown in Fig. 1. This muffler consists of three chambers separated by perforated baffles. Chamber 3 is an acoustically lined duct (a perforated tube surrounded by a cavity filled with acoustically absorptive material). There are two small diameter (d) pipes (pipe 1 and pipe 2); pipe 1 passes through chamber 1 and opens in chamber 2 and pipe 2 starts from chamber 2, takes a U-bend in chamber 1 and passes through chambers 2 and 3 as shown in Fig. 1.

These three chambers are not separate from each other; the sound waves in these chambers interact with each other through pipes as well as perforations in the intermediate plates (baffles). Thus the muffler elements do not form a cascade of isolated elements; they are multiply connected. So we have to change the analysis from the simple transfer matrix multiplication approach (as would apply to cascaded muffler) [22] to the Integrated Transfer Matrix (ITM) approach [14].

1-D ANALYSIS (PLANE WAVE ANALYSIS)

Mufflers are used to attenuate noise at the engine firing frequency and a few multiples of it, which dominate in the frequency spectrum. Plane wave analysis yields reasonably accurate results at such low frequencies and can be used to predict the behavior of the muffler; given the restriction that it is suitable only below the cut-off frequency of the muffler [22]. In the 1D analysis the acoustic effect of the intrusion of the bent pipe in chamber 1 has been neglected (This assumption will be shown to be justified when we compare the 1D results with the FEM solution).

Integrated Transfer Matrix Approach

In the Integrated Transfer Matrix (ITM) approach [14], the given muffler is notionally divided into a number of sections based on area discontinuities, perforates, baffles or absorptive linings. This approach relates the state variables across the entire cross-section of the muffler shell, as one moves along the axis of the muffler, and can be partitioned appropriately in order to relate the state variables of different elements constituting the cross-section. The different muffler sections are then combined appropriately into a single final matrix, with the relevant boundary conditions, to get a matrix relating the upstream state variables to the downstream state variables. This method is particularly suited for the perforated-element mufflers involving perforated annular baffles. Following the above scheme of partitioning, the muffler in consideration can be notionally divided in 11 segments as shown in Fig. 2.

Derivation of the Overall Transfer Matrix

As described above, we have notionally divided the whole muffler in 11 different segments based on the relative changes in the muffler geometry along the length, as shown in Fig. 2. There will be two sound fields at any section, each of which is defined by two state

variables namely acoustic pressure (p) and mass velocity (v). So there will be four state variables needed to describe the two fields.

Therefore a 4×4 matrix will be relating the upstream state variables to the corresponding downstream state variables. Using this approach, we get two 4×4 matrices, one relating the state variables of section 1 to those at section 6, and another relating section 8 to section 13 [14]. Besides, state variables at section 6 are connected to those at section 8 using the conservation of mass and momentum equations as follows:

$$p_a = p_{2,6} = p_{1,6} \quad (1) - (2)$$

$$p_b = p_{3,7} = p_{4,8} \quad (3) - (4)$$

$$v_a = v_{1,6} + v_{2,6} \quad (5)$$

$$v_b = v_{4,8} + v_{3,7} \quad (6)$$

where p_a is acoustic pressure at point 'a' and v_a is acoustic mass velocity at point 'a', and $X_{i,j}$ denotes the state variable X in domain 'i' at section 'j'; X may be acoustic pressure or mass velocity, depending on the context.

In Fig. 2, from point 'a' to point 'b', it is a uniform pipe element with the equivalent shell diameter $d_{eq2} = \sqrt{D^2 - 2d^2}$ and length $l_{1ext} - l_{2ext}$ (see Fig. 2). Thus,

$$\begin{bmatrix} p_a \\ v_a \end{bmatrix} = [P_{a,b}] \begin{bmatrix} p_b \\ v_b \end{bmatrix} \quad (7) - (8)$$

where $[P_{i,j}]$ is transfer matrix for the uniform pipe element connecting section 'i' to section 'j'. From point 7 to 8, in the bent pipe portion, it is again a uniform pipe element with diameter d and length $l_{eq_bent_pipe}$ (which is the length of the bent pipe from point 7 to 8).

$$\begin{bmatrix} p_{3,7} \\ v_{3,7} \end{bmatrix} = [P_{7,8}] \begin{bmatrix} p_{5,8} \\ v_{5,8} \end{bmatrix} \quad (9) - (10)$$

