

A Unified Framework for Representing Product Validation Testing Methods and Conducting Reliability Analysis

Zhigang Wei, Michael Start, Jason Hamilton, and Limin Luo
Tenneco Inc.

ABSTRACT

Durability and reliability performance is one of the most important concerns for vehicle components and systems, which experience cyclic fatigue loadings and may eventually fail over time. Durability and reliability assessment and associated product validation require effective and robust testing methods. Several testing methods are available and among them, three basic testing methods are widely used: life testing, binomial testing (bogey testing), and degradation testing. In fact, their commonalities, differences, and relationships have not been clearly defined and fully understood. Therefore, the maximum potential of these testing methods to generate efficient, optimized, and cost-effective testing plans, consistent results, and meaningful results interpretation have been significantly limited.

In this paper, a unified framework for representing these testing methods and conducting reliability analysis in a single damage-cycle ($D-N$) diagram is provided. The representation of the associated accelerated testing methods in the damage-cycle diagram is also discussed. The effectiveness of the framework for conducting validation testing is demonstrated with several worked examples and case studies. With the framework, more efficient and economical testing plans and procedures could be possible by combining and optimizing these various testing methods.

CITATION: Wei, Z., Start, M., Hamilton, J., and Luo, L., "A Unified Framework for Representing Product Validation Testing Methods and Conducting Reliability Analysis," *SAE Int. J. Mater. Manf.* 9(2):2016, doi:10.4271/2016-01-0269.

1. INTRODUCTION

Vehicle components and systems are subjected to mechanical, thermal or combined thermomechanical fatigue loading during operation. Their durability and reliability performance is a significant consideration in product design, validation, and reliability demonstration [1,2,3]. Durability describes the ability of a product to sustain required performance over a period of time or cycles without undesirable failure. Reliability describes the ability and probability of a component or system to function under stated conditions. Testing is still an indispensable and, oftentimes, the last check of a product validation and reliability demonstration processes. The commonly used testing methods include life testing (test-to-failure), binomial testing (pass or fail), and degradation testing [1, 3]. The binomial (Bogey) testing method is often used in reliability demonstration, with which the customers' specifications must be met for acceptance into service [3].

However, in practice, these testing methods are often used separately and independently because of customers' requirements, historical treatments, preferences of the engineers, etc. The lack of understanding of the interrelationships among these methods is another reason. In order to maximize the potential of these methods to achieve the best possible testing plans and consistent results, these testing methods must be described and interpreted in the same framework, which is still lacking. Additionally, the associated accelerated testing methods [3, 4], i.e. accelerated life testing,

accelerated binomial testing (bogey testing), and accelerated degradation testing, should also be representable in the unified framework in order to embrace multiple applications. Accelerated testing methods are commonly used to shorten the development time and reduce the associated cost while not significantly sacrificing the accuracy of the assessment [3,4,5,6]. Virtually, all laboratory life tests are accelerated in one manner or another [6].

In this paper, a unified framework for representing the product validation testing methods is established. The core of the framework is the damage-cycle ($D-N$) diagram [7], which consists of the following major constituents: (1) test data, either test-to-failure, binomial, degradation, or combined, for estimating the continuous probabilistic distribution function. (2) damage accumulation rules, such as the linear or nonlinear damage accumulation rules, for data interpolation and extrapolation, and (3) a variable transformation technique, which converts a probabilistic distribution of a variable into a probabilistic distribution of another variable. For a given set of testing constraints, the most efficient and economical testing strategies could be possible by combining and optimizing the testing methods with the help of the damage-cycle diagram.

This paper is organized as follows: Section 2 describes the damage-cycle diagram which is a tool that can effectively interpret the commonly used fatigue testing methods. Section 3 provides a

variable transformation technique, which can seamlessly reveal the interrelationship among these testing methods in the damage-cycle diagram. Section 4 provides some examples demonstrating the concepts and techniques developed in the two previous sections, with emphasis on degradation testing and the application of the Weibull distribution functions. Discussion on other possible further development and cautions are provided in Section 5. Finally, Section 6 summarizes the paper with several key observations.

