

# Resonance of a Spring Opposed Free Piston Engine Device

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## ABSTRACT

Recent free piston engine research reported in the literature has included development efforts for single and dual cylinder devices through both simulation and prototype operation. A single cylinder, spring opposed, oscillating linear engine and alternator (OLEA) is a suitable architecture for application as a steady state generator. Such a device could be tuned and optimized for peak efficiency and nominal power at unthrottled operation. One of the significant challenges facing researchers is startup of the engine. It could be achieved by operating the alternator in a motoring mode according to the natural system resonant frequency, effectively bouncing the translator between the spring and cylinder, increasing stroke until sufficient compression is reached to allow introduction of fuel and initiation of combustion. To study the natural resonance of the OLEA, a numeric model has been built to simulate multiple cycles of operation. The MATLAB® model combines differential relationships (for cylinder pressure and translator dynamics), empirical relationships (for heat transfer and friction), and simplifications (for gas exchange and a mechanical spring) to describe the primary dynamic elements acting on the translator. The translator is excited from rest by a sinusoidal forcing function which is defined by frequency and force amplitude. As an example, an engine with a 1 kg translator mass, 30 mm bore, maximum stroke of 45 mm, atmospheric intake pressure, and spring stiffness of 250 kN/m is given. When the motor applies a sine wave at 85 Hz with peak force at 400 N, the steady state stroke is found to be 30 mm with a compression ratio of 20 after 70 strokes. For the same system motored at a forcing frequency of 75 Hz, the resulting steady state stroke is reduced to 17 mm and the compression ratio to 3.2. Knowledge of the natural resonance for a given device will enable controlled motoring to achieve a desired stroke and compression ratio. This paper investigates the resonant nature of the OLEA and parametrically explores the forcing amplitude and frequency.

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## INTRODUCTION

In recent years, interest in the free piston linear engine generator has grown at a rapid pace because of its potential to directly convert fuel energy to electrical energy without the high complexity and mechanical losses associated with conventional crankshaft engine technology. Some of the leading researchers in the field from Newcastle University in the UK have published a recent review of commercial developments around the free piston engine [1]. The article focuses largely on patented devices. In addition, many configurations, from two to four stroke, single piston to dual to opposed, and from electric power output to hydraulic to kinetic are all pursued as feasible. Through the review, a number of significant challenges are seen that need to be overcome. Control of the free piston and combustion events continues to be one of the most significant hurdles and has been the focus of many studies [2, 3, 4, 5, 6]. Additionally, current development reports that commercialization will depend on solutions to stability, cooling, and start-up. Stability, as well as range of power output, depends on control while cooling needs are yet to be determined and will depend on experimental data to validate any proposed strategy. Start-up of the free piston OLEA, whether cold or hot, could be handled by operating the alternator as a

motor to successively bounce the translator between the engine cylinder and the opposing mechanical spring. This is the method proposed for the OLEA presented below.

The bounced starting method has been suggested by various researchers and explored in both analytical and numerical spaces for a dual cylinder free piston device. In 2006, an investigation from Kyungnam University in South Korea considered the use of a three phase permanent magnet linear generator as a motor to force oscillation in a linear engine at resonance to achieve sufficient compression [7]. They concluded that feedback control from translator position sensing would be required for precise control of the alternator inverter. In 2014, researchers from Newcastle University in the United Kingdom and the Beijing Institute of Technology in China published developmental and experimental results which showed the motored resonance approach can achieve sufficient compression in a dual piston free piston engine generator device [8]. The Beijing researchers then extended the bouncing startup investigation in [9], where a constant starting force, which changed sign with velocity, was employed to achieve a pulse width modulated square wave. For a 3kg translator mass with bore of 50mm, they simulated and achieved a compression ratio of ~20 and frequency of ~21 Hz with a forcing amplitude of 35 N. They also parametrically explored the effects of cylinder bore and translating

mass, concluding that decreased stroke-to-bore ratio increases steady state frequency response. Reducing the translator mass can also increase frequency but additionally results in lower force capabilities of the electric motor due to design limitations.

For the proposed OLEA system, the single engine cylinder is opposed by a mechanical spring to return the translator for the compression stroke. This implies the stiffness of the spring is high enough to balance against the force applied on the piston head from the cylinder pressure. The application of the OLEA as a steady state generator enables this simplicity - in contrast to examples which have employed compressed air bounce chambers for control authority over a potentially broader operating envelope [5, 10, 11]. The application and configuration also enable single point optimization of device components for efficiency, power output, and cost. Based on prior experience [12, 13], the present authors believe that high efficiency and power density are most readily achieved at high frequency operation. A stiff mechanical spring can be designed for the application and brings with it two design parameters for system optimization: the stiffness profile and the neutral force point of the spring. In this paper, the effect of the spring on system resonance is compared with that of the stroke-to-bore ratio for the single cylinder device. Also, the effects of energy loss to heat transfer and friction are explored. Finally, the shape of the force profile will likely vary from the modulated square wave, so both sinusoidal and square waves are compared in terms of OLEA excitation effectiveness in the paper below.

