

# Multi-Disciplinary Tolerance Optimization for Internal Combustion Engines Using Gaussian Process and Sequential MDO Method

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## ABSTRACT

The internal combustion engine (ICE) is a typical complex multidisciplinary system which requires the support of precision design and manufacturing. To achieve a better performance of ICEs, tolerance assignment, or tolerance design, plays an important role. A novel multi-disciplinary tolerance design optimization problem considering two important disciplines of ICEs, the compression ratio and friction loss, is proposed and solved in this work, which provides a systematic procedure for the optimal determination of tolerances and overcomes the disadvantages of the traditional experience-based tolerance design. A bi-disciplinary analysis model is developed in this work to assist the problem solving, within which a model between the friction loss and tolerance is built based on the Gaussian Process using the corresponding simulation and experimental data. In addition, the formulation of the compression ratio considering those non-critical dimensions which actually affect the friction loss is proposed. Finally the multi-disciplinary tolerance design optimization problem is formulated and solved using a recently developed sequential MDO (S-MDO) method.

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## INTRODUCTION

The internal combustion engine (ICE) is a complex nonlinear multidisciplinary system, which requires the support of precision design and manufacturing. The development of precision design and manufacturing of ICEs contribute a lot to the improvement of engine performance, such as fuel consumption efficiency and emission reduction. In this case, a small change in the dimensions of key components may have great impacts on the cost and system performance of ICEs. Thus more strict requirements have been imposed on tolerance design.

Tolerance design is the process of determining the machining tolerances and manufacturing precision of the corresponding components during the detailed design stage of a system. If the tolerance is set at relatively large values, although this would make the manufacturing procedure easier, it will lead to the degradation of assembling and performance, large fluctuations of system performance, and decrease of system reliability, thus failing to meet the expected targets. On the other hand, if tolerances are set too tight, it will lead to the increase of manufacturing costs and rejection rates. From this insight, if tolerances of those dimensions which have little or small effects on the system performance could be released to some extent while tolerances of those dimensions which have larger effects are shrunk reasonably, the manufacturing cost will decrease and the system performance will be improved simultaneously. As a result, how to determine critical and non-critical tolerances and then how to differentiate tolerance designs are very important to improve the system performance.

Traditional tolerance design is mainly based on the designers' experiences, or determined by the manufacturing precision of the machines available, or just based on the standards. Thus for the traditional methods, the tolerances are not determined systematically and globally considering the effects that the tolerances may have on the system performances. Besides, although the designers of a certain component do not have a comprehensive understanding of the relationship between the tolerances and the system performance, their design experiences take dominant roles in the tolerance decisions, which will result in unreasonable designing of manufacturing processes and wasting of limited resources. Meanwhile, it is difficult for the system designers to determine the tolerance distributions from the large number of corresponding dimensions purely based on experience. Although existing dimension and tolerance design optimization methods can determine the tolerance distributions for a certain dimension chain or for a simple subsystem, there are rare studies on tolerance design for a system based on optimization. This is because that the effects of tolerances at the system level are not easy to be measured or estimated, and as more and more components are considered, the tolerance assignments become much more complex. The tolerance design problem can be considered as a complex multi-disciplinary optimization problem. As a result, this work focuses on the application of multi-disciplinary optimization methods on the tolerance design of a gas engine based on simulation and experimental data.

There are quite a few researches on how to improve the system performance and to decrease manufacturing costs considering tolerance for ICEs [1, 2, 3, 4]. However, there are few studies on how

to solve for multi-disciplinary tolerance design optimization problems. There are few quantitative analysis models either, studying the relationship between tolerances which belong to the micro perspective and the system performances which belong to the macro perspective. The macro system performances for an ICE include the output power, torque, fuel consumption, emission, manufacturing costs, and so on.

There are a number of factors that relate to the tolerances and the system performances, of which the compression ratio and friction loss are two of the most important quantities. For example, low compression ratios may result in incomplete combustion of gases, low output torque and power, and high fuel consumption. On the other hand, high compression ratios will result in high noise and large vibration [5]. Moreover, for multi-cylinder engines, the variation of the compression ratios among multiple cylinders should be constrained, otherwise the uniformity of the engine performances will be largely degraded. Many researchers have proposed corresponding mathematical models on dimensions and the compression ratio, but they are limited to the calculation of the compression ratio, and did not research in depth of how sensitive the compression ratio is to tolerances and how the compression ratio varies with the change of tolerances. There are some researchers studying the relationship between system performance and the compression ratio [6, 7, 8], but most of them optimize the compression ratio by selecting the nominal values of key dimensions but do not consider their tolerances. In addition, current researches only consider a few numbers of dimensions, but do not identify the critical dimensions out of all dimensions that have major effects on the system performance. In this regard, it is indeed necessary to determine critical and noncritical dimensions. The method that is able to tackle this problem is sensitivity analysis, so as to study how variations of the inputs may affect the outputs. In [9], sensitivity analysis is applied for the selection of critical and non-critical dimensions and then optimization of tolerances for critical dimension is conducted. In their work, the optimization considers six dimensions which are only critical to the compression ratio, but not non-critical dimensions which may be important for other factors. Based on [9], several non-critical dimensions that may affect the friction loss are also taken into consideration in this work.

