

Non-Linear Bifurcation Stability Analysis for Articulated Vehicles with Active Trailer Differential Braking Systems

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ABSTRACT

This paper presents nonlinear bifurcation stability analysis of articulated vehicles with active trailer differential braking (ATDB) systems. ATDB systems have been proposed to improve stability of articulated vehicle systems to prevent unstable motion modes, e.g., jack-knifing, trailer sway and rollover. Generally, behaviors of a nonlinear dynamic system may change with varying parameters; a stable equilibrium can become unstable and a periodic oscillation may occur or a new equilibrium may appear making the previous equilibrium unstable once the parameters vary. The value of a parameter, at which these changes occur, is known as “bifurcation value” and the parameter is known as the “bifurcation parameter”. Conventionally, nonlinear bifurcation analysis approach is applied to examine the nonlinear dynamic characteristics of single-unit vehicles, e.g., cars, trucks, etc. Little attention has been paid to investigate the feasibility and effectiveness of the bifurcation analysis method for nonlinear stability analysis of articulated vehicles under varied operating conditions, e.g., varied forward speed and trailer payload. This motivates the research to examine stability boundaries of equilibrium and limit cycles in the parameter space and to predict qualitative changes in system’s behaviour (bifurcations) occurring at their equilibrium points. To this end, a nonlinear yaw-roll model with 6 degrees of freedom (DOF) is generated to simulate the nonlinear dynamics of articulated vehicles. The nonlinear yaw-roll model is also used to design an ATDB controller based on a fuzzy logic control technique. The bifurcation analysis based on the phase-plane method is conducted to evaluate the yaw and roll stability of the articulated vehicle. Built upon the conventional bifurcation analysis for single-unit vehicles, an innovative bifurcation analysis technique is developed in order to effectively assess the nonlinear stability of articulated vehicles. The applicability and effectiveness of the newly developed technique is examined and demonstrated.

KEYWORDS: nonlinear bifurcation analysis method, phase-plane analysis method, fuzzy logic control, active trailer differential braking, articulated vehicles, lateral stability, nonlinear yaw-roll car-trailer model

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INTRODUCTION

A car-trailer (CT) system consists of a towing unit, e.g., a car, and a towed unit, i.e., a trailer. The car and trailer are connected at an articulated point by a hitch [1]. With respect to single-unit cars, car-trailer systems may exhibit poor yaw and roll stability at high speeds because of their double-unit structure. For these vehicles, three unique unstable motion modes have been identified, including the trailer swing (oscillatory instability), jack-knifing, and rollover. These unstable motion modes are the common causes for fatal accidents occurring on highways [2]. In order to evaluate the yaw and roll stability of articulated vehicles, a stability evaluation method based on eigenvalue analysis of a linear vehicle model was reported [1, 2]. However, typical fatal accidents of articulated vehicles are mainly resulted from the nonlinear dynamics of these vehicles, e.g., saturation of lateral tire forces, large articulation angles between adjacent vehicle units, etc. [3]. In general, the behaviors of a nonlinear dynamic system may change with varying parameters. Equilibrium can become unstable and a periodic solution may appear or a new stable equilibrium could appear making the previous

equilibrium unstable. The value of a parameter, at which these changes occur, is known as “bifurcation value”, and the parameter, which is varied, is called the “bifurcation parameter” [4]. Therefore, the bifurcation analysis technique may be applied as an effective dynamic analysis approach to the evaluation of the yaw and roll stability of articulated vehicles considering the unique nonlinear dynamic features of these vehicles with varying parameters.

