

Methods to Find Best Designs Among Infeasible Design Data Sets for Highly Constrained Design Optimization Problems

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ABSTRACT

In recent years, the use of engineering design optimization techniques has grown multifold and formal optimization has become very popular among design engineers. However, the real world problems are turning out to be involved and more challenging. It is not uncommon to encounter problems with a large number of design variables, objectives and constraints. The engineers' expectation, that an optimization algorithm should be able to handle multi-objective, multi-constrained data is leading them to apply optimization techniques to truly large-scale problems with extremely large number of constraints and objectives. Even as newer and better optimization algorithms are being developed to tackle such problems, more often than not, the optimization algorithms are unable to find a single feasible design that satisfies all constraints. It is common to see designers spending large amounts of computational resources in evaluating infeasible designs mainly because either the algorithms take time to get to the feasible regions of the design space, or no feasible designs are obtained after all the allowed optimization iterations.

Even with surrogate based or meta-model based optimization, which enables designers to evaluate thousands of solutions with negligible computational efforts, finding a feasible design remains a challenge due to large number of optimization parameters and the highly constrained nature of the design problem. In such a case where no feasible design is available, it becomes very important to analyze the available data and find the best designs among a large number of infeasible designs.

The aim of this study is to research applicability of methods such as Multi-Criteria Decision Making (MCDM) and constraint relaxation to identify best designs among infeasible designs. Such methods aid the time-constrained engineer to make sense of the already available, albeit infeasible design data to better understand the design space. It is possible to identify regions of design space by slightly relaxing a few constraints. In this paper, two methods - Constraint Relaxation Technique and Multi-criteria Decision Making - are first applied to mathematical test problems and then to a real world automotive problem, involving different load cases with the objectives of weight reduction and performance improvement.

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INTRODUCTION

Most discussions of engineering design optimization problems, assumes existence of a feasible region of the design space and further assumes it is easy to find this region of feasibility. However, this is not the case always.

Finding the feasible region of the design space can prove to be a difficult task for a few cases. This difficulty is further compounded when the design evaluations prove to be expensive. There are situations where a certain computational budget allocation is not enough to find a feasible design when the problem to be solved is highly constrained. This happens when the number of constraints are high or when the non-linear nature of the response/constraint functions slow down the optimization algorithm.

When faced with such situations the optimization expert/ design engineer has very limited options. Facing time constraints and hard deadlines to meet, increasing the computational budget or spending more time exploring the design space is a difficult proposition and most times is not an option.

The study presented in this paper aims at providing some insights into how to proceed in such situations and to present the possible options available. Two of the possible ways to proceed when faced with infeasible design evaluations are to perform a Multi-Criteria Decision Making (MCDM) analysis or to use Constraint Relaxation Tool (CRT) to gain some insight into the design space. A brief

introduction and description of these two tools are given in the following two sections. The methodology adopted to use these tools on infeasible design data sets are described in further sections.

Ideally, the methods presented in this paper should be applied on data that is generated from unbiased input design of experiments data set. However, we can also apply it on infeasible data sets that are a result of an optimization run if we are left with no choice because of the expensive nature of the design evaluations.

MULTI-CRITERIA DECISION MAKING (MCDM)

The Multi-criteria Decision Making (MCDM) tool is included as a part of the modeFRONTIER process integration and design optimization software platform [1]. It is provided as a data post-processing tool that enables decision making to find the best design among the design data generated by a multi-objective optimization run. In this section, some details about this post processing method is described along with how it can be used to analyze infeasible design data from a highly constrained design optimization run.

When performing a multi-objective design optimization study, we usually see design options that are a tradeoff between multiple objectives. To understand this further, let us consider a two-objective design optimization study. Most of the times, the result of such an optimization run does not result in a single optimum design or a single answer that satisfies the two objectives we have at hand. This is especially true when the objectives are competing in nature. We find that a design that has a good performance for one of the objectives may not be better than another design in terms of the second design objective. This means that we have to choose the level of importance to give each of the two objectives to select the final candidate design.

