

Effect of Combustion Timing and Heat Loss on Spring-Assisted Linear Engine Translator Motion

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ABSTRACT

The free piston linear engine has the potential to achieve high efficiency and might serve as a viable platform for robust implementation of low temperature combustion schemes (such as homogeneous charge compression ignition - HCCI) due to its ability to vary compression and stroke in response to cylinder and load events. A major challenge is control of the translator motion. Lack of geometric constraint on the piston leads to uncertainty about its top dead center position and timing. While combustion control depends on knowledge of the piston motion, the combustion event also affects the motion profile of the piston. To advance understanding of this coupled system, a numeric model was developed to simulate multiple cycles of a dual cylinder, spring assisted, 2-stroke HCCI, free piston linear engine generator. The MATLAB®/Simulink model depends on simplifications (for scavenging, springs, and alternator), differential relationships (for cylinder pressure and translator dynamics), and empirical relationships (for friction, heat transfer, and combustion). To address the complexity of HCCI combustion within a multi-cycle simulation, an auto-ignition integral, a heat release profile, a heat loss model, and assumptions for combustion duration and efficiency are needed. In this investigation, two knock integral methods are examined along with a Wiebe heat release function. Finally, the effect of total heat release and total heat transfer on translator dynamics are demonstrated. Aside from contributing to the overall energy balance, combustion timing advanced ahead of TDC is shown to allow for adverse work. Increasing the heat transfer, along with advanced combustion timing, leads to increased adverse work and consequentially lower overall efficiency.

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INTRODUCTION

Relative to a slider-crank engine and traditional rotating generator, a free piston linear engine generator has the potential to operate with a reduced number of moving parts, with an increased power density, and higher efficiency while directly producing electrical energy without translation to rotary motion. Linear engines are also suited to piston compressor and pump design, where conventional systems translate linear motion to rotary motion and back to linear motion again.

This paper present original simulation data produced during the examination of a spring assisted, linear engine generator (SALEG), where electrical power is produced via a linear alternator, symmetrical about an axially oscillating shaft which rests within a sleeve and cylinder assembly. This shaft also acts to directly connect two pistons facing in opposite directions. For modeling purposes, these pistons are situated within two partitioned chambers of the grounded cylinder housing. Together, the shaft and pistons are called the translator. In the simulation, the motion of the translator is driven by a two-stroke cycle combustion process, using homogeneous charge compression ignition (HCCI) cycles in both combustion chambers and supported by mechanical or pneumatic springs located within central chamber of the cylinder housing. The spring mechanisms act to provide potential energy storage which serves to buffer the system against adverse occurrences such as misfire or

sudden load changes. The stiffness profile of the springs can be used to affect the dynamics of the translator and to raise frequency of oscillation to increase power density of the engine.

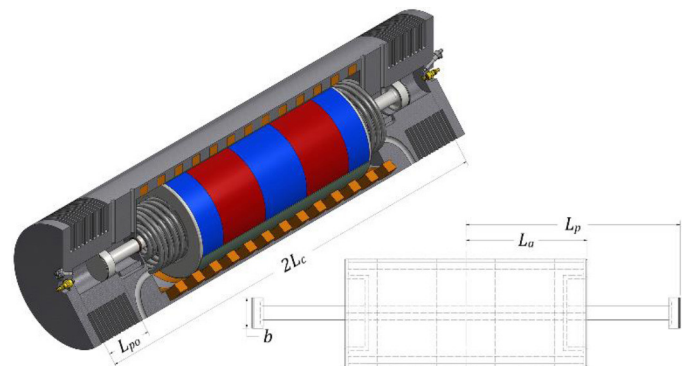


Figure 1. Spring-Assisted Linear Engine Generator Illustration

In the free piston SALEG (shown in Figure 1), the stroke length of the translator and compression ratio are not geometrically constrained as in a crankshaft driven engine. Instead, they are allowed to vary from stroke to stroke. This variability, while presenting difficulty in terms of system control, allows for a unique marriage between cycle-to-cycle flexibility and the HCCI combustion cycle. Cycle-to-cycle variations are sympathetic with the freedom of the translator to

increase or decrease its stroke length which acts as feedback for subsequent cycles of operation. In addition, because a generator has narrow power band requirements, HCCI operation is well suited for the application. As is discussed in this paper, the freedom of motion of a linear engine must be addressed carefully in simulation, to insure that the system is properly constrained. Parametric study of a linear engine leads to conclusions that are strongly dependent on the variables that are held constant.