The past two decades have witnessed the application of the phase-plane analysis approach and its theoretical base, bifurcation theory, to the stability analysis of road vehicles [5]. The side-slip angle phaseogram stable zone boundary was examined, which has been applied to the design of stability control systems for road vehicles [6]. The phase-plane analysis method was used to evaluate the performance of a stability controller based on results of simulations and tests [7]. Bifurcation analysis was used to evaluate the robustness of a vehicle stability controller and to examine the nonlinear dynamic characteristics of the vehicle system [8]. To date, the phase-plane analysis method and the bifurcation analysis technique are mainly

applied to the assessment of handling characteristics and lateral stability of single-unit vehicles, such as cars, trucks, and sport utility vehicles (SUVs), etc. Little attention has been paid to investigating the feasibility and effectiveness of the phase-plane analysis method and the bifurcation analysis technique for yaw and roll stability analysis of articulated vehicles. Articulated vehicles, such as car-trailer systems, have multiple vehicle units. Multi-unit vehicles have the unique dynamic features compared against single-unit vehicles [9]. Considering the unique configurations and dynamic features of articulated vehicles, it is questionable whether the bifurcation analysis based on single-unit vehicles is feasible, applicable and effective for analyzing of the yaw and roll stability of articulated vehicles.

Under variable operating conditions, such as forward velocity and trailer payload, the behaviors of articulated vehicle systems may change. Therefore, the main objectives of the paper are: (1) examining equilibrium and periodic orbits of articulated vehicles with variable parameters, (2) determining stability boundaries of equilibrium and limit cycles in the parameter space, and (3) predicting qualitative changes of system's behaviors (bifurcations) occurring at these equilibrium points.

In order to achieve these goals, a nonlinear yaw-roll model with 6 degrees of freedom (DOF) is generated to simulate the nonlinear dynamics of articulated vehicles. An ATDB controller is designed to improve the lateral stability of a CT system using a fuzzy logic control technique. The bifurcation analysis based on the phase-plane method is conducted to evaluate the yaw and roll stability of the articulated vehicle. Built upon the conventional bifurcation analysis for single-unit vehicles, an innovative bifurcation analysis technique is developed in order to effectively assess the nonlinear stability of articulated vehicles. The applicability and effectiveness of the newly developed technique is examined and demonstrated.

The remainder of this paper is organized as follows. The second section introduces the nonlinear 6-DOF yaw-roll car-trailer model. The third section presents nonlinear stability analysis based on the bifurcation analysis method. The ATDB controller based on the fuzzy logic control technique for the 6-DOF model is presented in the fourth section. Finally, conclusions are drawn in the fifth section.

NONLINEAR MODEL FOR CAR-TRAILER COMBINATIONS

In this paper, the CT system with a car-trailer combination is represented by the nonlinear 6-DOF yaw-roll model to identify the unstable motion modes and predict the critical speed(s) of the nonlinear CT system. As shown in Figure 1, each axle is represented by a single wheel. Based on the body fixed coordinate systems, $X_c - Y_c - Z_c$ and $X_t - Y_t - Z_t$ for the car and trailer, respectively, the governing equations of motion can be derived.

In the vehicle modeling, it is assumed that the car front wheel steering angle δ is given. The pitch and bounce motions of the car and trailer as well as the aerodynamic forces are ignored. The tire dynamics is simulated using the Magic Formula tire model, which specifies the nonlinear relationship between the tire force and the tire

side-slip angle. The roll stiffness and damping coefficients of the vehicle suspension systems are constant when roll motions are involved. The motions considered in the 6-DOF yaw-roll model are: (1) car forward speed U_c , (2) car lateral speed V_c , (3) car yaw rate r_c , (4) car roll angle ϕ_c , (5) trailer yaw rate r_t , and (6) trailer roll angle ϕ_t . Based on Newton's law of dynamics, the equations of motion for the car and trailer can be derived and are shown as:

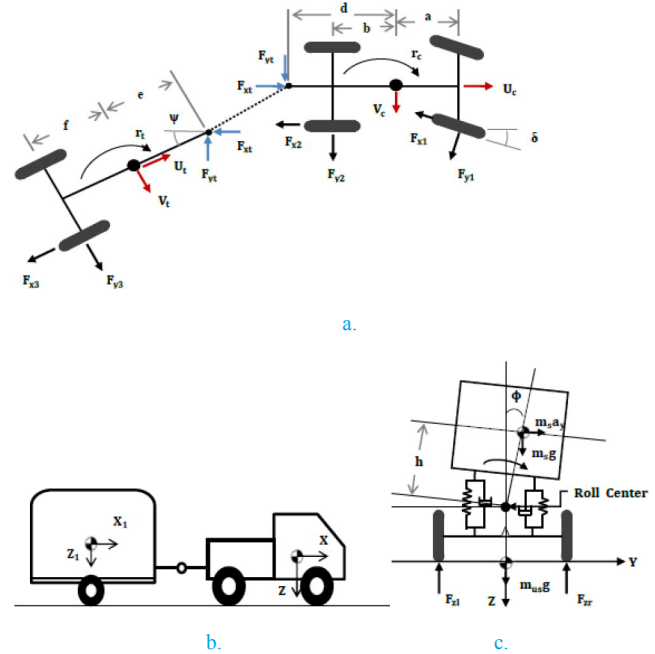


Figure 1. The schematic representation of the nonlinear 6-DOF yaw-roll model: (a) top view, (b) rear view, and (c) side view

The equations of motion for the car are

$$m_c(\dot{U}_c - r_c \cdot V_c) - m_{cs} \cdot h_1 \cdot \dot{r}_c \cdot \phi_c = -F_{x1} \cdot \cos(\delta) - F_{y1} \cdot \sin(\delta) - F_{x2} + F_{xt} \quad (1)$$

$$m_c(\dot{V}_c + r_c \cdot U_c) + m_{cs} \cdot h_1 \cdot \dot{\phi}_c = -F_{x1} \cdot \sin(\delta) + F_{y1} \cdot \cos(\delta) + F_{y2} + F_{yt} \quad (2)$$

$$I_{z1} \cdot \dot{r}_c - I_{xz1} \cdot \dot{\phi}_c = (F_{y1} \cdot \cos(\delta) - F_{x1} \cdot \sin(\delta)) \cdot a - F_{y2} \cdot b - F_{yt} \cdot d \quad (3)$$

$$(I_{xx1} + m_{cs} \cdot h_1^2) \cdot \ddot{\phi}_c - I_{xz1} \cdot \dot{r}_c + m_{cs} \cdot h_1 \cdot (\dot{V}_c + r_c \cdot U_c) = (m_{cs} \cdot g \cdot h_1 - k r_1) \cdot \phi_c - c r_1 \cdot r_c \cdot \dot{\phi}_c + F_{yt} \cdot z_1 \quad (4)$$

The equations of motion for the trailer are

$$m_t(\dot{U}_t - r_t \cdot V_t) - m_{ts} \cdot h_2 \cdot \dot{r}_t \cdot \phi_t = -F_{x3} - F_{xt} \quad (5)$$

$$m_t(\dot{V}_t + r_t \cdot U_t) + m_{ts} \cdot h_2 \cdot \dot{\phi}_t = F_{y3} - F_{yt}$$

$$I_{z2} \cdot \dot{r}_t - I_{xz2} \cdot \dot{\phi}_t = -F_{y3} \cdot e - F_{yt} \cdot f$$

$$(I_{xx2} + m_{ts} \cdot h_2^2) \cdot \ddot{\varphi}_t - I_{xz2} \cdot \dot{r}_t + m_{ts} \cdot h_2 \cdot (\dot{V}_t + r_t \cdot U_t) = (m_{ts} \cdot g \cdot h_2 - kr_2) \cdot \varphi_t - cr_2 \cdot r_t \cdot \dot{\varphi}_t - F_{yt} \cdot z_2 \quad (8)$$

In summary, the nonlinear equations describing the CT model can be expressed in a state-space representation as follows:

$$M \cdot \dot{X} = F(X, \delta) \quad (9)$$

The state variable vector is defined as

$$X = [\varphi_c \ \dot{\varphi}_c \ \varphi_t \ \dot{\varphi}_t \ r_c \ r_t \ V_c \ V_t \ U_c \ U_t]^T \quad (10)$$

Table 1 lists the notations and the values of the primary parameters of the CT system. All parameters of the Magic Formula tire model are listed in Table 2 and the lateral tire force is determined using the equation (11) as follows:

$$F(x) = D \cdot \sin[C \cdot \arctan\{B \cdot x - E \cdot (B \cdot x - \arctan(B) \cdot x)\}] \quad (11)$$

where x is the tire side-slip angle.

