

A New Interval Inverse Analysis Method and Its Application in Vehicle Suspension Design

Xingxing Feng, Kaimin Zhuo, Jinglai Wu, Vikas Godara, and Yunqing Zhang
Huazhong University of Science and Tech

ABSTRACT

Interval inverse problems can be defined as problems to estimate input through given output, where the input and output are interval numbers. Many problems in engineering can be formulated as inverse problems like vehicle suspension design. Interval metrics, instead of deterministic metrics, are used for the suspension design of a vehicle vibration model with five degrees of freedom. The vibration properties of a vehicle vibration model are described by reasonable intervals and the suspension interval parameters are to be solved. A new interval inverse analysis method, which is a combination of Chebyshev inclusion function and optimization algorithm such as multi-island genetic algorithm, is presented and used for the suspension design of a vehicle vibration model with six conflicting objective functions. The interval design of suspension using such an interval inverse analysis method is shown and validated, and some useful conclusions are reached.

CITATION: Feng, X., Zhuo, K., Wu, J., Godara, V. et al., "A New Interval Inverse Analysis Method and Its Application in Vehicle Suspension Design," *SAE Int. J. Mater. Manf.* 9(2):2016, doi:10.4271/2016-01-0277.

INTRODUCTION

The optimum selection of the suspension parameters of a vehicle is of great practical importance, because suspension system plays a significant role in ride and handling of vehicles. Many methods have been developed and employed to the optimum selection of the suspension stiffness and damping parameters of a vehicle.

Georgiou et al. [1] applied a systematic methodology to select optimum values for the suspension stiffness and damping parameters of a quarter-car model, subjected to road excitation. The optimization process is based on three suitable performance criteria related to ride comfort, suspension travel and road holding of the vehicle and yielding the most important suspension stiffness and damping parameters. Alkhatib et al. [2] applied the genetic algorithm to the optimization of a linear quarter-car model. The optimum solution is obtained numerically by using genetic algorithm and employing a cost function that sought minimization of absolute acceleration root mean square (RMS). Nariman-Zadeh et al. [3] applied a new multiobjective uniform-diversity genetic algorithm to Pareto optimization of a five-degree of freedom vehicle vibration model considering the five conflicting functions simultaneously. Such multi-objective optimization of vehicle model could unveil very important design trade-offs between conflicting objective functions which would not have been found otherwise. Pazooki et al. [4] studied sensitivity of the ride response to suspension parameters of a 13-degree-freedom ride dynamic model.

In all of these researches, the ride responses of the vehicle model are chosen as multi-objectives to be optimized. However these objectives often conflict with each other so that as one objective improves,

another deteriorates. Moreover it has been shown that the trade-off between ride and handling is difficult to achieve [5]. In this case, the objective functions can be released. That is, the values of objective functions are constrained in given reasonable intervals instead of being minimized, and the design variables are intervals instead of deterministic values. It actually forms an interval inverse problem of suspension design in which the uncertainty of the system parameters are described by interval numbers.

Interval inverse problems can be defined as problems to estimate input through given output, where the input and output are interval numbers. Many problems in engineering can be formulated as inverse problems. Zhang et al. [6] proposed a computational inverse technique to determine the encounter condition from the given projectile state in the projectile/multilayer medium system. Based on the deterministic inverse technique, interval analysis and the intergeneration projection genetic algorithm are employed as the uncertainty inverse procedure. Jiang et al. [7] proposed a novel method to solve uncertainty inverse problems, in which the uncertainty is modeled using intervals. An interval forward model is constructed and the inverse analysis process is formulated as an optimization problem based on an error function.

In order to solve the interval inverse problems, some interval analysis methods have to be used. The interval analysis methods, established on the basis of interval arithmetic [8] generally experience overestimation due to the intrinsic wrapping effect of the interval arithmetic. To this end, many other schemes are required to be incorporated into the interval analysis methods to control the overestimation, such as the Interval Taylor series method [9, 10] and Taylor model method [11].

Wu et al. [12] proposed a Chebyshev inclusion function method for multibody systems with interval uncertainties. This method is a type of non-intrusive method and can control overestimations much better than the Taylor methods. Furthermore Wu et al. [13] proposed a new design optimization framework for suspension systems in the presence of uncertainties, in which an interval bisection method was used to reduce the overestimation.

This study explores a new efficient interval inverse analysis method based on Chebyshev inclusion functions for suspension design of vehicle vibration model. In contrast with traditional optimum suspension design in which vehicle ride response should be minimized or maximized, interval inverse analysis assumes the vehicle ride response as an interval vector and the suspension parameter are also interval numbers to be solved. It can ensure ride responses of vehicle vary in a reasonable range and it can reduce the conflicts between the multi-objectives in some degree. On the other hand the stiffness and damping parameters of suspension have inherent uncertainties in geometry and material parameters, as well as assembly process and manufacturing tolerances, wear, ageing and so on. It is more reasonable to express the suspension parameters using interval uncertain parameters. Therefore it is meaningful to apply interval inverse analysis for suspension design.