The friction loss takes up nearly 70% of all mechanical loss of an ICE, so the friction loss between the friction pairs is a major factor that has great impacts on the system performance. There are mainly three groups of friction pairs, the piston and the cylinder, the crankshaft bearing, and the connecting-rod bearing. The friction loss at the piston takes up 60% to 75% of the total friction loss [10]. Without changing the lubricating system, the friction loss can be minimized by optimizing the tolerances and thus improving the fuel efficiency can be achieved. The literature reports research work on the design of piston rings and the reduction of the friction [11, 12, 13], and current studies on the friction at the piston ring and the crankshaft bearing are mature. However, there is rare work studying how tolerances can affect the friction.

In this work, simulation models for the three groups of friction pairs are built. By using sensitivity analysis, the critical dimensions are also determined. However, since three friction pair models are

independent to each other physically, it is difficult to apply them together for further tolerance optimization on the friction pairs. In addition, there is also a big challenge for the construction of tolerance and performance relationships at the engine system level, because there are a large amount of parts and components with a great number of dimensions and dimension chains which are coupled to each other. It is not easy to integrate different simulation models together while doing lots of experiments is also time and resource consuming. Multiple dimensions and complex couplings among different subsystems and components make it difficult to obtain an explicit relationship between the tolerances and the performances of the engine. To overcome this difficulty, it is necessary to use surrogate models, which can be obtained from implicit nonlinear multidisciplinary mathematical formula, simulation models, or experimental data using advanced data fitting techniques. There are several modeling techniques that have been reported in the literature, such as response surface modeling (RSM) [14], Kriging modeling [15] and Gaussian process (GP) modeling [16]. Gaussian process is able to model systems with multiple inputs and multiple outputs, and provides the prediction of the mean value along with the uncertainty around it. Due to these advantages of, it is utilized in this work.

The relationship between the compression ratio and the performances of the engine is molded first in this work. For the friction loss, the relationship between the tolerances and friction loss is built using Gaussian process based on the simulation results from three individual models as well as experimental data. The two aspects, i.e., the compression ratio and friction loss, contribute to two disciplines which also have couplings. A structured analysis model between the tolerances and the performances of the system, considering both the compression ratio and the friction loss, is then established. Finally an MDO problem based on the analysis model is formulated. In [17], a novel sequential MDO (S-MDO) method is proposed. Since S-MDO is shown to be computationally efficient, it is chosen to solve the formulated multi-disciplinary tolerance design optimization problem.

The rest of this paper is organized as follows. [Section 2](#) describes the background and necessary illustrations used in this work. [Section 3](#) provides the formulated problems and solutions, followed by the conclusions provided in [Section 4](#).

## BACKGROUND

In this section, the background of this work is presented. The tolerance optimization for the compression ratio is reviewed briefly, the Gaussian process modeling approach is presented in detail, and the applied S-MDO method is also introduced.

### *Tolerance Optimization of Compression Ratio for Gas Engine*

In [9], the compression ratio of a gas engine  $\epsilon$  is shown in [Eq. \(1\)](#):

$$\epsilon = \frac{V_c + V_s}{V_c} = 1 + \frac{V_s}{V_1 + V_2 + V_3 + V_4 + V_5}$$

where  $V_c$  is the minimum volume of the cylinder,  $V_s$  is the piston swept volume,  $V_1$  is combustion chamber volume of the piston,  $V_2$  is combustion chamber volume of the cylinder cover,  $V_3$  is hole volume of the cylinder cover gasket,  $V_4$  is combustion chamber volume of the cylinder, and  $V_5$  is volume of clearance between the piston and cylinder. The corresponding structure of a typical gas engine is illustrated in Figure 1.

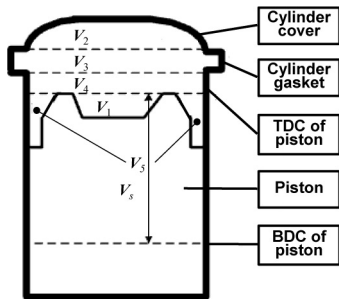


Figure 1. Illustration for an ICE (modified from [9])

Some of the volumes in Eq. (1) can be calculated based on sizes of corresponding components, and the compression ratio with respect to related dimensions can be obtained. After performing sensitivity analysis with respect to each of those dimensions, the critical and non-critical dimensions can be determined as discussed in [9]. The compression ratio with respect to critical dimensions is calculated as in Eq. (2):

$$\varepsilon = 1 + \frac{91526.412r_c}{12(V_1 + V_2) + 45763.208R + 25252.946} \quad (2)$$

Where  $r_c$  is crank radius,  $R = H - r_c - l - h - 0.04$ ,  $H$  is the distance from the upper surface of the cylinder to the center of the crankshaft hole,  $l$  is the center-to-center distance of the connecting rod, and  $h$  is the compression height of the piston.

