

Review and Assessment of Frequency-Based Fatigue Damage Models

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ABSTRACT

Several popular frequency-based fatigue damage models (Wirsching and Light, Ortiz and Chen, Larsen and Lutes, Benasciutti and Tovo, Benasciutti and Tovo with $\alpha_{.75}$, Dirlik, Zhao and Baker, and Lalanne) are reviewed and assessed. Seventy power spectrum densities with varied amplitude, shape, and irregularity factors from Dirlik's dissertation are used to study the accuracies of these methods. Recommendations on how to set up the inverse fast Fourier transform to synthesize load data and obtain accurate rainflow cycle counts are given. Since Dirlik's method is the most commonly used one in industry, a comprehensive investigation of parameter setups for Dirlik's method is presented. The mean error and standard deviation of the error between the frequency-based model and the rainflow cycle counting method was computed for fatigue slope exponent m ranging from 3 to 12. The results showed Ortiz and Chen, Benasciutti and Tovo with $\alpha_{.75}$, Larsen and Lutes, Dirlik, and Benasciutti and Tovo to be significantly more accurate than Lalanne and Zhao and Baker. These five models have a tendency for the error variation to increase as the fatigue exponent m increases. For this study using Dirlik's seventy spectra, Ortiz and Chen, Benasciutti and Tovo with $\alpha_{.75}$, and Larsen and Lutes Single Moment methods had lower mean error than Dirlik's method.

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INTRODUCTION

Fatigue and durability of automotive components mainly focuses on two types of mechanical loadings: deterministic cyclic loading from the periodical engine firing events and random loading from the road excitations. Fatigue for engine related components/structures typically adopt an infinite life approach based on a sampled time series loading from engine idling to fuel cut off [1]. While for frame/body mounted components, due to the random nature of the loading, fatigue analysis is generally accomplished in two ways: the time series approach or the spectral method based on the power spectral density of the load inputs. For the time series approach, an adequate duration of the time-domain mechanical loading covering all random loading conditions is required. Rainflow cycle counting [2, 3] and the linear damage rule [4] are then used to estimate the damage and fatigue life. There are drawbacks for applying the time series approach on random loading. It is costly to record a sufficiently long time history of load data to capture all frequency contents especially the less frequent but highly damaging events. Based on the study by Bishop [5], it is also computationally expensive to evaluate stresses and the fatigue damage from time series data of long duration.

The most common way to represent the dynamic response of the random loading input is by spectral methods in the frequency domain using a power spectral density with the assumption that the loading is random, stationary and Gaussian. Following Rice's [6] description of

the probability density function for load peaks, Bendat [7] showed how to calculate the fatigue damage for a narrowband random loading input. However, it was found that the narrow band approach is too conservative for estimating the damage because the random loadings on structures are more or less wide band in nature. Approaches were developed by Wirsching and Light [8], Ortiz and Chen [9], and Larsen and Lutes [10] to account for the difference between narrow-band and wide-band fatigue damages, by introducing an empirical correction factor which is a function of irregularity factor and fatigue slope exponent (a negative reciprocal of the fatigue strength exponent in Basquin's equation). A refined method which claimed having sound theoretical framework was introduced recently by Benasciutti and Tovo [11]. Both numerical and experimental verifications of the method have been reported elsewhere [12, 13, 14]. The other approach was originally introduced by Dirlik [15] to empirically describe the probability density function for approximating the rainflow amplitude distribution, which uses a combination of an exponential and two Rayleigh probability densities. Following a similar concept, Zhao-Baker [16] estimated the amplitude distribution using a combination of one Weibull and one Rayleigh density, and Lalanne [17] proposed a method using Rice's formula which Halfpenny [18] found gave excellent results.

The purpose of this study is three-fold: (1) to present the frequency-to-time domain process for random loading, (2) to establish the guidance for estimating the parameters in Dirlik's approach, and (3) to review all the mentioned frequency-based fatigue damage models and recommend the best one for use.

Applications of the spectral methods have been presented in numerous papers [5, 10, 11, 18, 19, 20, 21]. It was concluded that the Dirlik method is considered by far the most accurate and superior method in estimating rainflow fatigue damage. However, the papers do not address in detail the parameter setups for the spectral moment and damage calculation processes. A significant error could result if inaccurate parameters are used. Therefore, in this paper a thorough study of all the parameters in order to achieve accurate damage results from Dirlik's and other methods is presented. Seventy power spectral densities (PSD's) from Dirlik's dissertation were used for the calculations. Explanations are given on how to numerically integrate Dirlik's method and get results with high precision. A method to calculate the upper bound for integration is given and the effect on accuracy of integration step size is discussed.