Linear engine devices with springs can vary from the extreme where the spring and mass dominate the motion (the energy added per stroke is small in comparison to the energy stored in the system), to the extreme where the energy delivered by the engine and absorbed by the alternator has substantial effect on the motion of the immediate stroke.

The numeric simulation used in this paper was built to represent the interactions of the thermodynamic and dynamic energy exchanges. The model is based on the application of Newton's second law - the summation of forces acting on the translator to cause an acceleration of the translator in proportion to its mass. The translator oscillates within a stationary housing according to forces from the engine chambers, springs, friction, and integrated alternator system. These forces are represented by mathematical models which depend on known physical relationships, empirical constants, and instantaneous state values of the system.

LITERATURE REVIEW

This application of free piston technology over the years is summarized in a comprehensive timeline provided by Aichlmayr [1] with more details reviewed by Mikalsen and Roskilly [2]. Within the last two decades, linear engine research has moved to emphasize electrical generation and hydraulic pumping. The concept of reduced mechanical losses and increased simplicity has attracted many researchers and innovators truly beginning in the mid-1990s. One of these examples is the 1995 US Army funded project to explore the feasibility of the free piston linear engine alternator device as an auxiliary power device for hybrid electric vehicles [3].

A short time later (1998), work at Sandia National Laboratories (SNL) and West Virginia University (WVU) began in earnest. Investigations at SNL examined the use of HCCI combustion within a free piston engine through the use of a rapid compression machine which achieved nearly constant volume combustion [4]. From this, they explored the idea of HCCI combustion within a free piston linear engine alternator through numeric modeling of the system [5]. Their work shows the potential of such a device and the importance of cylinder recharging processes. Continuing in this vein, researchers at SNL extended their modeling towards optimization of scavenging and the alternator to maximize thermal efficiency [6]. They found that cycle-to-cycle variations in compression did not greatly affect scavenging performance and that steady, low temperature charge produced best results [7]. Today, development of their device continues under the sponsorship of the Department of Energy's (DOE) Vehicle Technologies Office in the form of a twin, opposed piston arrangement, which includes a single, centrally located combustion chamber and outlying pneumatic bounce chambers

(analogous to mechanical springs, but with variable spring rate) to aid in control. From this program, valuable data is being gathered from both modeling and prototype efforts [8].

Work on a dual cylinder device at WVU began with a parallel approach where an ad-hoc, off the shelf benchtop prototype model was built for experimentation while a numeric investigation was conducted for verification and experimental simulation [9, 10]. Valuable insight was gained from these efforts, particularly with regard to understanding reverse (or adverse) work around TDC and the implications of cycle-to-cycle energy balance for either stall or runaway compression ratio rise. In throttled operation, with no spring, it was found that adverse work might be necessary to raise cylinder pressure to control the compression ratio. This initial gasoline fueled prototype produced 309 W over a sustained period of operation during which it was seen that stability depended greatly on combustion timing. The simulation provided the basis for a parametric study of the device and helped to close the loop in understanding the interactions between heat addition, combustion duration, peak pressure, engine frequency, and compression ratio. The effects of translator mass were also explored to show proportional relationships to stroke and compression and an inverse relationship to engine frequency.

A second round of linear engine development at WVU applied simulation and artificial intelligence techniques to gain broader understanding of the effects of fuel/air ratio, moving mass, frictional losses, injection timing, and geometry on frequency and efficiency [11]. This fed the development of a second generation diesel fueled prototype. In the diesel linear engine, a boosted, direct injected, port scavenged, dual cylinder engine operated between 50 and 60 Hz [12] without spring assistance. Among the insights gained was the need for increased engine speed, which could be achieved by adding springs and reducing the translator mass.

In 2004, the history and current state of the art in linear engine technology was reviewed [13], and then operational data from the diesel prototype were analyzed and presented in [14]. This work corroborated the findings from the first generation prototype, such as the mass relationship to frequency, and showed that raising the amount of fuel and advancing the injection timing both increased power output, efficiency, and frequency suggesting that the engine operated near the HCCI combustion regime. Fundamental analysis of the dual piston linear engine [15, 16] supported the research reported in this paper, as well as the recently announced effort to build a spring assisted, single cylinder linear engine generator prototype [17].