2. COMMON FATIGUE TESTING METHODS AND THEIR REPRESENTATION IN THE DAMAGE-CYCLE DIAGRAM

The life testing (test-to-failure) method tests a component to the occurrence of failure under the specified loading. The cycle data from the test of a certain sample size, either complete data or censored data, can be used to estimate the life distribution of the product. The determination of sample size is often determined by balancing the cost, timing, accuracy, and the samples available. Usually, a limited amount of samples, e.g. 6, are specified for a specific program. Two-parameter and three-parameter Weibull or lognormal distribution probabilistic functions are often used to estimate the parameters, i.e. mean and standard deviation for two-parameter lognormal (or normal in a log plot) distribution, and shape and scale parameters for two-parameter Weibull distribution. A threshold or location parameter is introduced in the three-parameter distributions to model the threshold physical phenomena, such as crack initiation and propagation [3]. Below the threshold value, often positive for a well-manufactured product, there will be no failure. The least squares (LS) methods and maximum likelihood estimation (MLE) methods are often used to estimate the parameters. For a normal distribution function, both LS and MLE methods result in exactly the same values of estimated parameters [8]. For other distribution functions, the estimated results of the parameters are different. The durability and reliability performance of a product can be sufficiently assessed by comparing the estimated distribution of failure against the performance targets set in the design and validation specifications.

The binomial testing method tests the product to the target, e.g. a specific applied cycle or time, under specified loading conditions. Only two possibilities occur in the binomial test: pass or fail. Test-to-success, also referred to as success-run method, is a special binomial test method, in which no failure occurs. Clearly, the binomial testing method provides less information as the testing to failure method does. However, it proves to be effective in reliability demonstration, in particular for complex systems [3]. Bernoulli trials for independent random events is the basis of the binomial testing and the discrete probability distribution generated from the Bernoulli trial is the binomial distribution. Similar to life testing, the reliability as a unknown parameter at a given applied cycle or time can be estimated from the binomial testing results.

If life testing takes too long for a product to fail operating under normal or even accelerated stress conditions, degradation testing method can be used or as a complementary [3]. If a failure mechanism can be directly linked to the degradation of part of the product, and degradation analysis allows to extrapolate to a failure

time based on the measurements of degradation or change in performance over time [5]. Degradation data may even yield more accurate life estimates than life test data with few failures and yield better insight into the degradation process and how to improve it [4]. A widely cited example of product degradation is the wear of brake pads, by which the life of the brake pads can be accurately estimated based on the observation by degradation testing made in a relatively short time [3, 5]. Fatigue crack propagation due to material fatigue is also a widely used example [9].

To better understand the advantages and disadvantages of these testing methods for a given product validation project, a general framework, under which the three methods can be evaluated is required. The damage-cycle diagram [7] is a tool to bring all of these three testing methods together in the same picture and under the same framework.

Fig.1 schematically depicts the damage-cycle (D - N) diagram and the three testing methods in the damage-cycle diagram. In Fig.1, the horizontal axis represents the applied fatigue cycle N while the vertical axis represents the damage D . The intersection of the two axes represents the very beginning of the testing process, at which the applied cycle $N = 0$ and damage $D = 0$. The damage D is always an increasing function of N because a damage process is often assumed to be an irreversible process, i.e. $dD/dN \geq 0$, so that any possible damage healing processes are ignored. When the applied cycle $N = N_f$, the damage $D = 1$ indicating a complete failure of the product. The dashed lines shown in Fig.1 represent the evolution trajectory of the damage bounds (lower and upper), which can be linear or nonlinear depending on the assumption of the damage process. The dashed lines above $D = 1$ are physically meaningless, however, its presence can facilitate the mathematical operation of the damage distribution at a given cycle as shown in the following sections. It is called “virtual damage trajectory” here to differentiate it from the trajectory for $D \leq 1$.