The way to evaluate any frequency-based models is to compare the estimated damage value from the frequency domain method with that calculated from rainflow cycle counting histogram of the original time domain loading. Since no detailed study on the subject of converting a power spectral density to a time-domain loading has been reported, it is another purpose of the this paper to present the frequency-to-time domain conversion process including the details such as sampling rate, duration, resolution.

The third objective of this paper is to review the above mentioned frequency-based fatigue damage models and to present our findings based on the damage error assessment process and results. Fatigue damage versus the fatigue slope exponent is also studied for each method. The results indicate the accuracy of spectral methods from previously published sources may not hold if a different fatigue slope exponent is used.

PROCESSES FOR CALCULATING FATIGUE DAMAGE FROM RANDOM LOADS

Fatigue damage calculations for random loading are initially conducted in the time domain followed by rainflow cycle counting and the linear damage rule. By relating a random load's power spectral density in the frequency domain to the load's amplitude distribution in the time domain, fatigue damage calculations are then extended to the frequency domain. In order to verify the accuracy of any proposed frequency spectral methods, a synthesized time domain approach is developed. The general calculation process and methods for all three approaches are illustrated in Figure 1. In this paper, each approach will be considered briefly but the synthesized time series approach and primarily the frequency spectral approaches are discussed in detail.

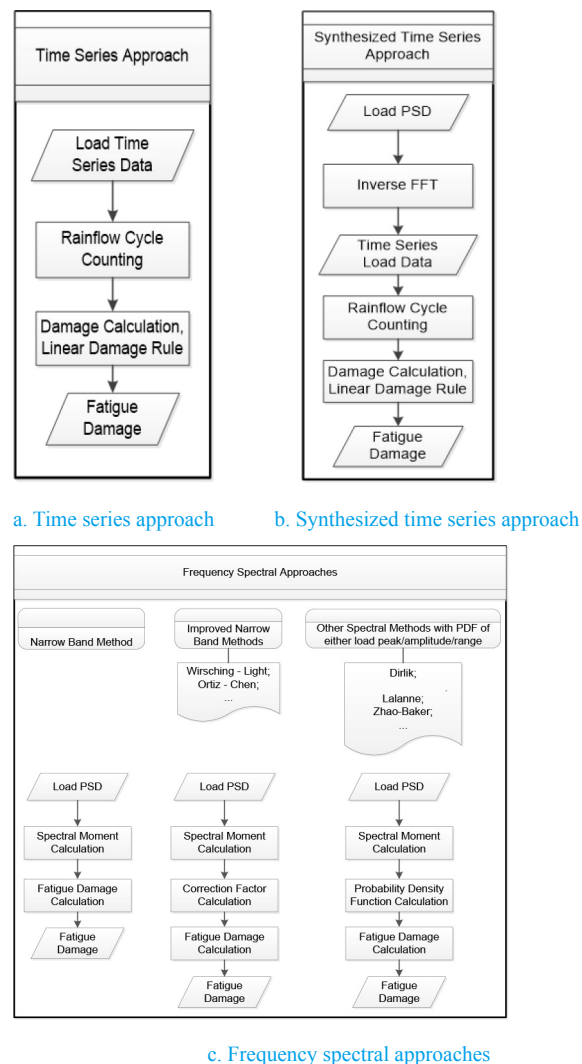


Figure 1. Approaches for estimating fatigue damages for random loadings

Case 1: Time History Fatigue Damage Calculations

During data collection, obeying Nyquist's criteria of using sampling rate higher than two times the highest frequency of interest will reveal the maximum frequency of the loading. For accurate rainflow cycle counting one needs to identify the peaks and troughs and use a sampling rate that is at least ten to twenty times higher than the highest frequency of interest [22]. Time history tests are the most costly since they require real time data recording on actual hardware.

Case 2: Synthesized Time History Fatigue Damage Calculations

Taking a power spectral density of random loading specification, a time history can be synthesized on a computer by performing an inverse fast Fourier transform for a given time duration. As with time history calculations, a rainflow cycle counting of the peaks and troughs is extracted from the synthesized load data, followed by a damage calculation using Miner's linear damage rule. Again, for accurate rainflow cycle counting results, one needs to synthesize a time-domain loading history with a sampling rate that is at least ten to twenty times higher than the highest frequency of interest [22]. With

the assumption that the data is stationary and Gaussian, this approach can simulate the results obtained from costly data collection. If the test duration is long enough, the damage rate will converge towards a mean value. The process of performing inverse fast Fourier transforms and rainflow cycle counting on long synthesized time histories can be computationally expensive. However, this approach has commonly been used in the development of frequency domain methods for comparisons with rainflow cycle counting histograms.

There are various ways to convert the load PSD input to time history load data. In this paper a process as shown in Figure 2 is used. A custom Fourier Filter is used which multiplies the digitized PSD curve and the Fourier transform of the input white noise. Then, an inverse Fourier transform is used to obtain the time series which is Gaussian and has the same PSD shape and amplitude as the original input. A confirmation is conducted by comparing the original PSD and the synthesized PSD for each load case.