Researchers at Newcastle University have been exploring the free piston linear engine device with a single combustion chamber opposed by a pneumatic bounce chamber. Their endeavors in simulation were preceded by the extensive review of free piston technology, already mentioned [2], which concluded that the most difficult challenge remaining for free piston devices is the control of the piston and that the free piston engine offers a viable platform for HCCI combustion. Since then, they have studied a variety of operating conditions including spark ignition [18] and compression

ignition [19, 20]. Their work continues as they pursue the control of the device through various approaches including tailoring the bounce chamber pressure profile to the dynamics of the translator [21, 22].

The researchers at Newcastle have recently published a review of the latest free piston publications to gain insights into current technical challenges and commercial expectations [23]. Cited challenges including piston motion, heat transfer, vibration, scavenging, and lubrication are impeding the viability of the free piston engine, and despite its known potential, there are few operational prototypes and no clear path to commercialization.

A team from the German Aerospace Center (DLR) is working toward production a single piston, 8 kW prototype which includes a mechanical spring for rebounding and operates around 20 Hz within a battery backup hybrid electric vehicle. Beginning in 2012, the team at DLR started with individual component testing by using controlled hydraulics to emulate the motion of a free piston engine [24]. A two stroke, gasoline fueled, head scavenged engine system was modeled and tested [25], and the linear electric generator was investigated for optimization [26, 27]. Today, a functional demonstrator has provided up to 10 kW at a frequency of approximately 20 Hz, and one of the major challenges remains to be robust control of the free piston. Just as in the prototype development at WVU, slow translator speed is associated with high linear alternator moving mass (the DLR alternator provides a maximum force of 12 kN). Their prototype demonstrates a total efficiency of 17.9% with 5.9% fuel energy lost to friction and 68.2% lost to combustion inefficiency [28].

In addition to these more expansive bodies of work are a number of interesting patents and research pieces aimed at developing the linear engine concept in various forms. Pempek Systems (Australia) developed the FP3 concept device which fitted three free piston linear engines into a single package [29]. Researchers at Chalmers University of Technology simulated a dual free piston device to investigate its multifuel capabilities [30]. Researchers from IFP, France undertook simulation work which showed the use of high compression and EGR can yield linear engine indicated efficiencies around 50% [31].

Toyota has announced and is currently working towards operation of a single cylinder, 10 kW, bounce chamber (gas spring) opposed device for use in a hybrid electric vehicle system [32, 33]. A research group in South Korea at the University of Ulsan has an operational prototype and has modeled a spring supported, propane fueled, spark ignited, dual cylinder free piston linear engine generator to explore the control of the device and possibilities for transitioning from spark to HCCI operation [34].

From the literature reviewed, it is clear that the proposed implementation of springs to raise frequency and augment the performance of a free piston linear engine is well supported but in need of refinement. Further, it is believed that the springs (or spring-equivalent devices) offer the dual benefit of tailoring the

piston dynamics for HCCI operation. In the next section, the influence of springs within an engine system is explored at a fundamental level which has been amplified in the authors' prior publications [15, 16].

FUNDAMENTAL DESCRIPTION

Modeling of the dual cylinder system began with a basic model that incorporated simplifying assumptions, neglecting friction and heat transfer, and assuming ideal properties for the working gas air and the springs.

Geometric parameters can be related to find the instantaneous volume of either chamber (Eq. 1).

$$L = L_c - L_p$$

$$V_l = (L + x) \frac{\pi b^2}{4} \quad V_r = (L - x) \frac{\pi b^2}{4}$$

Eq. 1

A polytropic relationship was used for compression and expansion of the gas. In the basic analysis, the pressure in either chamber was equal when the translator was at its midpoint position ($x = 0$). Combining the volume relationships with the polytropic equality produces the following instantaneous pressure relationships (Eq. 2).

$$P_{mp} \left(L \frac{\pi b^2}{4} \right)^\gamma = P_l \left[(L + x) \frac{\pi b^2}{4} \right]^\gamma$$

$$= P_r \left[(L - x) \frac{\pi b^2}{4} \right]^\gamma$$

Eq. 2

The subscript *mp* indicates the midpoint condition and γ is the specific heat ratio, which is assumed to be constant. Solving for the pressure forces on the translator yields Eq. 3.

