

Analysis of the Turbocharger Compressor Surge Margin Using a Hurst-Exponent-based Criterion

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ABSTRACT

Turbocharger compressors are limited in their operating range at low mass flows by compressor surge, thus restricting internal combustion engine operation at low engine speeds and high mean effective pressures. Since the exact location of the surge line in the compressor map depends on the whole gas exchange system, a safety margin towards surge must be provided. Accurate early surge detection could reduce this margin. During surge, the compressor outlet pressure fluctuates periodically. The Hurst exponent of the compressor outlet pressure is applied in this paper as an indicator to evaluate how close to the surge limit the compressor operates. It is a measure of the time-series memory that approaches zero for anti-persistence of the time series. That is, a Hurst exponent close to zero means a high statistical preference that a high value is followed by a low value, as during surge. Maps of a passenger-car sized turbocharger compressor with inlet geometries that result in different surge lines are measured on a cold gas stand. It is demonstrated that the Hurst exponent in fact decreases as the compressor moves towards surge, and that a constant value of the Hurst exponent can be used as a threshold for stable operation. Transient pressure signals of the compressor entering surge are analyzed in order to evaluate the time lag until surge can be detected using the Hurst exponent. Two surge cycles are usually needed to detect unstable operation. However, since the amplitude of these oscillations is relatively small for the first cycles, detection is possible before the oscillations grow into deep surge.

CITATION: Kerres, B., Nair, V., Cronhjort, A., and Mihaescu, M., "Analysis of the Turbocharger Compressor Surge Margin Using a Hurst-Exponent-based Criterion," *SAE Int. J. Engines* 9(3):2016, doi:10.4271/2016-01-1027.

INTRODUCTION

A common way of increasing the efficiency of internal combustion engines is downsizing, where the displaced volume is reduced, and this reduction is compensated for by a higher boost pressure. The boost pressure is often provided by a turbocharger, consisting of a turbine that utilizes the exhaust gas enthalpy flow to drive a centrifugal compressor in the engine intake system. In many engine applications, e.g. vehicle propulsion, the engine needs to be able to operate over a wide range of speeds and loads. The mass flow through the compressor scales approximately linearly with the engine speed, while the required boosting pressure increases with the torque demand. This translates into the requirement of a wide operating map for the turbocharger compressor.

Centrifugal compressor operation is limited by the flow choking at high mass flows, and by compressor surge at low mass flows. In surge, compressor operation becomes unstable as the volume between the compressor and a flow resistance downstream (e.g. the throttle in SI engines or intake valves in CI engines) is filled and emptied periodically. The pressure and mass flow oscillations during surge create noise, affecting the combustion process negatively and damage the compressor if their amplitude becomes too large. Surge operations of the turbocharger compressor must therefore be avoided, and can limit the available engine torque at low speeds.

The location of the surge line in the compressor map depends on many properties, not only of the compressor itself, but the whole compression system. Greitzer [1, 2] showed for axial compressors, that there are three main parameters determining when the compressor enters surge: The slope of the compressor characteristic, the slope of the throttle characteristic, and the so-called B parameter. The compressor and throttle characteristics (ψ_C and ψ_T) are the pressure rise parameter ψ as a function of the mass flow parameter ϕ . ψ is defined as

$$\psi = \frac{\Delta p}{0.5 \rho_2 U^2} \quad (1)$$

where Δp is the change in pressure over the compressor or throttle, respectively, ρ_2 is the compressor outlet density, and U the impeller tip speed. The mass flow parameter ϕ is defined as

$$\phi = \frac{\dot{m}}{\rho_2 A_2 U} \quad (2)$$

where A_2 is the compressor outlet duct cross-section. The B parameter is defined as

$$B = \frac{U}{2\omega_H L_c} \quad (3)$$

where L_c is the intake duct length and ω_H is the eigenfrequency of a Helmholtz resonator with an equivalent duct length and plenum volume, shown in Figure 1. The stability criteria are then

$$\frac{\partial \psi_c}{\partial \phi_c} < \frac{\partial \psi_T}{\partial \phi_T} \quad (4)$$

$$\frac{\partial \psi_c}{\partial \phi_c} < \frac{1}{B^2} \left(\frac{\partial \psi_T}{\partial \phi_T} \right)^{-1} \quad (5)$$

The throttle characteristic slope $\partial \psi_T / \partial \phi_T$ is generally positive. Therefore, the compressor is in the stable operating regime for a negative characteristic slope, while the systems stability depends on the value of B and the throttle characteristic slope for a positive compressor characteristic. Fink et.al. [3] confirmed that this analysis also holds for centrifugal compressors.

