

Study on Repeated-Root Modes in Substructure Modal Composition Analysis

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ABSTRACT

The dynamic properties of disc rotor play important role in the NVH performance of a disc brake system. Disc rotor in general is a centrosymmetric structure. It has many repeated-root modes within the interested frequency range and they may have significant influence on squeal occurrence. A pair of repeated-root modes is in nature one vibration mode. However, in current complex eigenvalue analysis model and relevant analysis methods, repeated-root modes are processed separately. This may lead to contradictory result. This paper presents methods to deal with repeated-root modes in substructure modal composition (SMC) analysis to avoid the contradiction. Through curve-fitting technique, the modal shape coefficients of repeated-root modes are expressed in an identical formula. This formula is used in SMC analysis to obtain an integrated SMC value to represent the total influence of two repeated-root modes. Finally the presented method is applied to analysis a high frequency squeal problem of a disc brake. SMC analysis results are given, where each pair of repeated-roots have one unique SMC. The impacts of these modes on squeal can be clearly examined.

CITATION: Gao, P., Du, Y., Wang, Y., and Lv, Y., "Study on Repeated-Root Modes in Substructure Modal Composition Analysis," *SAE Int. J. Passeng. Cars - Mech. Syst.* 9(1):2016, doi:10.4271/2016-01-0477.

INTRODUCTION

With growing driving speed and the introduction of new friction materials, NVH performance of brake system becomes more and more important when evaluating automobile overall performance. During braking, the kinetic energy is transformed to thermal energy due to frictional sliding. Drastic vibration and annoying squeal may be stimulated. Brake squeal is one of the main sources of urban sound pollution. What's worse, severe, continuous vibration may lead to early failure of relevant components. Therefore, research on the method to further understand the mechanism of brake squeal, to formulate effective measures to suppress squeal and to avoid the phenomena in product design are of great importance [1, 2].

Complex eigenvalue analysis is one of the most used frequency domain method to build and to analysis the stability characteristics of brake system. Liles[3] was the first person who built a frequency-domain model for disc brake squeal analysis. A complex mode formulation was derived for the system, where the friction effect was reflected as the asymmetric non-diagonal element of system stiffness matrix. Nack[4, 5] presented a detailed instruction on how to build the frictional stiffness matrix and clearly stated the meaning of using complex eigenvalue analysis.

Guan and her research team[6~12] considered brake squeal as a self-excited vibration problem due to the instability of the frictional closed-loop coupling of the system. It's caused by the mismatch of

structural parameters of brake components. Adjusting the structural parameters matching properly is an effective way to solve this problem. Based on modal synthesis and FE method, a closed-loop coupling model of disc brake was established. The dynamics characteristics of the components are represented by their modal parameters. The contacts between the components are presented in the form of coupling stiffness matrix. Based on this model a set of analysis methods were developed, including substructure modal composition analysis, substructure modal parameter sensitivity analysis and feed-in energy analysis, with which the influential structural parameters to squeal can be extracted and structural modification strategy can be put forward. Their capacities to solve squeal problems have been demonstrated in practice.

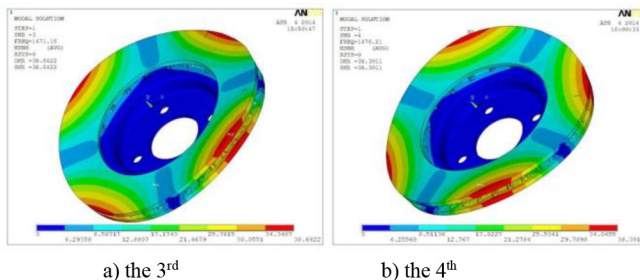
In closed-loop coupling model, the dynamic characteristics of components are expressed in the form of modal parameters. Concerning the centro-symmetry characteristic of the disc rotor, it is no doubt that a number of repeated-root modes will be obtained by performing the modal analysis of its finite element (FE) model with partial constraints. Many of these repeated-roots modes are in the interested frequency range. For example, Table 1 lists modal parameters of modes below 5kHz of a disc rotor of a car front brake corner[13].