This decision or choice of assigning importance to one of the objectives becomes more and more difficult when an optimization problem is setup to have greater number of objectives that form a tradeoff. Having more than three objectives makes it difficult to plot the Pareto front that in turn makes the decision to choose the best design, quite hard. However, MCDM is a tool that assists making easy decisions by boiling down the choices to an ordered rank list of designs. In order to create this ranked order of designs, the MCDM tool employs quantities such as risk aversion index and weights assigned for each objective used in the MCDM decision making. These quantities are further used in the calculation using the utility for each objective. These utilities are then added to get the global utility for each design. Based on these global utility values for each design we are able to rank the designs.

The use of the MCDM tool for analyzing infeasible design data is described in detail using examples described later.

CONSTRAINT RELAXATION TECHNIQUE (CRT)

Using the Constraint Relaxation Technique (CRT) we can gain insights into the data generated by an optimization run especially when the designs turn out to be infeasible. As the name suggests, the method allows relaxation of the constraints in incremental amounts systematically and then analyze the tradeoffs of getting a feasible design against the number and amount of constraint violations. A tool that implements the CRT is included as part of the SOMO enterprise solution platform [2] used for multidisciplinary and collaborative optimization.

CRT analyzes the given design data set and suggests relaxing the constraints to get feasible designs. Initially the tool asks for the maximum amount of relaxation allowable for each of the constraints as a percentage of the maximum violation seen for each constraint in the provided data. Using this information, and using internal strategies to find the combinations of constraints to relax, the tool comes up with five configurations that have progressively increasing amount and number of relaxed constraints. From the resulting configurations, we are able to pick the best designs that satisfies the relaxed constraints. Moreover, we can even decide to run further studies using the relaxed constraints, knowing that they will allow for feasible designs.

APPLICATION 1: ONE-DIMENSIONAL MATHEMATICAL OPTIMIZATION PROBLEM

In this section, we apply the two techniques, the MCDM and CRT, to a simple test mathematical problem that is highly constrained. The mathematical problem is defined as below. This mathematical problem is easy to graph and hence it is very straightforward to visualize the optimization problem. This makes it simple to illustrate the benefits of applying the MCDM and the CRT method on infeasible design data. [Figure 1](#) plots the output response versus the optimization parameter free variable along with the constraints delineating the feasible region of the design space.

$$\begin{aligned}
 &\min f(\mathbf{x}) = x^2 \\
 \text{s. t.} \quad &(1) 1. f(\mathbf{x}) - 312.583 - 94.7222 * x < 0 \\
 &(1) 2. (x - 6)^2 + (f(\mathbf{x}) - 10)^2 - 8 < 0 \\
 &(1) 3. f(\mathbf{x}) - 30 + 3 * x < 0 \\
 &\text{for } x \in [-100, 100]
 \end{aligned} \tag{1}$$

From [Figure 2](#), it is clear that the solution to the optimization problem defined in (1) is the lower point on the intersection of $f(x)$ and the circle constraint defined in (1)2. Since this optimization problem is highly constrained even for a single variable problem, an optimization algorithm such as the NSGA-II (a variant of the Genetic algorithm) takes some design evaluations before getting to the feasible region of the design space. Considering this fact, we first run a DOE study of a 100 designs generated using the Uniform Latin Hypercube (ULH) method. All the 100 designs turn out to be infeasible due to the highly constrained nature of the problem. To find

the best designs among these 100 infeasible designs the MCDM and the CRT methods are applied and the results are explained in the following sub-sections.

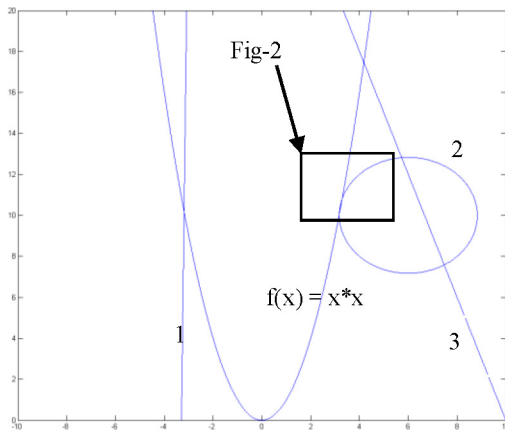


Figure 1. Plot of the response function along with constraints.