## NUMERIC MODEL

To study the resonant dynamics of the OLEA, a numeric model has been constructed based on Newton's 2<sup>nd</sup> law of motion (Eq. 1).

$$\sum \vec{F} = m\vec{a} \quad \text{Eq. 1}$$

The forces acting on the OLEA translator are due to four main sources: the alternator (or motor depending on operating mode), the cylinder pressure from the engine, the mechanical spring, and friction. These are shown on Figure 1 and the applied forces are summed according to Eq. 1 and shown in Eq. 2.

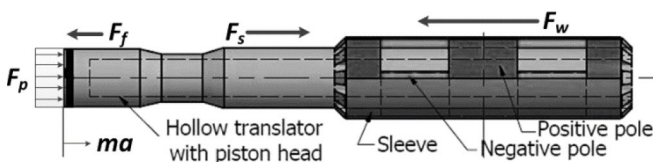


Figure 1. Illustration of single cylinder OLEA translator with forces applied

$$\vec{F}_p + \vec{F}_s - \vec{F}_f - \vec{F}_w = m\ddot{x} = m\vec{a} \quad \text{Eq. 2}$$

In the equation and illustration,  $F_p$  represents the force caused by engine cylinder pressure,  $F_s$  is the force from the spring,  $F_f$  is the frictional force,  $F_w$  is the thrust force caused by the alternator/motor, and  $m$  is the total moving mass.

## Spring and Friction Forces

While the design of the mechanical spring involves the challenges of reducing mass and managing fatigue, it is reasonable to assume an ideal spring with constant stiffness within the numeric model. This results in the linear deflection model given in Eq. 3.

$$\vec{F}_s = k\delta = k(x - L_{k0}) \quad \text{Eq. 3}$$

In Eq. 3,  $k$  is spring stiffness,  $x$  is translator position, and  $L_{k0}$  is the zero force point of the spring. The zero force point refers to the free length of the spring, but is given with respect to the coordinate system defined for the numeric model.

Modeling the friction force acting on a free piston translator is often characterized by a viscous damping term where the force is proportional to the translator velocity ( $v$ ). The relationship given in Eq. 4 provides for this proportionality through the damping constant ( $C_f$ ). As an alternative, the authors in [9] include both static and dynamic damping in the friction force based on the numerical correlation found in [14].

$$\vec{F}_f = c_f \dot{x} = c_f \vec{v} \quad \text{Eq. 4}$$

The value for the friction coefficient is a modified form of the mean frictional pressure calculation found in [15] which correlates the piston ring friction to the resulting force. The calculated friction value is then applied as a constant force through the length of the stroke. In its original form proposed by Blair in [16], the correlation was based on stroke (a known, fixed length) and engine speed (in rpm) and applied to industrially sized (large bore) engines. In a study at WVU, a ring down experiment showed that a velocity relationship is sufficient for friction modeling, though the correlation coefficient is not given [13]. Thus, Blair's correlation is adapted to estimate the friction coefficient according to Eq. 5.

$$c_f = \left( \frac{\pi b^2}{8} \right) 300 S_{max} (60 f_i) \quad \text{Eq. 5}$$

In where  $F_i$  is the cyclic operating frequency of the translator (Hz), and bore ( $b$ ) and max stroke ( $S_{max}$ ) are in meters. For a given simulation, steady state operation implies that all of the energy added to the system is dissipated. If heat transfer is neglected, the energy must be lost to friction. For the case where heat transfer is included, the energy exchange at steady state depends on the heat transfer mechanism and is explored in a later section. In any case, the resonant dynamics are strongly affected by the energy lost to friction which is parameterized for a range of frictional losses as a part of this investigation.

## Alternator/Motor Model

During start-up, the tubular permanent magnet linear alternator operation is reversed to operate as a tubular brushless DC motor. In order to accomplish this, a three-phase inverter is designed to drive the motor in linear motion back and forth. Similar to other systems

systems, it is anticipated that it will not be the magnitude of force applied to the translator but its frequency that will drive the system to resonance [12]. The inverter starts at relatively low frequency and achieves in-phase, harmonic motion with the translator at steady state. High gauge magnetic wire is embedded along the side of the armature windings and used as a hall-effect device for position sensing. The position sensor data is then fed into a micro-controller to gate the three-phase inverter for driving the motor. With knowledge of the system resonance, the inverter's frequency can be gradually increased from just below the expected resonant frequency until resonance is achieved. When steady state frequency, stroke length, and compression ratio reach target operation, fuel is injected and combustion replaces the motor as the source for energy input for the OLEA. The linear electric machine then acts as an alternator where the inherently built-in inverter transistors' diodes (shot-through diodes) act as full-wave rectifiers for rectified DC power output.