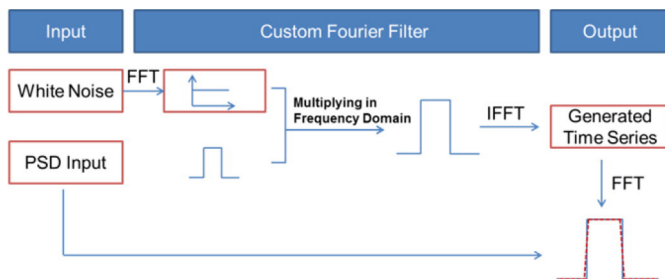


Figure 2. Process to convert PSD load input to load in time domain

Several parameters such as the sampling rate, duration, and custom filter buffer size are the keys for generating a representative time history load data from a PSD input. A pseudo damage analysis was conducted for various generated time series loads with different sampling rates. The analysis result is shown in Figure 3 where the X axis is the ratio between sampling rate and the highest frequency in the original PSD input and the Y axis is the pseudo damage of the time series load assuming a constant S-N curve. As shown, the sampling rate should be higher than ten times the peak frequency content to achieve a converged pseudo damage value. In this paper, the sampling rate was set as 5000 Hz for all the PSDs cases, which is more than 20 times higher than the highest loading frequency.

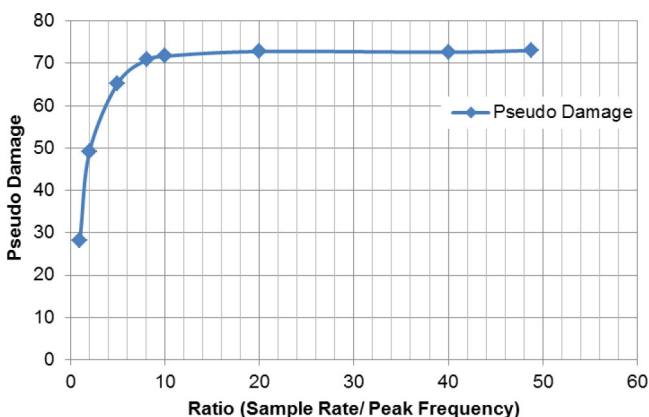


Figure 3. Effect of sampling rate to pseudo damage of the generated time series load

Duration of the generated time history load also affects the accuracy. Figure 4 shows the original versus synthesized PSD comparisons with varied durations for the same rectangular PSD input. As shown, the PSD of the generated time series load is much closer to the actual PSD input for a longer duration data than for a short data. To better represent the actual PSD shape, time series data with long enough duration is required. In this paper, the duration of the generated time series load is 10,000 seconds.

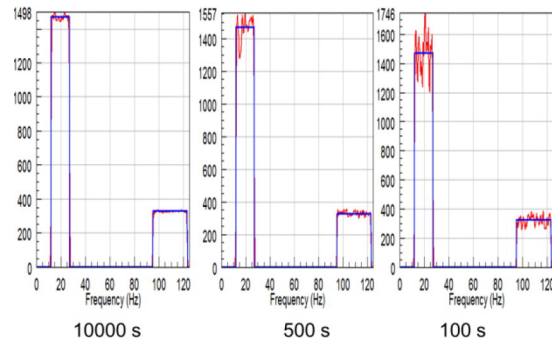


Figure 4. Effect of duration of generated time series load to PSD correlation

The accuracy of the Fourier transforms and inverse Fourier transforms is also affected by the buffer size of the analysis, which determines resolution when digitizing the PSD input and FFT of the white noise, and also the IFFT process. As shown in Figure 5, the higher the buffer size of the analysis, the better the correlation turns out to be. All analysis used in this paper has a buffer size of 8,192 for all PSD inputs.

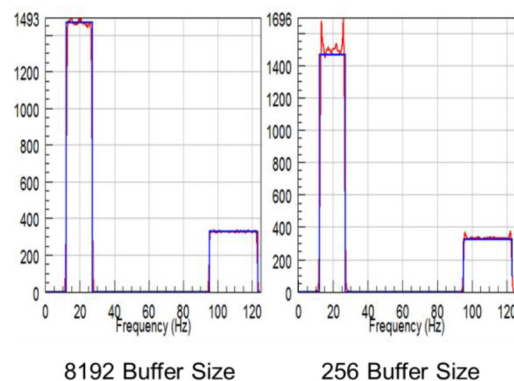


Figure 5. Effect of buffer size to PSD correlation

Case 3: Frequency Domain Fatigue Damage Calculations

Frequency methods utilize parameters calculated from the spectral moments about the origin and integrated over the power spectral density. One advantage is that all frequency domain methods find a single mean damage rate regardless of the test duration. The usual assumption is that the power spectral density is stationary and Gaussian and is representative of the expected field loading condition. Some fundamental vibration and fatigue theories can be found in References [17, 23, 24, 25].