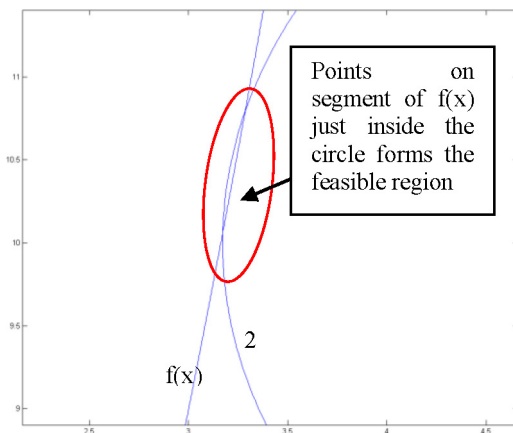


Figure 2. Zoomed-in section of the feasible region.

MCDM Applied to the One-Dimensional Mathematical Problem

The MCDM methodology is applied to the infeasible design data set generated from the DOE evaluations to find the best designs among the infeasible designs. To do this, the data requires to be transformed in order to be suitable for MCDM analysis.

When setting up an MCDM analysis, we need to specify the objectives that are to be considered and the direction (minimize or maximize) is to be specified for each objective. In this case, as our objective is to find the best designs that violate constraints the least, the evaluations of the constraint functions have to be transformed into the objectives for the MCDM analysis. In addition, to not mislead the MCDM tool, all constraints that are not violated need to be set to 0 so that they are treated equally. As an example, let us consider the constraint (1)1. If $x = 0$, the constraint function on the left-hand side evaluates to -312.583 and hence the constraint is not violated. We could say the constraint violation is 0 in this case and hence set the constraint value that is provided to MCDM analysis, to 0. If however, $x=100$, then the left-hand side evaluates to 215.197. In this case, the constraint is violated by this value 215.197. We would like this to be

as close to 0 as possible and the MCDM will rank the designs based on how small this number is. The transformation for the constraint function then becomes:

$$\max(f(x) - 312.583 - 94.7222 * x, 0) < 0 \quad (2)$$

In inequality (2), the left-hand side always evaluates to a positive number or zero. Specifying this LHS function as an MCDM objective will enable the method to seek designs that have a low amount of constraint violation.

Performing the above-mentioned transformation for all the constraints, we get:

$$\max((x - 6)^2 + (f(x) - 10)^2 - 8, 0) < 0 \quad (3)$$

$$\max(f(x) - 30 + 3 * x, 0) < 0 \quad (4)$$

These results in 10 designs (listed in Table 1) identified as the best by the linear MCDM analysis that used equal weights and equal preference:

Table 1. Best Designs from an MCDM analysis of infeasible DOE designs: Application 1.

Design ID	x	f(x)
3	3.113575099	9.694349899
74	0.865413858	0.748941146
42	-1.233902773	1.522516054
73	4.827358736	23.30339237
57	6.247114693	39.02644199
97	-3.76119734	14.14660543
7	8.180831929	66.92601105
80	-4.675441855	21.85975654
31	10.73762022	115.2964881
75	12.67594808	160.6796597

Plotting these points and shading the data points according to rank (Figure 3) clearly indicates, that the best-ranked point (Design ID:3) are the closest to the feasible region/optimum of the design space.

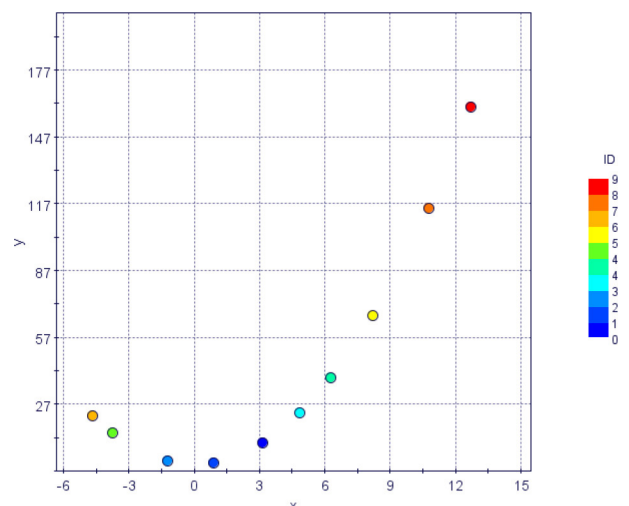


Figure 3. Best 10 DOE points ranked according to MCDM analysis and plotted on the design space. Demonstrates that the highest ranked designs are close to the feasible region of the design space.