Table 1. The notations and the primary parameters of the CT system

Car total mass	m_c	1521 kg
Car sprung mass	m_{cs}	1306 kg
Trailer total mass	m_t	2000 kg
Trailer sprung mass	m_{ts}	1864 kg
Yaw moment of inertia of the total mass of the car	I_{x1}	1816 kgm ²
Yaw moment of inertia of the total mass of the trailer	I_{x2}	3000 kgm ²
Roll moment of inertia of the sprung mass of the car	I_{xx1}	846.6 kgm ²
Roll moment of inertia of the sprung mass of the trailer	I_{xx2}	707.3 kgm ²
Roll-yaw product of inertial of the sprung mass of the car	I_{xz1}	0
Roll- yaw product of inertial of the sprung mass of the trailer	I_{xz2}	0
Longitudinal distance between the CG of the car and front axle of the car	a	1.063 m
Longitudinal distance between the CG of the car and rear axle of the car	b	1.716 m
Longitudinal distance between the CG of the car and hitch	d	2.937 m
Longitudinal distance between the CG of the trailer and hitch	e	6.0 m
Longitudinal distance between the CG of the trailer and axle of the trailer	f	0
Height of the CG of car sprung mass above roll axis	h_1	0.325 m
Height of the CG of trailer sprung mass above roll axis	h_2	0.676 m
Vertical distance between car roll center and hitch	z_1	0.305 m
Vertical distance between trailer roll center and hitch	z_2	0.391 m
Roll damping coefficient of the car suspension	cr_1	19000 Nms/rad
Roll damping coefficient of the trailer suspension	cr_2	30000 Nms/rad
Roll stiffness of the car suspension	kr_1	110000 Nm/rad
Roll stiffness of the trailer suspension	kr_2	230000 Nm/rad

Table 2. Tires parameters for using the Magic formula tire model

The front tire coefficient of the car	B_{c1}	0.2583
The front tire coefficient of the car	C_{c1}	1.3
The front tire coefficient of the car	D_{c1}	2588.4
The front tire coefficient of the car	E_{c1}	-0.2567
The rear tire coefficient of the car	B_{c2}	0.2793
The rear tire coefficient of the car	C_{c2}	1.3
The rear tire coefficient of the car	D_{c2}	2013.7
The rear tire coefficient of the car	E_{c2}	-0.0318
The tire coefficient of the trailer	B_t	0.1787
The tire coefficient of the trailer	C_t	1.3
The tire coefficient of the trailer	D_t	4626.7
The tire coefficient of the trailer	E_t	-1.1189

NON-LINEAR BIFURCATION STABILITY ANALYSIS

In this section, a systematic nonlinear analysis method is presented to analyze the yaw and roll stability of the nonlinear yaw-roll CT model. The nonlinear vehicle stability analysis is based on the phase-plane method and Lyapunov stability analysis theory. The phase-plane state trajectories are used to predict qualitative changes of system's behaviors (bifurcations) occurring with variable parameters. The phase-plane state trajectories' envelopes are used to provide an approximate stability range. The Lyapunov stability analysis can be used to judge whether the equilibrium point of the nonlinear system is stable.

Phase-Plane Stability Analysis

For articulated vehicle systems, varied steering input, forward velocity and trailer payload are regarded as external input variables and important factors on the yaw and roll stability of the CT system. Therefore, to conduct the bifurcation analysis and to investigate the effect of variable driving conditions on the yaw and roll stability of the nonlinear CT model, the numerical simulations of the nonlinear 6-DOF yaw-roll model with varying forward velocity are conducted. Under the simulated maneuver, the car front wheel steer input maintains constant at an angle of 0.0175rad, the vehicle forward speed will take different values (for different cases) from the speed series of 60, 65, 70, ..., 130km/h (with a constant increment step of 5 km/h), and the road friction coefficient μ takes the value of 0.55.