$$F_{p,l} = \frac{\pi b^2}{4} P_{mp} \left(\frac{L}{L + x} \right)^\gamma \quad F_{p,r} = \frac{\pi b^2}{4} P_{mp} \left(\frac{L}{L - x} \right)^\gamma$$

Eq. 3

The relationships for each spring are given in Eq. 4.

$$F_{spring,l} = k(L_{fl} - |x - L_p|) = k(L_{fl} + (x - L_p))$$

$$F_{spring,r} = k(L_{fl} - |x + L_p|) = k(L_{fl} - (x + L_p))$$

Eq. 4

Summing the forces yields a second order differential equation in time for the position of the translator (Eq. 5).

$$\frac{\pi b^2}{4} P_{mp} \left[\left(\frac{L}{L + x} \right)^\gamma - \left(\frac{L}{L - x} \right)^\gamma \right] - 2kx = m\ddot{x}$$

Eq. 5

Eq. 5 represents the time dependent motion of the translator as it travels from some position left of the origin to the equal but opposite position right of the origin.



Eq. 5 is made nondimensional by substitution and manipulation with the following terms:

- The ratio of specific heats:

$$\gamma = C_p/C_v$$

- Dimensionless translator position bounded by -1 to the left and 1 to the right:

$$x^* = x/L$$

- Dimensionless pressure, in this case, the midpoint pressure:

$$P_{mp}^* = \frac{\pi b^2 P_{mp}}{4Lk}$$

- Characteristic time:

$$t^* = \frac{t}{\sqrt{m/k}}$$

$$P_{mp}^* \left[\frac{1}{(1+x^*)^\gamma} - \frac{1}{(1-x^*)^\gamma} \right] - 2x^* = \frac{d^2 x^*}{dt^{*2}} = a^*$$

Eq. 6

In Eq. 6, the dimensionless acceleration is introduced as the second time derivative of the dimensionless position. A substitution (shown in Eq. 7) introduces the dimensionless velocity.

$$v^* = \frac{dx^*}{dt^*} \Rightarrow \frac{d^2 x^*}{dt^{*2}} = v^* \frac{dv^*}{dx^*}$$

Eq. 7

$$P_{mp}^* \left[\frac{1}{(1+x^*)^\gamma} - \frac{1}{(1-x^*)^\gamma} \right] - 2x^* = v^* \frac{dv^*}{dx^*}$$

Eq. 8

Eq. 8 can be solved to produce a closed form solution of the dimensionless velocity with respect to the dimensionless position of the translator. Performing the integration yields Eq. 9.

$$\frac{1}{2} v^{*2} = \frac{-P_{mp}^*}{\gamma-1} [(1+x^*)^{1-\gamma} + (1-x^*)^{1-\gamma}] - x^{*2} - C^*$$

$$C^* = \frac{-P_{mp}^*}{\gamma-1} [(1+x_l^*)^{1-\gamma} + (1-x_l^*)^{1-\gamma}] - x_l^{*2}$$

Eq. 9

The constant of integration comes from the initial condition: at some initial position to the left of the origin ($x = x_l^*$), the velocity is equal to zero ($v^* = 0$). The solution in Eq. 9 is also limited by a similar boundary condition which recognizes that the translator velocity will return to zero at some point to the right of the origin (x_r^*). Eq. 9 presents the case where no energy is added to or removed from the system, so the final position of the translator will be an equal distance from the origin but opposite of the starting position. The first of the publications [15] explores this system in greater detail (spring versus pressure limiting cases), but in any case the velocity profile is symmetric with respect to the position origin such that $x_l^* = -x_r^*$.

To maintain simplicity in pursuing a more realistic analysis, it is assumed that: (1) Heat is added and rejected at top and bottom dead center respectively; (2) Heat addition and rejection occur instantaneously; (3) Isentropic expansion and compression of cylinder gases is maintained; and (4) Work output is represented as a constant energy removal over the length of the stroke.