Table 1. Modal parameters of a disc rotor below 5kHz

Order	Modal frequency(Hz)	Modal shape
1	1300.1	1st OPD
2	1306.2	
3	1471.2	2nd OPD
4	1476.2	
5	1762.1	0th OPD
6	2836.0	3rd OPD
7	2847.9	
8	3468.7	0th IPC
9	4110.1	1st IPR
10	4111.4	
11	4491.2	4th OPD
12	4500.1	

OPD: out-of-plane diametric
IPC: in-plane circumferential
IPR: in-plane radial

It can be seen that the disc has 12 modes within 5kHz and there are 4 pairs of modes which the modal frequency are very close and share a same modal shape. Figure 1 shows the modal shapes of the 3rd and 4th modes. Obviously, if one of them is rotated 45 degrees then the two shapes will become identical. So they are orthogonal. Due to the modeling error, repeated-root modes calculated by FEM have the following characteristics: (1) the modal frequencies are very close but not completely equal; (2) the modal shapes are orthogonal.

Figure 1. The 3rd and 4th order modal shapes of a disc brake FE model

According to the theory of modal analysis, the existence of repeated-root modes is only a kind of mathematical descriptions of a special modal characteristic of symmetric structures. A pair of repeated-root modes is in nature one physical vibration mode. However, in current closed-loop coupling disc brake model, the repeated-root modes are treated as two independent modes and they are involved in the modelling and analyzing calculations separately. When performing substructure modal composition (SMC) analysis or other analyses, the SMC values of each repeated-root modes are also listed out separately. For example, Table 2 lists the top SMC values of disc rotor modes of an unstable mode at 13kHz of a squealing disc brake[12]:

Table 2. SMC values of disc rotor modes

Order	Modal Freq.(Hz)	SMC
39	13047	0.7080
40	13076	0.6889
42	13276	0.0611
44	13595	0.0485
37	11617	0.0208
41	13242	0.0191
43	13569	0.0149
35	11411	0.0130

Table 2 shows both the 39th and 40th disc modes contribute most to the squeal mode. Their modal frequencies are 13047Hz and 13076Hz respectively. And by examining their modal shapes we know they are a pair of 2nd IPC modes. But the two repeated-root modes have different SMC values. SMC of the 39th mode is greater than that of the 40th. Yet this shouldn't be interpreted as the 39th contributes more than the 40th to the noise mode because they are physically one mode. None of the two SMC values in table 2 can represent the real contribution of this physical mode. Same problem exists when dealing with other repeated-roots such as the 37th/38th, 41st/42nd and 43rd/44th.

This speciousness also appears in other substructure modal parameter related analysis methods such as sensitivity analysis. So, it is quite necessary to perform study on the repeated-root modes in closed-loop coupling model and relevant analyses.

This paper presents methods to deal with repeated-root modes in SMC analysis to avoid the contradiction. First a brief introduction of closed-loop coupling model and SMC analysis method is given. Then the equivalency of repeated-root modes in mathematics and single physical mode is proved through comparing frequency response function (FRF) of disc at two circumstances. Then, curve-fitting technique is introduced to obtain a unique formula to represent the modal shape coefficients of repeated-root modes. Then this formula is used to calculate the modal response of an unstable mode to derive a single expression of SMC to represent the total influence of two repeated-root modes to the noise mode. Finally the presented method is applied to analysis a high frequency squeal problem of a disc brake to show the advancement of the presented method.

CLOSED-LOOP COUPLING MODEL AND SMC ANALYSIS METHOD

Closed-Loop Coupling Model [8]

Neglect the damping of substructures, the dynamic equation of disc brake system is:

$$M\ddot{u} + (K - K_f)u = 0$$

where $\mathbf{M}$ and $\mathbf{K}$ are the discretized mass and stiffness matrix respectively, $\mathbf{u}$ is nodal displacement vector in physical coordinates, and $\mathbf{K}_f$ is the coupling matrix referring to the relationship between components, which includes spring and friction coupling. $\mathbf{K}_f$ is an asymmetric matrix.