Fatigue test data often show large scatters which come from many sources such as material uncertainty, loading uncertainty, and other uncertainties. In many cases, especially for single failure mechanisms and modes, continuous probability density functions (PDFs) $f(N)$ and cumulative distribution functions (CDFs) $F = \int f(N) dN$ can be effectively used for characterizing the scattered test data.

The PDF and CDF as obtained from fitting the test-to-failure data, which are generated from life testing, can be described as $f[N_f(D = 1)]$ and $F[N_f(D = 1)]$. In Fig.1 the probabilistic distributions are assumed to be representative of the population, so that the uncertainty caused by the sample size is ignored. The obtained probabilistic distribution of failure can be compared against the established reliability criterion or metric, and the reliability performance of the product can be evaluated. The most appropriate reliability metric is simply the reliability function $R = 1 - F$. For example, a product specification R99 states that 99% of the product is expected to pass a specified target.

The scatter and uncertainty of the fatigue behavior can also be described by $f[D(N)]$ and $F[D(N)]$ which are the PDF and CDF of the damage at a specific applied cycle, respectively, also shown in

Fig.1. As compared to the life data from the life testing, obtaining the damage distribution below $D < 1$ is more difficult. Even worse, to obtain the damage distribution for $D > 1$ is physically and conceptually impossible. Again, the only purpose of keeping the “virtual” damage distribution is to understand the mathematical structure of the damage distribution. Eventually, in lieu of the more detailed continuous distribution, a practical way to simplify $f[D(N)]$ and $F[D(N)]$ is to use a coarser quantitative measure: pass or fail, which is exact the measure used in the binomial testing. Mathematically, all products with $D < 1$ are characterized as “pass” and all others are characterized as “fail”.

The distribution of the cycles at any given damage level below $D = 1$ i.e. $f[N(D_d)]$ shown in Fig.1 and the damage distribution at any applied cycle below N_F , i.e. $F[D(N_d)]$ also shown in Fig.1 can be represented in the damage-cycle ($D - N$) diagram. The lower case d stands for degradation.

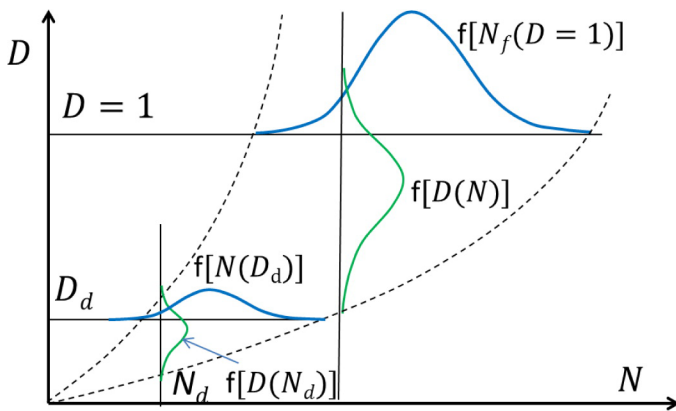


Fig.1. The representation of population as revealed in life testing, binomial testing, and degradation testing in the damage-cycle ($D-N$) diagram

It should be noted that significant amounts of test data are required to estimate the distribution of the population. However, in practice it is difficult and often impossible to get the distribution of the population because of either budget constraints, timing, or both. In the automotive industry, the number of samples used for product design and validation is usually very limited [1]. However, durability and reliability engineers have to find a way to effectively validate a product from a limited sample size. The testing procedure with limited sample size is schematically illustrated in Fig. 2. The dashed distribution lines indicate that the true population distributions are essentially unknown. The circles on the horizontal line $D = 1$ represent the cycles to failure of the samples. The solid circles stand for the samples below a specified cycle N , and the hollow circles stand for the samples above the cycle N . The diamonds on the vertical line at the specific cycle N represent the pass/fail status of the samples. The solid diamonds stand for pass, i.e. $D < 1$ and the hollow diamonds stand for fail, i.e. $D > 1$. A previous paper [10] presented some commonly used techniques to validate a product from tests with limited sample size. specified