With these assumptions, an initial position to the left of the origin ($-1 < x_l^* < 0$), and a known midpoint pressure (equal for each cylinder), the pressure and volume in each chamber can be calculated. Focusing attention on the left chamber at this initial state, pressure and volume are given in Eq. 10.

$$P_{l,1} = P_{mp} \left(\frac{L}{L+x_l} \right)^\gamma \quad V_{l,1} = (L+x_l) \frac{\pi b^2}{4} \quad \text{Eq. 10}$$

For any given cycle, when the translator reaches its left-most position, constant volume heat addition occurs within the left cylinder at this "top dead center" position. Thermodynamic first law energy analysis for a closed system allows the cylinder pressure to be related to the heat addition (Eq. 11).

$$Q_{in} = \Delta U = m_{gas} C_v (T_2 - T_1) \quad \text{Eq. 11}$$

In Eq. 11, T represents the temperature of the gas with subscripts 1 and 2 representing the first and second state respectively. Also, the heat added (Q_{in}), change in internal energy (ΔU), and mass of the gas (m_{gas}) are represented. Because the properties and mass of the ideal gas within each chamber remain constant, the following substitutions can be made to produce Eq. 12.

$$Q_{in} = m_{gas} C_v \left(\frac{P_{l,2} V_{l,2}}{m_{gas} R} - \frac{P_{l,1} V_{l,1}}{m_{gas} R} \right) = \frac{V_{l,1} C_v}{R} (P_{l,2} - P_{l,1}) \quad \text{Eq. 12}$$

The ideal gas constant (R) can be related to the specific heat according to Eq. 13.

$$R = C_p - C_v \quad \text{Eq. 13}$$

This enables the substitution shown in Eq. 14.

$$Q_{in} = \frac{V_{l,1} C_v}{C_p - C_v} (P_{l,2} - P_{l,1}) = \frac{V_{l,1}}{\gamma - 1} (P_{l,2} - P_{l,1}) \quad \text{Eq. 14}$$

Then, the initial pressure and volume relationships from Eq. 10 are applied to yield Eq. 15.

$$Q_{in} = \left(\frac{\pi b^2}{4} \right) \left(\frac{L+x_l}{\gamma-1} \right) \left[P_{l,2} - P_{mp} \left(\frac{L}{L+x_l} \right)^\gamma \right]$$

Rearranging Eq. 15 to solve for the pressure at state two yields Eq. 16.

$$P_{l,2} = Q_{in} \frac{4(\gamma - 1)}{\pi b^2 (L + x_l)} + P_{mp} \left(\frac{L}{L + x_l} \right)^\gamma \quad \text{Eq. 16}$$

With the cylinder pressure known at top dead center, the polytropic relationship can be applied to calculate the pressure (Eq. 17), and then the related force on the translator (Eq. 18), at any position.

$$P_{l,2} V_{l,i}^\gamma = P_l V_l^\gamma$$

$$P_l = \left[Q_{in} \frac{4(\gamma - 1)}{\pi b^2 (L + x_l)} + P_{mp} \left(\frac{L}{L + x_l} \right)^\gamma \right] \left(\frac{L + x_l}{L + x} \right)^\gamma \quad \text{Eq. 17}$$

$$F_{p,l} = \frac{Q_{in}(\gamma - 1)(L + x_l)^{\gamma-1}}{(L + x)^\gamma} + \frac{\pi b^2}{4} P_{mp} \left(\frac{L}{L + x} \right)^\gamma \quad \text{Eq. 18}$$

A work term, representing a combination of friction and useful work out of the system, is added to the force balance for the system which yields Eq. 19.

$$\frac{Q_{in}(\gamma - 1)(L + x_l)^{\gamma-1}}{(L + x)^\gamma} + \frac{\pi b^2}{4} P_{mp} \left[\left(\frac{L}{L + x} \right)^\gamma - \left(\frac{L}{L - x} \right)^\gamma \right] - 2kx - F_w = m \frac{d^2 x}{dt^2} \quad \text{Eq. 19}$$

This is similar in form to Eq. 5 but includes heat and work components. Again forming nondimensional terms, Eq. 19 can be reduced to Eq. 20

$$Q_{in}^* \left[\frac{(\gamma - 1)(1 + x_l^*)^{\gamma-1}}{(1 + x^*)^\gamma} \right] + P_{mp}^* \left[\frac{1}{(1 + x^*)^\gamma} - \frac{1}{(1 - x^*)^\gamma} \right] - 2x^* - F_w^* = \frac{d^2 x^*}{dt^{*2}} = v^* \frac{dv^*}{dx^*} = a^* \quad \text{Eq. 20}$$

The nondimensional heat addition and work are given by the following two terms respectively (Eq. 21).