According to the kinematic analysis based on the yaw-roll model shown in Figure 1, the relationship among the side-slip angles of the car and the trailer, the yaw rate of the car and the trailer, the roll angles of the car and the trailer, and the articulation angle between the car and the trailer can be expressed as:

$$\beta_c - \beta_t - \frac{h_c \cos(\varphi_c)}{U} d\varphi_c/dt + \frac{h_t \cos(\varphi_t)}{U} d\varphi_t/dt + \frac{d}{U} r_c - \frac{e}{U} r_t + \psi = 0 \quad (12)$$

For a zero initial condition, the articulation angle can be derived as:

$$\dot{\psi} = -(r_c - r_t) \quad (13)$$

where the notations and the primary parameters of the CT system are shown in Table 1. The β_c and β_t are the side-slip angles of the car and trailer, $h_c \cos(\varphi_c)$ and $h_t \cos(\varphi_t)$ are the vertical distances from the center of gravity (CG) of the car and the trailer to the roll axis, respectively.

are the vertical distance between the body CG to the roll center of the car and the trailer, respectively. Note that the relationship shown in Equation (12) will be used to aid the phase-plane analysis to evaluate the yaw and roll stability of the nonlinear model.

The nonlinear stability analysis based on the phase-plane method examines the set of relation between: (1) the leading unit's side-slip angle and side-slip angular velocity ($\beta_c - d\beta_c/dt$), (2) the trailing unit's side-slip angle and side-slip angular velocity ($\beta_t - d\beta_t/dt$), (3) the side-slip angle of the leading unit, the yaw rate of the leading unit, and the articulation angle between the leading and trailing units ($\beta_c - r_c - \psi$), (4) the side-slip angle of the trailing unit, the yaw rate of the trailing unit, and the articulation angle between the leading and trailing units ($\beta_t - r_t - \psi$), and (5) the articulation angle and the articulation angular velocity ($\psi - d\psi/dt$). Results of the simulation are analysed to assess the lateral stability of the articulated vehicle. Similarly, the leading unit's roll angle and roll angular velocity ($\phi_c - d\phi_c/dt$) and the trailing unit's roll angle and roll angular velocity ($\phi_t - d\phi_t/dt$), are examined to evaluate the roll stability of this combination.

Figures 2 and 3 show the phase-plane trajectories of the side-slip angle and side-slip angular velocity of the car and the trailer, i.e., $\beta_c - d\beta_c/dt$ and $\beta_t - d\beta_t/dt$, respectively. Each of the state trajectories is associated with a specified value of the car forward speed. The X axis denotes the variable quantity of the car and trailer's side slip angle. The Y axis represents the variable rate of car and trailer's side slip angle rate. According to the phase-plane analysis method, if a state trajectory may return to the origin of the coordinate system or the state trajectory could converge to a final steady-state, this means that the system is stable. On the other hand, if the state trajectory can't return to the origin of the coordination system or the state trajectory can't converge a steady-state, this means that the system will loss stability. The effect of the vehicle forward speed on the yaw stability of the CT system is clearly illustrated by the trajectories of $\beta_c - d\beta_c/dt$ and $\beta_t - d\beta_t/dt$.

As shown in Figures 2 and 3, all the state trajectories may reside in 3 regions:

1. Yaw Motion Stable Region. If the vehicle forward speed is less than 115km/h (i.e., within the range of 60km/h to 110km/h), all the state trajectories will converge to the corresponding points on the horizontal axis of the coordination system. All those points are the respective final steady-states of the vehicle units. Thus, the CT system remains yaw stability. This region swept by the state trajectories (in blue) is defined as the yaw motion stable region of the CT system. As shown in Figure 2, the stable region of the leading unit is enclosed in the phase-plane range of $-0.003\text{rad} < \beta_c < 0.015\text{rad}$ and $-0.095\text{rad/s} < d\beta_c/dt < 0.045\text{rad/s}$, while the stable region of the trailing unit shown in Figure 3 is expanded to the phase-plane range of $0 < \beta_t < 0.1\text{rad}$ and $-0.1\text{rad/s} < d\beta_t/dt < 0.15\text{rad/s}$. This means that during the constant steering angle input maneuver, compared with the car, the trailer has amplified lateral motions. This phenomenon is usually called "Rearward Amplification", which is one of the unique dynamic features of articulated vehicles [3].