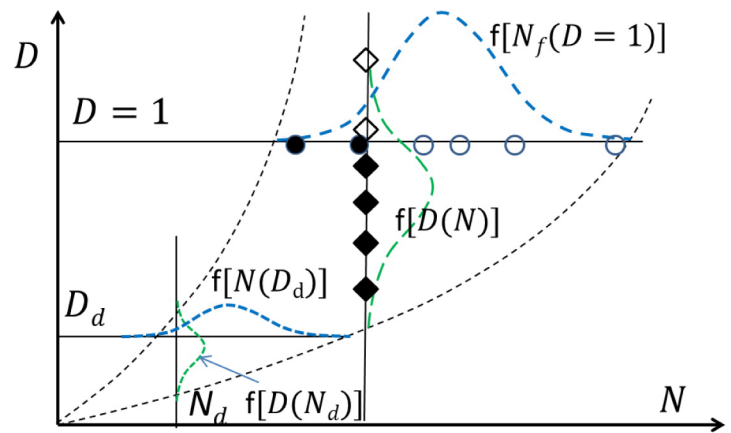


Fig 2. The representation of unknown population and samples in the damage-cycle ($D-N$)

3. INTERRELATIONSHIPS AMONG THE TESTING METHODS AS INTERPRETED BY A VARIABLE TRANSFORMATION TECHNIQUE

Intuitively, there should be close interrelationships among the distributions shown in Fig.1. The close relationship between the life distribution and the damage distribution has been conceived in a previous paper and proven for a Weibull distribution and the linear damage rule [10]. More rigorous mathematical treatment and discussion is provided in this section. The key to reveal the interrelationships among the distributions is to find the relationship between two different variables for a given damage evolution equation. Mathematically, the problem is equivalent to seeking a target distribution function $F_Y(y)$ for a given initial distribution function $F_X(x)$ and transformation functions: $y = \varphi(x)$ and $x = \psi(y)$. In fact, closed-form solutions can be obtained by the following procedure [11] which is well developed and described briefly below.

The target distribution function $F_Y(y)$ can be expressed as

$$F_Y(y) = P(Y \leq y) = P[\varphi(X) \leq y] \quad (1)$$

First, consider the case where $y = \varphi(x)$ is a strictly monotone increasing function. $x = \psi(y)$ is then a unique inverse function and

$$F_Y(y) = \int_{-\infty}^{\psi(y)} f_X(x) dx = F_X[\psi(y)] \quad (2)$$

The target PDF $f_Y(y)$ of Y is obtained as

$$f_Y(y) = \frac{dF_Y(y)}{dy} = f_X[\psi(y)] \frac{d\psi(y)}{dy} \quad (3)$$

For cases where $\varphi(x)$ is a strictly monotone decreasing function,

$$F_Y(y) = \int_{\psi(y)}^{\infty} f_X(x) dx = 1 - F_X[\psi(y)] \quad (4)$$

$$f_Y(y) = \frac{dF_Y(y)}{dy} = -f_X[\psi(y)] \frac{d\psi(y)}{dy} \quad (5)$$

The two cases in Eq.(3) and (5) can be combined as

$$f_Y(y) = \frac{dF_Y(y)}{dy} = f_X[\psi(y)] \left| \frac{d\psi(y)}{dy} \right| \quad (6)$$

The variable transformation technique shown in Eq.(1)-Eq.(6) is the essential part of the unified framework for representing the various testing methods.

With this simplified technique, the distribution of cycles to failure can be easily calculated from the damage distribution at a given cycle or the cycle distribution at a given damage. In reverse, if we know the final life distribution and the damage evolution equation, the distribution of damage at any given cycles and the cycle distribution at any given damage levels can be calculated. An important result derived from the variable transformation technique is that:

The failure percentage at a given cycle as predicted from the life distribution is exactly the same as that obtained from the damage distribution at the same given cycle and at $D = 1$.