The muffler has rigid end plates at sections 1 and 13 which give rigid boundary conditions (i.e., particle velocity u is zero). Thus,

$$u_{1,1} = 0, \text{ i.e., } v_{1,1} = 0 \text{ at section 1} \quad (11)$$

$$u_{4,13} = 0, \text{ i.e., } v_{4,13} = 0 \text{ at section 13} \quad (12)$$

The matrices relating the state variables at section 1 to those at section 6 are listed in Table 1. Using these transfer matrices, we formulate a 4×4 matrix which relate the state variables at section 1 to those at section 6 as follows:

$$\begin{bmatrix} p_{1,1} \\ v_{1,1} \\ p_{2,1} \\ v_{2,1} \end{bmatrix} = \begin{bmatrix} [\mathbf{P}_{1,1-2}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{P}_{2,1-2}] \end{bmatrix} [\mathbf{PE}_{2-3}] \begin{bmatrix} [\mathbf{P}_{1,3-4}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{P}_{2,3-4}] \end{bmatrix} \times \dots$$

$$\times \begin{bmatrix} [\mathbf{B}_{1,4-5}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{P}_{2,4-5}] \end{bmatrix} \begin{bmatrix} [\mathbf{P}_{1,5-6}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{P}_{2,5-6}] \end{bmatrix} \begin{bmatrix} p_{1,6} \\ v_{1,6} \\ p_{2,6} \\ v_{2,6} \end{bmatrix}$$

which on successive multiplication yields

$$\begin{bmatrix} p_{1,1} \\ v_{1,1} \\ p_{2,1} \\ v_{2,1} \end{bmatrix} = [\mathbf{T}_{1-6}] \begin{bmatrix} p_{1,6} \\ v_{1,6} \\ p_{2,6} \\ v_{2,6} \end{bmatrix} \quad (13) - (16)$$

Matrices correlating state variables at section 8 to those of section 13 are tabulated in Table 2. Using these transfer matrices, we formulate a 4×4 matrix which relates state variables at section 8 to those at section 13 as follows:

$$\begin{bmatrix} p_{4,8} \\ v_{4,8} \\ p_{5,8} \\ v_{5,8} \end{bmatrix} = \begin{bmatrix} [\mathbf{P}_{4,8-9}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{P}_{5,8-9}] \end{bmatrix} \begin{bmatrix} [\mathbf{B}_{4,9-10}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{P}_{5,9-10}] \end{bmatrix} \times \dots$$

$$\times \begin{bmatrix} [\mathbf{T}_{int}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{I}] \end{bmatrix} \begin{bmatrix} [\mathbf{P}_{4,10-11}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{P}_{5,10-11}] \end{bmatrix} \times \dots$$

$$\times [\mathbf{APE}_{11-12}] \begin{bmatrix} [\mathbf{P}_{4,12-13}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{P}_{5,12-13}] \end{bmatrix} \begin{bmatrix} p_{4,13} \\ v_{4,13} \\ p_{5,13} \\ v_{5,13} \end{bmatrix}$$

where $[\mathbf{X}_{n,i-j}]$ represents Transfer matrix $\mathbf{X}$ (which may be for pipe, baffle, perforated element, and the perforated element lined duct) connecting point 'i' to point 'j' across the 'nth' domain. Carrying out successive multiplication yields

$$\begin{bmatrix} p_{4,8} \\ v_{4,8} \\ p_{5,8} \\ v_{5,8} \end{bmatrix} = [\mathbf{T}_{8-13}] \begin{bmatrix} p_{4,13} \\ v_{4,13} \\ p_{5,13} \\ v_{5,13} \end{bmatrix} \quad (17) - (20)$$

However, in this case, we also have to consider transfer matrix for the air and absorptive material interface just behind the baffle 2. Transfer matrices of pipe element $[\mathbf{P}_{n,i-j}]$, 2-duct perforated element $[\mathbf{PE}_{i-j}]$, absorptive 2-duct perforated element $[\mathbf{APE}_{i-j}]$, air absorptive material interface $[\mathbf{T}_{int}]$ and baffle impedance $[\mathbf{B}_{n,i-j}]$ are detailed in Ref. 22, and $[\mathbf{0}]$ and $[\mathbf{I}]$ represent 2×2 null matrix and identity matrix, respectively.

Thus, we have 20 equations (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20) and 22 variables. We can represent these 20 variables in terms of the remaining two variables, namely, $p_{5,13}$ and $v_{5,13}$ (which are our downstream state variables). Hence, the required transfer matrix relation between the upstream and the downstream state variables (which are $(p_{2,1}, v_{2,1})$ and $(p_{5,13}, v_{5,13})$, respectively) can be written as,

$$\begin{bmatrix} p_{2,1} \\ v_{2,1} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}_{2 \times 2} \begin{bmatrix} p_{5,13} \\ v_{5,13} \end{bmatrix} \quad (21)$$

where $[\mathbf{T}]$ is the transfer matrix relating the upstream state variables to the downstream variables.

Using the above transfer matrix $[\mathbf{T}]$, we can calculate the Transmission Loss [22],

$$TL = 20 \log_{10} \left[\left(\frac{Y_{tp}}{Y_{ex}} \right)^{\frac{1}{2}} \frac{1}{2} \left| T_{11} + \frac{T_{12}}{Y_{tp}} + T_{21} Y_{ex} + T_{22} \frac{Y_{ex}}{Y_{tp}} \right| \right] \quad (22)$$

where Y_{tp} and Y_{ex} are characteristic impedances of the tail pipe and the exhaust pipe, respectively.