$$Q_{in}^* = \frac{Q_{in}}{kL^2} \quad F_w^* = \frac{F_w}{kL} \quad \text{Eq. 21}$$

Applying the velocity transformation given in Eq. 7 and integrating with the zero velocity initial condition ($v^*(x_l^*) = 0$), the closed form solution of Eq. 20 yields the relationship between position and velocity shown in Eq. 22.

$$\frac{1}{2} v^{*2} = -Q_{in}^* \left[\left(\frac{1 + x_l^*}{1 + x^*} \right)^{\gamma-1} \right] - \frac{P_{mp}^*}{\gamma - 1} [(1 + x^*)^{1-\gamma} + (1 - x^*)^{1-\gamma}] - x^{*2} - F_w^* x^* - C^* \quad \text{Eq. 22}$$

$$C^* = -Q_{in}^* - \frac{P_{mp}^*}{\gamma - 1} [(1 + x_l^*)^{1-\gamma} + (1 - x_l^*)^{1-\gamma}] - x_l^{*2} - F_w^* x_l^*$$

This relationship represents the travel of the translator effected by instantaneous heat addition in the left cylinder at TDC and constant work taken out over the length of the stroke. To accomplish a full cycle of engine operation, heat rejection is accomplished in the left cylinder by returning the pressure to the midpoint pressure polytropic curve which is analogous to a motoring curve. This instantaneous heat rejection acts as a highly idealized exhaust/scavenging process and is independent of position except that the motoring curve varies with position. The process effectively dissipates any energy built up within the cylinder in the form of internal gas energy and occurs when the left cylinder is at its bottom dead center (BDC) position. Left cylinder BDC position coincides with right cylinder TDC position. At this position, heat is added to the right cylinder in exactly the same fashion and amount as described above for instantaneous heat addition to the left cylinder. The right dead center position is identified as x_r^* , and, through nearly the same analysis as was applied to find Eq. 22, the following closed form solutions are found for the dimensionless position, velocity (Eq. 24), and acceleration (Eq. 23) of the translator as it travels from right to left.

$$a^* = v^* \frac{dv^*}{dx^*} = Q_{in}^* \left[\frac{(\gamma - 1)(1 - x_r^*)^{\gamma-1}}{(1 - x^*)^\gamma} \right] + P_{mp}^* \left[\frac{1}{(1 - x^*)^\gamma} - \frac{1}{(1 + x^*)^\gamma} \right] + 2x^* - F_w^* \quad \text{Eq. 23}$$

$$\frac{1}{2} v^{*2} = Q_{in}^* \left[\left(\frac{1 - x_r^*}{1 - x^*} \right)^{\gamma-1} \right] + \frac{P_{mp}^*}{\gamma - 1} [(1 + x^*)^{1-\gamma} + (1 - x^*)^{1-\gamma}] + x^{*2} - F_w^* x^* - C^* \quad \text{Eq. 24}$$

$$C^* = Q_{in}^* + \frac{P_{mp}^*}{\gamma - 1} [(1 + x_r^*)^{1-\gamma} + (1 - x_r^*)^{1-\gamma}] + x_r^{*2} - F_w^* x_r^*$$

The translator then returns with a negative velocity to a position left of the origin (according to Eq. 24). When the left dead center position is reached again, the instantaneous heat addition and heat rejection processes are applied to the left and right cylinders respectively.

NUMERIC MODEL

Although the basic analysis provides insight into the linear engine behavior, and can be extended to include early or late heat addition, successful simulation must include heat addition (combustion), heat loss, friction and load that varies over the stroke or a finite period of time. Closed solutions are not possible, so a time-based model in MATLAB®/Simulink was used instead. A more detailed description of the model was presented by Robinson along with more extended analyses of the sub-models described in the current work [35]. Numeric models in the literature have demonstrated successful simulations with a time step on the order of 10^{-5} seconds [36], and sensitivity analysis of the present model supports the use of a time step of $0.5 \cdot 10^{-5}$ seconds. The force balance and the force from the springs has already been presented in the fundamental analysis, and can be applied in a differential sense in a time-based model. The incylinder pressures are described by the differential pressure equation which includes heat transfer as shown in Eq. 25.

$$\frac{dP}{dt} = \left(-\gamma \frac{P}{V}\right) \frac{dV}{dt} + \left(\frac{\gamma + 1}{V}\right) \left(\frac{dQ_{ht}}{dt} + \frac{dQ_{HR}}{dt}\right) \quad \text{Eq. 25}$$

In Eq. 25, P is cylinder pressure, V is volume, γ is the specific heat ratio of the gas, Q_{ht} is the heat transfer from the cylinder, and Q_{HR} is the heat released in the cylinder during combustion. Pressure is related to gas temperature (T) and volume through the ideal gas law.