Proof

First, the fatigue damage can be generally considered as an irreversible process as stated before, i.e., $dD/dN \geq 0$. Therefore, the damage $D = \varphi(N_f)$ is a monotone decreasing function for a given cycle N . The linear damage rule $D = N/N_f$ is a typical example. Eventually, Eq.(4) and Eq.(5) should be used for variable transformation if the distributions of cycles to failure $f[N_f(D = 1)]$ and $F[N_f(D = 1)]$ are converted to the distributions of damage $f[D(N)]$ and $F[D(N)]$ for a given cycle. For a given distribution of cycles to failure, $F(N_f)$, the distribution function of damage at any cycle, according to Eq.(4), is $F_D(D) = 1 - F_N[N_f(D)]$. Clearly, when $D = 1$, $F_D(1) = 1 - F_N[N_f(1)]$, indicating that the complementary part of the accumulative distribution of damage is equal to the accumulative distribution of cycles to failure. The subscript D and N related to functions $F()$ represent damage and cycle, respectively. In other words, there is one-to-one correspondence (exactly the same) between the failure percentages of the life distribution and the damage distribution accumulative no matter if the damage evolution is linear or nonlinear. In Fig.1, the geometric representation of the transition process from the life distribution to damage distribution is the left side turns up and the right side turns down. The fact that the two reliability values are the same for a population is critical to the assumption used for the extended binomial test method [3, 10]. Several examples provided below will demonstrate the variable transformation procedure. Two distribution functions, i.e. uniform distribution and the two-parameter Weibull distribution, are used. Both linear and nonlinear functions are considered.

Example-1: The damage distribution at a specific given applied cycle N obtained from the uniform life distribution with linear damage rule, $D = N/N_f$

The PDF and CDF of the uniform distribution function are given in Eq. (7) and Eq. (8), respectively.

$$f(N_f) = \begin{cases} \frac{1}{b-a}; & a \leq N_f \leq b \\ 0; & N_f < a \quad \text{or} \quad N_f > b \end{cases} \quad (7)$$

$$F(N_f) = \begin{cases} 0; & N_f < a \\ \frac{N_f - a}{b - a}; & a \leq N_f \leq b \\ 1; & N_f > b \end{cases} \quad (8)$$

For linear damage rule, $D = \varphi(N_f) = N/N_f$, $N_f = \psi(D) = N/D$ and $d\psi(D)/dD = -N/D^2$. D is a monotone decreasing function of N_f . With the help of the variable transformation technique, Eq.(4) and Eq.(5), the PDF, Eq.(9), and CDF, Eq.(10), of damage D at a given applied cycle N can be obtained.

$$f(D) = \begin{cases} N/[D^2(b-a)] & N/b \leq D \leq N/a \\ 0; & D > N/a \quad \text{or} \quad D < N/b \end{cases} \quad (9)$$

$$F(D) = \begin{cases} 0; & D < N/b \\ (Db - N)/[D(b - a)] & N/b \leq D \leq N/a \\ 1; & D > N/a \end{cases} \quad (10)$$

Clearly, the PDF shown in Eq. (9) is not a uniform function of D as it does in Eq. (7). Additionally, when $D = 1$, the complementary part, i.e., $1 - F[D(1)]$ of Eq. (10) is exactly the same as Eq. (8), $F(N_f)$.

Example-2: The damage distribution at a specific given applied cycle N obtained from the Weibull life distribution with linear damage rule, $D = N/N_f$

The two-parameter Weibull distribution $f[N_f(D = 1)]$ and $F[N_f(D = 1)]$ are shown in Eq. (11) and Eq. (12).

$$f(N_f) = \left(\frac{\beta}{\eta} \right) \left(\frac{N_f}{\eta} \right)^{(\beta-1)} \exp \left[- \left(\frac{N_f}{\eta} \right)^{\beta} \right], \quad N_f \geq 0 \quad (11)$$

$$F(N_f) = 1 - \exp \left[- \left(\frac{N_f}{\eta} \right)^{\beta} \right], \quad N_f \geq 0 \quad (12)$$