MEAN FLOW DISTRIBUTION CALCULATION

In this configuration, the mean flow Q_0 enters through pipe 1, and while passing through perforations, divides into a bias component Q_2 and a grazing component Q_1 . Bias flow Q_2 passes through the annular chamber and the perforated baffle plate 1 and mixes with the grazing flow Q_1 coming from pipe 1 at section 6. Then the entire flow reverses its direction and passes through the bent pipe as shown in Fig. 2 (Note: negligible flow would go into chamber 3 because it is filled with absorptive material). Fig. 3 shows an equivalent lumped flow resistance network [13, 18]

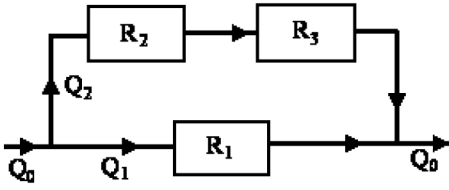


Figure 3. Equivalent electric network.

Considering continuity of volume flux, we can write,

$$Q_0 = Q_1 + Q_2 \quad (23)$$

The pressure drop consideration (Kirchhoff's loop equation) yields

$$(R_2 + R_3)Q_2^2 = R_1Q_1^2 \quad (24)$$

where R_1 , R_2 and R_3 are flow resistances of sudden area expansion at section 6, resistance of the perforated portion of pipe 1, and resistance of the perforated baffle plate 1, respectively.

Solving the two Eq. (23) and Eq. (24) simultaneously yields

$$Q_1 = Q_2 \sqrt{\frac{R_2 + R_3}{R_1}} \quad (25)$$

$$Q_2 = Q_0 / \left(1 + \sqrt{\frac{R_2 + R_3}{R_1}} \right) \quad (26)$$

$$\text{Here, } R_1 = \frac{\rho_0}{2S_1^2}, R_2 = \frac{\rho_0}{2\sigma_2^2 C_D^2 S_2^2} \text{ and } R_3 = \frac{\rho_0}{2\sigma_3^2 C_D^2 S_3^2}$$

where S_1 , S_2 and S_3 represent cross-sectional areas of pipe 1, open area of the perforated portion of pipe 1 and upstream area of the baffle plate 1, respectively, C_D is the coefficient of discharge ($C_D = 0.8$, assumed), ρ_0 is exhaust gas density, σ_2 and σ_3 are porosities of the perforated pipe and baffle 1, respectively.

The bias flow Mach number and grazing flow Mach number in the perforated section are given by,

$$M_{bias} = \frac{Q_2}{c_0 S_2 \sigma_2} \quad (27)$$

$$M_{grazing} = \frac{Q_0 + Q_1}{2c_0 S_1} \quad (28)$$

The bias flow Mach number through the baffle plate holes,

$$M_{bias} = \frac{Q_2}{c_0 S_{open}} \quad (29)$$

where c_0 is the sound speed and S_{open} is the total area of the holes in baffle, $S_{open} = S_3 \sigma_3$.

Here pressure drop due to the wall friction has been neglected.

3-D FEM ANALYSIS

The entire fluid domain has been discretized into small finite elements. Use of the variational formulation for governing equations of acoustics yields [22]

$$([M] - k^2[K] + j\omega\rho[C])\{p_n\} = -j\omega\rho\{F\} \quad (30)$$

where $[M]$, $[K]$ and $[C]$ are inertia, stiffness, and damping matrices, respectively. k , ω and ρ are wave number, angular frequency and density of fluid, respectively, $\{F\}$; the forcing vector is determined by boundary conditions and vector $\{p_n\}$ comprises nodal acoustic pressures. This system has the following boundary conditions,

1. $u = 1$ m/s (arbitrarily chosen value of the particle velocity at inlet end, i.e., inlet of pipe 1);
2. acoustic impedance $\xi = \rho_0 c_0$ at the anechoic outlet end of pipe 2, where ρ_0 is density of medium and c_0 is the speed of sound in medium.

Once the matrix Eq. 30 is solved for the complete domain, we can calculate values of the pressure and particle velocity throughout the domain, at each frequency, and assuming planar wave propagation in the inlet and outlet pipes, the Transmission Loss (TL) of the muffler can be calculated as,

$$TL = 20 \log_{10} \left| \frac{p_i + Y_i v_i}{2Y_o v_o} \right| \quad (31)$$

where p is acoustic pressure, v is mass velocity and Y is characteristic impedance. Subscripts 'i' and 'o' indicate the inlet and outlet, respectively.

Grid Independence

The effect of the element size on results (i.e., TL) can be seen from Fig. 4, which shows that for element size of 2 mm and 5 mm, the TL values for the coarse mesh of the element size 5 mm differ very marginally from those for the 2 mm element size for frequency values of 1000 Hz onwards. So, considering the hardware and execution