To be representative of the literature, as well as more realistic permanent magnet linear electric machine operation, the forcing function is modeled by both a time-based square wave and a sine wave for comparison. The time-based models are chosen to avoid the assumption that translator position and velocity data are accurately available. Both are parameterized by input frequency and input amplitude force. The sine wave formulation is given in Eq. 6 showing the two parameters of interest: forcing amplitude ( $A$ ) and forcing frequency ( $f$ ) in Hz.

$$\vec{F}_w = A \cdot \sin(2\pi \cdot f \cdot t) \quad \text{Eq. 6}$$

### Cylinder Pressure

Engine cylinder pressure modeling is key to capturing the dynamic interaction between the engine and translating free piston. The investigation reported here focuses on start-up of the device prior to the initiation of combustion, so the cylinder model must only involve three processes. Compression and expansion are described by the differential pressure equation which includes heat transfer as shown in Eq. 7.

$$\frac{dP}{dt} = \left(-\gamma \frac{P}{V}\right) \frac{dV}{dt} + \left(\frac{\gamma + 1}{V}\right) \frac{dQ_{ht}}{dt} \quad \text{Eq. 7}$$

In Eq. 7,  $P$  is cylinder pressure,  $V$  is volume,  $\gamma$  is the specific heat ratio of the gas, and  $Q_{ht}$  is the heat transfer between the cylinder gas and walls. For the one dimensional pressure model, the cylinder contents are considered homogeneous. Pressure is related to gas temperature ( $T$ ) and volume through the ideal gas law (Eq. 8) and it is assumed that no mass is lost from the cylinder except during the gas exchange process.

$$PV = mRT \quad \text{Eq. 8}$$

The specific heat ratio is dependent on temperature ( $T$  in K) according to the polynomial fit shown in Eq. 9 [17].

$$\gamma = (-9.967 \times 10^{-12}) \cdot T^3 + (6.207 \times 10^{-8}) \cdot T^2 + (-1.436 \times 10^{-4}) \cdot T + 1.396 \quad \text{Eq. 9}$$

Heat transfer depends on the instantaneous surface area ( $A$ ) of the cylinder and difference between gas and wall temperatures ( $T$  and  $T_{wall}$  respectively) as shown in Eq. 10.

$$\frac{dQ_{ht}}{dt} = hA(T - T_{wall}) \quad \text{Eq. 10}$$

The heat transfer coefficient ( $h$ ) can be calculated using a number of empirically correlated formulae, each with strengths and weaknesses according to application. Piston velocity is the usual reference for the in-cylinder charge motion that enhances the heat transfer. The difficulty is that many of these correlations, Woschni's [18] and Annand's [19] included, are formulated against limited data sets from crankshaft driven engines. This makes them difficult to relate to a free piston engine cylinder. Many of the available relationships are compared in a recent article [20]. Hohenberg's formulation [21] - given in Eq. 11 - was chosen for simplicity. In Eq. 11,  $V$  is cylinder volume ( $\text{m}^3$ ),  $P$  is pressure (Pa),  $T$  is temperature (K), and  $\bar{U}$  is mean piston speed (m/s).

$$h = 130V^{-0.06} \left(\frac{P}{10^5}\right)^{0.8} T^{-0.4} (\bar{U} + 1.4)^{0.8} \quad \text{Eq. 11}$$

The third and final process, gas exchange, is assumed to be perfect, instantaneous, and set to begin and end when the translator position coincides with the port location ( $L_p$ ). In other words, the conditions in the cylinder are immediately replaced with intake conditions at the moment the ports open. While this is a significant simplification, the method is shown to be effective in recent literature [2, 15].

### Simulation procedure

For the free piston linear engine, unconstrained by connecting rod and crankshaft geometry, the simulation is most conveniently dependent on time rather than position. With known acceleration and a sufficiently small time step ( $\Delta t$ ), integration of Eq. 1 yields translator velocity and position.

The simulation is initialized by finding the translator static equilibrium position based on cylinder pressure and spring force. Knowing that the translator velocity is zero and assuming an isentropic pressure change from port closing position, the initial position can be found numerically as the solution for  $x_0$  from Eq. 12.

$$k(x_0 - L_{k0}) = \frac{\pi b^2}{4} P_{in} \left(\frac{L_p}{x_0}\right)^\gamma \quad \text{Eq. 12}$$

In Eq. 12,  $P_{in}$  is the intake pressure,  $L_p$  is the location of the ports, and  $b$  is cylinder bore. Knowing the translator starting position and setting the initial velocity to equal zero, the forcing function ( $\vec{F}_w$ ) is applied to the translator. Numeric time-step integration is used to solve for the multiple cycles of operation.