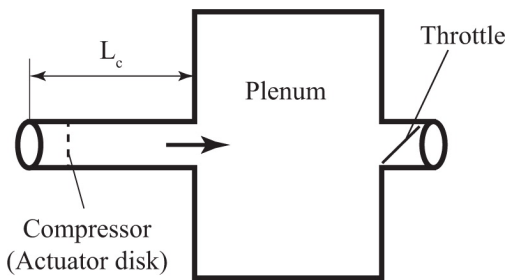


Figure 1. A Compression system as a Helmholtz resonator

The engine developer selects the turbocharger based on the compressor map, which is measured and delivered by the turbocharger supplier. These measurements are carried out on a gas stand, where the value of B and the throttle characteristic are different from the values on the engine. Furthermore, pulses at the compressor outlet as generated by the engine intake valves, as well as the pipe geometry at the compressor inlet have been shown to influence the onset of surge in [4, 5]. The surge limit given by the supplier is therefore not generally valid on the engine, and a safety margin must be incorporated. This safety margin could be reduced by utilizing onengine detection of the onset of surge, thus allowing higher torque at low engine speeds.

This paper is organized as follows: First, a short overview over the methods used to determine compressor stability is given. Then, the Hurst exponent and the computing methodology are described using a step-by-step calculation for artificially generated signals. In the section after that, the experimental setup is described. Then, the time scales of interest are established for this compression system, and it is shown that the Hurst exponent can be used for this system as an indicator of compressor surge. The stability of the compressor used is

then analyzed using the Hurst exponent criterion. Thereafter, two transients during which the compressor enters surge are evaluated in order to determine if surge detection using the Hurst exponent can be employed for transient compressor signals. Finally, the paper is concluded.

BACKGROUND

Surge Criteria for Automotive Turbocharger Compressors

Compressor surge is characterized by a filling and emptying process of the volume between the compressor and some other flow resistance downstream. This process leads to oscillations in the compressor outlet pressure and the compressor mass flow. Measurements of these variables are therefore an obvious starting point. Since time-resolved pressure measurements are much easier to implement than time-resolved mass flow measurements, they are usually preferred.

Based on compressor outlet pressure measurement, [6] and [7] use the standard deviation of the signal (p_{2c}) as a surge indicator for compressor in steady-state operation. It is often normalized with the impeller dynamic pressure ($0.5\rho U^2$), i.e.

$$\hat{\sigma}_{p2} = \frac{\text{std}(p_{2c})}{0.5\rho_{01}U^2}, \quad (6)$$

in order to obtain a criterion that is less dependent on the impeller speed. This criterion includes oscillations at all frequencies.

Since surge oscillations have a distinct frequency that can be estimated by an equivalent Helmholtz resonator as shown by Greitzer [1], one can also estimate the surge frequency and then only consider oscillations in this frequency range. For example, in [6] and [8], the height of peak in the frequency spectrum, obtained via discrete Fourier transform (DFT) of the outlet pressure signal, was used.

A closely related approach was taken in [9], where instead of the linear spectrum obtained by $|\text{DFT}\{p\}|$, the power spectral density S_{pp} obtained by $|\text{DFT}\{p\}|^2$ is considered; furthermore, the inlet pressure signal measured using a microphone is used instead of the outlet pressure signal. The power spectral density is integrated over an interval from 10 Hz to 30 Hz in order to better capture the signal power in this bandwidth. This paper uses a similar approach,

$$P_p = \int_{f_p - 10\text{Hz}}^{f_p + 10\text{Hz}} S_{pp} df, \quad (7)$$

where f_p is the frequency with the highest power density between 20 and 100 Hz. This gives essentially the variance of the pressure in the given frequency interval, and is therefore related to the standard deviation in Equation (6).

Since these oscillations also generate noise, a subjective surge criterion based on the driver's noise comfort was tested in [6].

Another parameter of interest in surge detection is the near-wall inlet temperature of air upstream the impeller. Close to and in surge, there is usually some backflow present along the shroud. This fluid will have an increased temperature. The backflow becomes the stronger, the closer to surge the compressor operates. Therefore, the rise of the temperature at the compressor inlet can be used as an indicator of the proximity to surge, e.g. for turbochargers in [6] and [9].

The oscillations of the compressor discharge pressure and mass flow lead to a change in torque on the compressor shaft. This results in speed oscillations, which will be small for most turbochargers compared to the mean speed, since the moment of inertia of the turbine wheel is relatively large. However, there are cases when they also can be used as an indicator or even influence compressor behavior during surge as observed in [3].

In this paper, the Hurst exponent based on outlet pressure measurements will be evaluated. It was first proposed as an indicator of compressor surge in [10] for a part of a steady-state compressor map and will here be used to analyze the stability over the compressor map, as well as transient compressor signals.

Detrended Fluctuation Analysis and the Hurst Exponent

The Hurst exponent H^2 of a time series contains information about the correlations and memory present in a time series. It is calculated for a given range of time scales. A Hurst exponent close to zero means that the signal is negatively correlated in this range; a value of $H^2 = 0.5$ means zero autocorrelation, like in white noise, and a value of $H^2 > 0.5$ means a positive correlation. Thus, we can expect H^2 of a pressure signal of a compressor in surge to be near zero, while it will be higher for normal turbulent pressure fluctuations.