INTERVAL INVERSE ANALYSIS METHOD USING CHEBYSHEV INCLUSION FUNCTION

Consider a system containing uncertain interval parameters as follows:

$$\mathbf{Y} = \mathbf{Y}(\mathbf{X}, \mathbf{V}, \mathbf{P}) \quad (1)$$

where $\mathbf{Y}$ is the output and $\mathbf{X}$ is the input. And $\mathbf{V}$ is the vector composed by system deterministic parameters while $\mathbf{P}$ is the vector composed by system uncertain interval parameters.

As the vector $\mathbf{X}$ and vector $\mathbf{V}$ are deterministic, [equation \(1\)](#) can be simply expressed by

$$\mathbf{Y} = \mathbf{Y}(\mathbf{P}) \quad (2)$$

For a specific interval $\mathbf{Y}^I$ of system output, it can be modeled using midpoint-radius form, as follows

$$\begin{aligned} \mathbf{Y}^I &= [\mathbf{Y}^L, \mathbf{Y}^U] = \\ &[\mathbf{Y}^M - \mathbf{Y}^R, \mathbf{Y}^M + \mathbf{Y}^R] = \mathbf{Y}^M + [-1, 1] \mathbf{Y}^R \end{aligned} \quad (3)$$

where $\mathbf{Y}^L$ and $\mathbf{Y}^U$ are lower bound vector and upper bound vector; $\mathbf{Y}^M$ and $\mathbf{Y}^R$ are midpoint vector and radius vector, respectively. The corresponding interval $\mathbf{P}^I$ of system uncertain parameters can also be written in the form of

$$\begin{aligned} \mathbf{P}^I &= [\mathbf{P}^L, \mathbf{P}^U] = \\ &[\mathbf{P}^M - \mathbf{P}^R, \mathbf{P}^M + \mathbf{P}^R] = \mathbf{P}^M + [-1, 1] \mathbf{P}^R \end{aligned} \quad (4)$$

where $\mathbf{P}^L$ and $\mathbf{P}^U$ are lower bound vector and upper bound vector; $\mathbf{P}^M$ and $\mathbf{P}^R$ are midpoint vector and radius vector, respectively. According to [equation \(2\)](#), following equation can be obtained

$$\begin{aligned} \mathbf{Y}^L(\mathbf{P}^I) &= \min \mathbf{Y}(\mathbf{P}^I) = \min \mathbf{Y}(\mathbf{P}^M \pm \mathbf{P}^R) \\ \mathbf{Y}^U(\mathbf{P}^I) &= \max \mathbf{Y}(\mathbf{P}^I) = \max \mathbf{Y}(\mathbf{P}^M \pm \mathbf{P}^R) \end{aligned} \quad (5)$$

It can be seen from [equation \(5\)](#) that the interval inverse problem can be transformed into two deterministic inverse problems. One is inversely determining midpoint of the system interval parameter vector $\mathbf{P}^M$ from the given midpoint of the system output vector $\mathbf{Y}^M$, the other is inversely determining radius of the system interval parameter vector $\mathbf{P}^R$ from the given radius of the system output vector $\mathbf{Y}^R$.

The midpoint of the system interval parameter vector $\mathbf{P}^M$ can be achieved directly from the given midpoint of the system output vector $\mathbf{Y}^M$ using optimization technique.

$$\min_{\mathbf{P}^M \in \Omega^M} \left(\left| \mathbf{Y}(\mathbf{P}^M) - \mathbf{Y}^M \right| \right) \quad (6)$$

where Ω^M is the inverse analysis space for $\mathbf{P}^M$. After getting the midpoint vector $\mathbf{P}^M$, we need to calculate the radius vector $\mathbf{P}^R$ according to [equation \(4\)](#).

The radius of the system interval parameter vector $\mathbf{P}^R$ can be obtained by Chebyshev inclusion function in conjunction with the traditional optimization technique.

$$\min_{\mathbf{P}^R \in \Omega^R} \left(\left| \mathbf{Y}^L(\mathbf{P}^I) - \mathbf{Y}^L \right| + \left| \mathbf{Y}^U(\mathbf{P}^I) - \mathbf{Y}^U \right| \right) \quad (7)$$

where Ω^R is the inverse analysis space for $\mathbf{P}^R$; $\mathbf{Y}^L$ and $\mathbf{Y}^U$ are the lower bound and upper bound of the specific interval $\mathbf{Y}^I$. $\mathbf{Y}^L(\mathbf{P}^I)$ and $\mathbf{Y}^U(\mathbf{P}^I)$ are the lower and upper bounds of the system output, which can be estimated using the Chebyshev inclusion function.

$$\begin{aligned} \mathbf{Y}(\mathbf{P}^I) &= \sum_{i_1=0}^k \cdots \sum_{i_n=0}^k \left(\frac{1}{2} \right)^l \mathbf{Y}_{i_1, \dots, i_n} C_{i_1, \dots, i_n}(\mathbf{P}^I) \\ &= \frac{1}{2^n} \mathbf{Y}_{0, \dots, 0} + [-1, 1] \sum_{\substack{0 \leq i_1, \dots, i_n \leq k \\ i_1 + \dots + i_n \geq 1}} \left(\frac{1}{2} \right)^l \mathbf{Y}_{i_1, \dots, i_n} \end{aligned} \quad (8)$$