Constructing coordinate transformation equation as:

$$\mathbf{u} = \Phi \mathbf{q} \quad (2)$$

where Φ is transformation matrix, composed of substructures' modal shape matrices, which are modal mass normalized respectively. Then Eq.1 becomes:

$$\ddot{\mathbf{q}} + \mathbf{K}_{sys} \mathbf{q} = 0 \quad (3)$$

$$\mathbf{K}_{sys} = \begin{bmatrix} \lambda_A & & & & \\ & \lambda_B & & & \\ & & \lambda_C & & \\ & & & \lambda_D & \\ & & & & \lambda_E \end{bmatrix} - \Phi^T \mathbf{K}_f \Phi \quad (4)$$

where diagonal matrices $\lambda_A \sim \lambda_E$ are modal frequency matrices of each component respectively

Performing complex eigenvalue analysis to Eq.3, the system's complex eigenvalues s_i and corresponding eigenvectors ψ_i can be calculated, where $i = 1, \dots, n$, n denotes the order of character equation (3). Eigenvalue with a positive real part indicates an unstable squealing mode. Table 3 lists all the squeal modes of a disc brake obtained through the complex eigenvalue analysis, whose imaginary part represents the squeal frequency and the real part is system damping, which indicates the tendency to squeal[11].

Table 3. List of noise modes

Seq.	Real	Imag.
1	8.16	2782.69
2	20.24	7472.11
3	16.13	9536.72
4	29.35	9826.29
5	17.07	12517.48
6	48.16	13044.99

Substructure Modal Composition Analysis Method

By performing complex eigenvalue decomposition to Eq.3, the eigenvalue s_i and its corresponding right eigenvector ψ_i can be derived. According to Equation 2, the i^{th} modal shape of the synthesized system in nodal coordinate can be expressed as[8]:

$$\phi_i = \Phi \psi_i \quad (5)$$

Further, Equation 5 can be expanded as:

$$\phi_i = \Phi \psi_i = \sum_{j=1}^n \phi_j \psi_{ij} \quad (6)$$

where ϕ_j is the j^{th} column of matrix Φ and denotes the modal shape of the corresponding j^{th} substructure mode. And ψ_{ij} denotes the j^{th} element of corresponding eigenvector ψ_i . It can be seen that the modal shape of the synthesized system is a linear combination of all the corresponding substructures' modal shapes. Each element of ψ_i represents the contribution of the corresponding substructure's mode to the unstable mode of the system, which is called substructure modal composition (SMC).

REPEATED-ROOT MODES

According to vibration theory, repeated-root modes exist in symmetric structures such as disc rotor. A pair of repeated-roots has same modal frequency and orthogonal modal shapes. But when performing modal test, there are only one single vibrating mode at the frequency. Its modal shape is same as either of the repeated-roots, but the peak point will follow the exciting point[14, 15]. In this section, the equivalency of the two descriptions will be proved.

Suppose that through FE modal calculation, a disc got a pair of repeated-root modes at frequency ω_0 . The corresponding modal shapes are elliptical, which means they have 2 nodal lines. The peak points of two modal shapes locate at angular position θ_1 and θ_2 respectively, as shown in Figure 2.

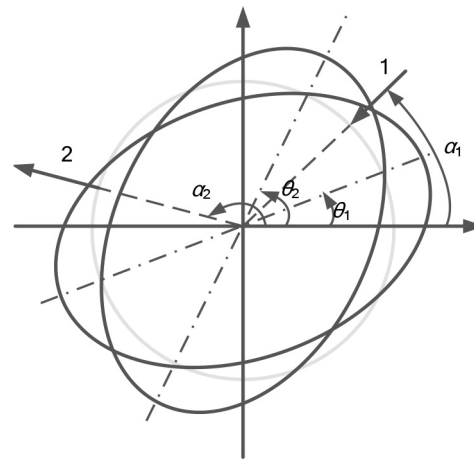


Figure 2. Elliptical repeated-root modal shapes

Assume the amplitude of the modal shape is A , and expand the modal shape along the central angle α , then the modal shapes of the two mode can be expressed as:

$$\phi_1(\alpha) = A \cos 2(\alpha - \theta_1)$$

$$\phi_2(\alpha) = A \cos 2(\alpha - \theta_2)$$