## SIMULATION RESULTS

Based on the desire of the researchers to design and eventually build an OLEA which produces 1-2 kW of electrical power (kWe) at full throttle, initial bore and maximum stroke parameters are chosen. In addition, port locations and intake conditions must be specified. While the maximum stroke to bore ratio are varied within this study, the intake parameters are held constant for this investigation.

- $P_{in} = 101325 \text{ Pa}$
- $T_{in} = 350 \text{ K}$
- $S_{max} = 0.045 \text{ m}$
- $L_p = 0.0315 \text{ m}$

For the sake of size and clarity, all referenced figures are placed in the [Appendix](#). In each simulation, a position of  $x = 0$  corresponds to the theoretical minimum where the piston head collides with the cylinder while a position of  $x = S_{max}$  is the theoretical maximum. [Figure 2](#) illustrates the first simulation case through the position-velocity plot on the left and change in stroke and frequency over time on the right. The change in stroke length over time clearly illustrates the second order response nature of the system. No controller is active within the system, so the second order response is due to spring-mass-damper system harmonics brought on by early phase difference between the forcing function and translator motion. A wide range of responses is available, so deviations from this initial case (the control case) are explored first.

### Heat Transfer and Friction

The previous control case simulated the engine with adiabatic compression and expansion. To approximate the heat transfer that might occur in the engine, [Eq. 10](#) and [Eq. 11](#) are applied. Empirical data for the OLEA device is not available, so the wall temperature is assumed to be fixed at 350 K and the heat transfer coefficient ( $h$ ) is parameterized with a scaling factor (e.g.  $0 \cdot h$  for the control case).

Two plots are given in [Figure 3](#) to show the effects of heat transfer on the motored OLEA. For each plot, all parameters are kept constant and equal to those given in the control case. In the left plot, mean friction power and mean cylinder pressure are shown to decrease with an increase of heat transfer rate. With a constant friction coefficient, reduced friction power implies reduced translator velocity. The right-hand plot, however, shows very small effects on translator stroke and frequency. In fact, the steady state response frequency is very dominated by the motoring frequency. The increasing heat transfer is offset by the reduced friction power and system stiffness resulting in negligible changes to frequency rise rate and steady state stroke response. In addition, the system is dominated by the spring which is relatively stiff compared to the engine cylinder.

Friction is parameterized in a similar fashion. For all cases presented above, the friction coefficient has been constant according to the calculation from [Eq. 5](#). [Figure 4](#) shows the effects of friction as it is varied from 0 - 125%. In the left-hand plot, increase in friction shows decreases in stroke length and cylinder pressure. Of particular interest is the apparent discontinuity which can be seen in both left- and right-hand plots. On the right, the discontinuity manifests as two distinct resonant regions. For low friction coefficient cases, convergence to a higher steady state stroke regime is seen. This upper regime is set apart because the stroke length of the translator enables the port to open. The instantaneous nature of the gas exchange process manifests as the bifurcation of the steady state response. Also of note, though not shown in the plotting, is the case of excessive damping which results in no travel of the translator - a trivial solution except perhaps in the event of mechanical component failure.

While only a single control case (demonstrated in [Figure 2](#)) was used in this study of heat transfer and friction, there are some important conclusions. First, heat transfer is not negligible, but the amount of heat transfer has low significance. Second, friction is significant and can act to bring about multiple resonant zones of operation depending on port placement. For the remainder of the study, 100% friction and heat transfer coefficients are applied.

### Sine vs Square Wave Motoring Force

The previously described study assumed a constant motoring force dependent on the sign of the translator velocity [9]. While the approach is convenient for simulation, it assumes the availability of translator position-velocity data as well as the ability of the motor to act against the translator with a constant force. Rectangular profiles have been studied for permanent magnet linear machines [22], showing that rectangular commutation is viable for the slow speed, dual cylinder device. However, accurate position-velocity sensing of the translator is not addressed despite the dependent nature of the control architecture.

To avoid the eventual complexity of position-velocity dependent control, the proposed starting strategy employs a time-based forcing function. For a given device, the limited control scheme would increase the motored frequency, eventually matching the natural resonance of the system, until sufficient stroke and compression are reached for fuel injection and combustion.

With various waveforms possible, it is important to understand the implications of the shape of the motoring force profile. [Figure 5](#) compares three wave shapes: square, triangular, and sinusoidal which are represented by the dotted, dashed, and solid plot lines respectively. The three simulation cases are given with all parameters equal except the shape of the motoring force profiles, and it is seen that the square wave results in higher stroke length and compression ratio followed by sinusoidal wave. This is consistent with the fact that the square wave force profile imparts the most energy into the system while the triangular wave imparts the least. It follows then that the square wave is more desirable because a lower force magnitude is required to achieve the same stroke/compression thus reducing the mass requirements of the motor.