$$D = \frac{E[\text{freq}] \cdot T \cdot E[S_a^m]}{C_a} \quad (1)$$

$E[\text{freq}]$ is the expected or average frequency, T is the test duration, $E[S_a^m]$ is the expected load or stress raised to the fatigue exponent and C_a is the Basquin material constant. The subscript "a" in the above equation stands for "amplitude" which is the convention used throughout this paper, however range can also be used. Also, stress can be replaced by strain or displacement with a suitable change in the material constant, C_a .

The spectral moments about the origin of any order are found by

$$M_j = \int_0^\infty f^j W_x(f) df \quad (2)$$

The term W_x is the PSDs with units (load²/Hz) or (stress²/Hz) and f^j is the frequency of order j .

The PSD is often a simplified envelope of the fast Fourier transform of time history data but with a reduced number of break points. Typically a PSD is given in the form of a discontinuous function shown as a table of values, or breakpoints. It is necessary to numerically integrate a discontinuous PSD to obtain the spectral moments. Lalanne [17] provides a number of equations for calculating spectral moments for various bases of the PSD axes. Most frequently, PSD's are defined on a log-log plot scale. The equation for integrating a segment on log-log scale from one break point, A, to the next one, break point B is

$$M_j = \frac{W_A}{f_A^k} \left(\frac{f_B^{(k+j+1)} - f_A^{(k+j+1)}}{(k+j+1)} \right) \quad (3)$$

$$k = \frac{\ln \frac{W_B}{W_A}}{\ln \frac{f_B}{f_A}} \quad (4)$$

In the above equations, W_A is the PSD magnitude at the first break point, k is the slope between breakpoints on the PSD plot, and j is the order of the spectral moment. Note that if $k = -(j+1)$ then

$$M_j = f_1^{-(j+1)} W_1 \ln \left(\frac{f_2}{f_1} \right) \quad (5)$$

The moments of any order are found by summing the individual segments between break points found using [equation 3](#).

Two spectral parameters that are derived from the spectral moment of considerable note are the zeroth order moment, M_0 , which is the variance, and $\sqrt{M_0}$, which is the standard deviation and also the area enveloped by the PSD. In addition, the expected rate of positive zero

crossings $E[0^+]$ (the rate that load or stress changes sign from negative to positive) and the average rate of peaks $E[P]$ (the average rate that load or stress reaches a peak value) are given by

$$E[0^+] = \sqrt{\frac{M_2}{M_0}} \quad (6)$$

and

$$E[P] = \sqrt{\frac{M_4}{M_2}} \quad (7)$$

Another important function is the irregularity factor which is given by

$$\gamma = \frac{E[0^+]}{E[P]} = \sqrt{\frac{M_2^2}{M_0 M_4}} \quad (8)$$

A narrowband process has an irregularity factor of close to one and a for wideband process it is close to zero.

If the mean is not zero, then the mean stress effect must be accounted for also by the Morrow, Walker, modified Walker, or Smith-Watson-Topper equations [26]. It will be assumed for purpose of this paper that the mean value is zero, the structure is linear, and the loading is stationary and Gaussian.

In his often cited paper, S.O. Rice [6] described the probability density function of peak values for a narrowband process follows a Rayleigh distribution.

$$f_{S_a}(S_a, \sigma_{S_a}) = \frac{S_a}{\sigma_{S_a}^2} \cdot e^{-\left(\frac{S_a^2}{2\sigma_{S_a}^2}\right)} = \frac{S_a}{M_0} \cdot e^{-\left(\frac{S_a^2}{2M_0}\right)} \quad (9)$$

σ_{S_a} is the standard deviation of the random process.

Rice stated further that for any random stationary Gaussian process, narrowband or wideband, the probability density function is the weighted sum of a Rayleigh and normal distribution. This function is known as the Rice distribution and is given in [equation 10](#). The function is plotted in [Figure 6](#) versus normalized load for various irregularity factors. Note that for an irregularity factor of 1, the shape is a Rayleigh distribution and for an irregularity factor of zero, the shape is Gaussian.

$$f(S_a) = \frac{\sqrt{1-\gamma^2}}{\sqrt{2\pi M_0}} \exp\left(-\frac{S_a^2}{2(1-\gamma^2)M_0}\right) + \gamma \frac{S_a}{M_0} \exp\left(-\frac{S_a^2}{2M_0}\right) \Phi\left(\frac{\gamma S_a}{\sqrt{(1-\gamma^2)M_0}}\right) \quad (10)$$

Φ is the standard normal distribution function.