The designed (nominal) values and the corresponding tolerances of these variables are listed in Table 1.

Table 1. Designed (nominal) values and corresponding tolerances for the gas engine

| Terms                    | Critical sizes | Tolerances | Variables | Actual Dimension intervals |
|--------------------------|----------------|------------|-----------|----------------------------|
| $V_1$ (mm <sup>3</sup> ) | 5600           | $\pm 300$  | $x_1$     | $5600 \pm 300 x_1$         |
| $V_2$ (mm <sup>3</sup> ) | 24400          | $\pm 400$  | $x_2$     | $24400 \pm 400 x_2$        |
| $r_c$ (mm)               | 39.5           | $\pm 0.05$ | $x_3$     | $39.5 \pm 0.05 x_3$        |
| $l$ (mm)                 | 121            | $\pm 0.05$ | $x_4$     | $121 \pm 0.05 x_4$         |
| $h$ (mm)                 | 25.1           | $\pm 0.05$ | $x_5$     | $25.1 \pm 0.05 x_5$        |
| $H$ (mm)                 | 186            | $\pm 0.05$ | $x_6$     | $0.5 \pm 0.05 x_6$         |

In the original design, the maximum variation of the compression ratio could reach 0.4388, which is too large and can significantly affect the consistency of the engine performance. The objective of the research work in [9] is to ensure that the maximum variation of compression ratio be within  $\pm 0.2$  by redesigning (shrinking) the

tolerances of these six critical dimensions. The design variables are such that a larger  $x_i$  means to change the tolerance a little, and a smaller  $x_i$  means to change the design a lot.

The optimization problem is formulated as:

$$\begin{aligned} \max \quad & f = \sum x_i^2 \\ \text{s.t.} \quad & \max |\Delta \varepsilon| \leq 0.2 \\ & x_i \in [0, 1], i = 1, 2, \dots, 6 \end{aligned} \quad (3)$$

where  $\varepsilon$  is as shown in Eq. (2),  $\Delta \varepsilon = \max[|\varepsilon_{\max} - \varepsilon_0|, |\varepsilon_{\min} - \varepsilon_0|]$  with  $\varepsilon_0$  being the nominal compression ratio, and the design variables are as shown in Table 1.

The objective of this problem is to maximize the squared sum of these design variables, meaning to keep the redesigned tolerances close to the original design as much as possible. Thus for critical dimensions that have larger impacts on the system performance, it does not necessary to make large changes but only necessary ones to ensure the system performance. By doing so, the performance of the system can be improved with the minimal cost (since the changes on the tolerances are the smallest). In this regard, cost and performance are both taken into consideration in this formulation.

### Gaussian Process Modeling

The Gaussian process can deal with multiple outputs, which makes it a good choice for modeling [18]. It is usually the case that the computer model is deterministic, that is, given the same inputs, the same outputs will always be obtained. However, experiments usually may not provide the same outputs even the experimental conditions are set to be the same. So there is a discrepancy between the experiments and simulation models. In [19], the computer model and the discrepancy function are modeled based on the Gaussian process, so that the simulation model can better fit the experiments. Due to these advantages of the Gaussian process, it is used as the surrogate modeling approach in this work.

It is shown in [16] and [19] that the relationship between the experiment and computer model can take the formulation as:

$$\mathbf{y}^e(\mathbf{x}) = \mathbf{y}^m(\mathbf{x}) + \delta(\mathbf{x}) + \varepsilon \quad (4)$$

where  $\mathbf{y}^e(\mathbf{x})$  is the vector of outputs (or response) from physical experiments,  $\mathbf{y}^m(\mathbf{x})$  is the vector of outputs from simulation model,  $\delta(\mathbf{x})$  is the discrepancy function, and  $\varepsilon$  is the measurement uncertainty.

The measurement uncertainty  $\varepsilon$  is usually modeled as a normal distribution with the mean zero and a small variance  $\lambda$ . Since this  $\varepsilon$  only affects the amount of the output variance in Eq. (4), it can be neglected in this work. The discrepancy function takes into consideration of uncertainties that may exist in the inputs, such as measuring errors of the inputs. Since the objective of this work is to establish a relationship between the tolerance and performance, thus