2. Yaw Motion Stability Boundary Region. When the vehicle forward speed reaches 115km/h, the state trajectories will also converge to the points located on the horizontal axis of the coordination system, but more time is needed for the cycles. When the vehicle forward speed is larger than 115km/h, all the state trajectories (in red) will diverge, thereby causing the unstable yaw motions of the CT system. Hence, the region around the envelope of the state trajectories associated with the forward speeds around 115 km/h is defined as the yaw motion stability boundary region, and the forward speed of 115 km/h may be simply considered as the critical speed of the nonlinear CT system, above which the CT system will lose the lateral stability.
3. Yaw Motion Unstable Region. If the vehicle forward speed is between 120km/h and 130km/h, all the state trajectories will diverge with increasing of time, thereby leading to the loss of yaw stability. The region that is swept by those state trajectories (in black) is defined as the yaw motion unstable region.

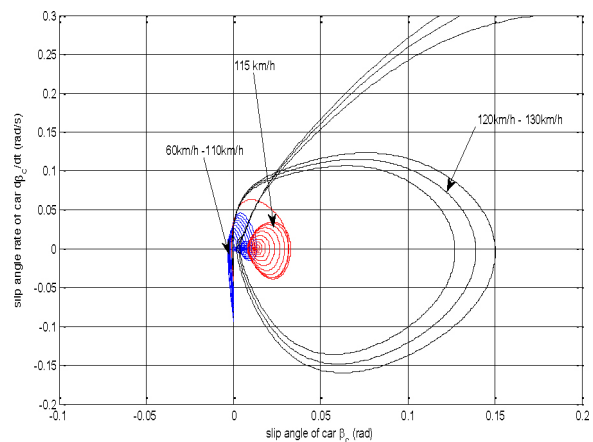


Figure 2. Phase-plane trajectories of $\beta_c - d\beta_c/dt$ with variable forward speed

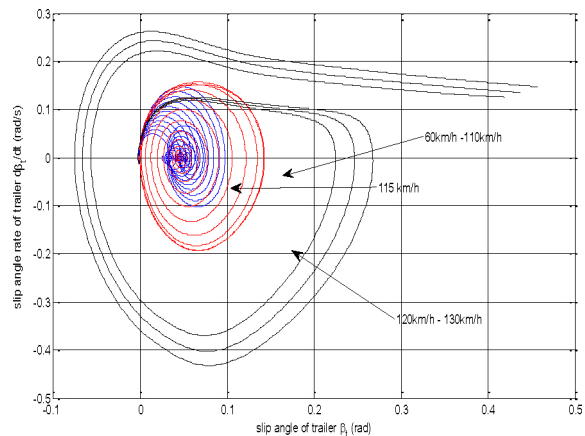


Figure 3. Phase-plane trajectories of $\beta_t - d\beta_t/dt$ with variable forward speed

Figures 4 show the phase-plane trajectories of the articulation angle and articulation angular velocity, $\psi - d\psi/dt$. Each of the state trajectories is associated with a specified value of the vehicle forward speed. All the state trajectories may also reside in 3 regions corresponding to those shown in Figures 2 and 3. The stable region swept by the state trajectories (in blue) is enclosed in the phase-plane range of $-0.03\text{rad} < \psi < 0.05\text{rad}$ and $-0.11\text{rad/s} < d\psi/dt < 0.15\text{rad/s}$. Similarly, the boundary region is around the envelope of the state