The specific heat ratio is dependent on temperature (T in °K) according to the polynomial fit shown in Eq. 26 which shows good correlation to HCCI engine data over a temperature range of 550 K - 2150 K in [37].

$$\gamma = (-9.967 \cdot 10^{-12}) \cdot T^3 + (6.207 \cdot 10^{-8}) \cdot T^2 + (-1.436 \cdot 10^{-4}) \cdot T + 1.396 \quad \text{Eq. 26}$$

Convective heat transfer depends on the instantaneous surface area (A) of the cylinder, assuming that the wall temperature (T_{wall}) is the same over the head, cylinder walls, and piston crown and fixed throughout a simulation. The differential relationship for heat transfer is given in Eq. 27.

$$\frac{dQ_{ht}}{dt} = hA(T - T_{wall}) \quad \text{Eq. 27}$$

The heat transfer coefficient (h) can be calculated using various empirically correlated formulae, each with strengths and weaknesses according to application. Unfortunately, these empirical relationships are based upon slider-crank motion rather than the unusual and variable motion of the linear engine system. Piston velocity is the usual reference for the in-cylinder charge motion that enhances the heat transfer, along with instantaneous gas temperature, pressure, and cylinder volume. While Hohenberg's formulation [38] is chosen for use in the base case due to its simplicity and recent application [39], correlations by Woschni [40], Annand [41], and Chang (referred to as

the HCCI modified Woschni correlation) [37] are employed in this work for comparison. Their formulations are summarized well in [42] and Hohenberg's unit-dependent relationship is provided in Eq. 28.

$$h = a \cdot V^b \cdot P^c \cdot T^d \cdot (S_p + 1.4)^e \quad \text{Eq. 28}$$

In the base case with Eq. 28 representing the heat transfer, V is volume in m^3 , P is pressure in bar, T is temperature in K, and S_p is average piston speed in m/s. The empirical constants are: $a = 130$, $b = -0.06$, $c = 0.8$, $d = -0.4$, and $e = 0.8$.

Heat addition depends on the rate of combustion, which is readily programmed into a time-based model. However, empirical models such as those based on Wiebe functions [43, 44], were also originally derived from slider-crank engine data. There are several possible approaches for modeling any single combustion event. Computationally expensive efforts have attempted to combine chemical kinetics with computational fluid dynamics across multiple zones within a cylinder [45, 46]. However, the amount of time necessary to complete the routine (on the order of hours for a relatively fast computing system) are not appropriate for multiple cycles of simulation at the current level of linear engine modeling sophistication. In addition, pressure wave dynamics, flame front propagation, quench, and knocking are all complex phenomena which compound the difficulty of combustion modeling. A model must be chosen which sufficiently approximates the combustion event but does not encumber the total effort with excessive computational needs.

Single zone HCCI combustion modeling of the free piston engine in the literature typically involves a knock integral to predict start of combustion and a time-based Wiebe function to represent the heat release profile. Later in this present work, heat transfer model comparison is used to demonstrate that the amount of energy exchanged is of much greater importance than the profile over which the energy is exchanged. This idea is corroborated by Lee in his dissertation [47]. In another dissertation, [48], Mo concludes that combustion timing is the single most important variable for the entire combustion process and presents a series of one dimensional relationships that sufficiently predicts HCCI engine performance. Unfortunately, these correlations are highly dependent on crankshaft position and cannot be adequately re-expressed for the free piston engine in the time domain. Such research indicates that single zone simulations can be sufficient for accurate prediction of engine cylinder pressure during HCCI combustion operation, but that such accuracy is dependent on proper correlation to the application. The current work employs a time-based Wiebe function of the form shown in Eq. 29.

$$\chi(t) = 1 - \exp \left[-a \left(\frac{t - t_{soc}}{C_d} \right)^{1+b} \right] \quad \text{Eq. 29}$$

$$\frac{dQ_{HR}}{dt} = \eta_{comb} E_{fuel} \frac{d\chi(t)}{dt}$$