There are different ways of estimating H^2 for signals without a trend, e.g. Hurst's original "Rescaled Range" approach [11], or the "Fluctuation Analysis" approach described in [12]. In this paper, where also transient pressure signals are analyzed, a trend in the signal is present. Therefore, the detrended fluctuation analysis approach (DFA) is used and will be described below. It was first described by Peng [13] and used to find correlations in DNA sequences. It has been applied to investigate the onset of combustion instabilities in [14], and in [10] for compression system instabilities.

Exemplary Estimation of the Hurst Exponent for Three Artificial Signals

In the following, the estimation of H^2 using DFA will be explained step by step for three artificially generated signals over time t :

1. A sine wave x^a with $f_1 = 2$ Hz

$$x^a = \sin(2\pi f_1 t)$$

2. White noise x^b

$$x^b \sim \mathcal{N}(0,1)$$

3. A superposition of 2 sine waves x^c with $f_1 = 2$ Hz and $f_2 = 50$ Hz

$$x^c = \sin(2\pi f_1 t) + \frac{1}{4} \sin(2\pi f_2 t)$$

As a first step, the cumulative of the time series is calculated, i.e. for x^a :

$$Y^a(t_j) = \sum_{i=1}^j x^a(t_i) , \quad (8)$$

and equivalently for x^b and x^c . This gives again a trigonometric wave $[1 - \cos(2\pi f t)]$ for the sine signals, and a Gaussian random walk for the white noise, as shown in Figure 2. The differences between Y^a and Y^c are small, since one period of the additional sine wave in x^c is small compared to the one in x^a .

Now, the time scales t_s need to be determined. It is assumed here that those are unknown; as recommended in [12], a lower limit of $t_s = 10/f_{\text{sample}}$ is chosen so that there are at least 10 data points in each window, while the upper limit is determined by the sampling period. As also recommended in [12], one should aim for at least 4 non-overlapping windows of size t_s , so that

$$t_{s,\max} \leq \frac{T_{\text{sample}}}{4} . \quad (9)$$

The time scales in between are then logarithmically spaced, as also shown in Figure 2. In this figure, not all time scales are drawn in order to improve clarity.

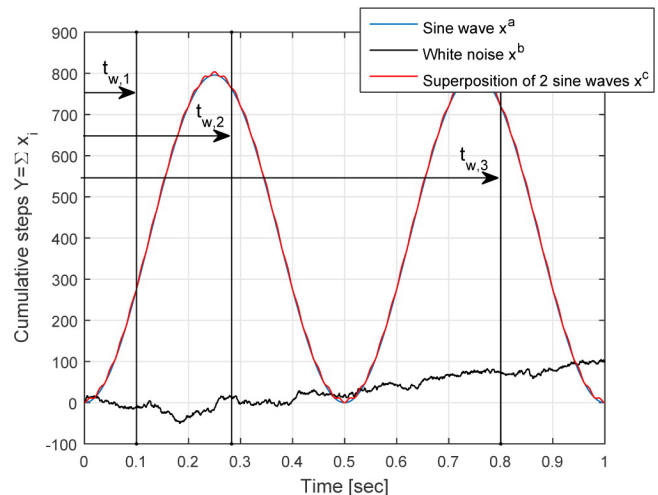


Figure 2. Cumulative series $Y(t)$ of a single sine wave x^a , white noise x^b , and a superposition of two sine waves x^c . Since the frequency of the superimposed sine wave is much higher than the one of the original sine wave, the difference between the integrated time series is barely visible.

For each time scale, the signal is split into windows of length t_s . In each window, the mean and the linear trend are removed¹:

$$y_w = Y_w - v_w, \quad (10)$$

where v_w is the first-order least-square fit of Y_w . After the removal of the linear trend and mean, the variance of each window is calculated,

$$F_w^2 = \text{var}[y_w], \quad (11)$$

and the root mean of the variances of the different windows is calculated for each time scale

$$F^2(t_s) = (\text{mean}[F_w^2])^{\frac{1}{2}}. \quad (12)$$

$F^2(t_s)$ is called the fluctuation function. A log-log plot of F^2 over the time scales t_s , Figure 3, is used to determine the Hurst exponent H^2 . In this graph, it corresponds to the slope of the linear fit of the fluctuation function over time scales. One can also generate a different order of the fluctuation function F^q ,

$$F^q(t_s) = \left\{ \text{mean} \left([F_w^2]^{\frac{q}{2}} \right) \right\}^{\frac{1}{q}}, \quad (13)$$

and calculate a more general Hurst exponent H^q . In the following section, H^2 will mainly be used, as suggested by [12], but other H^q will be considered in the analysis of the transient signals.

For a single sine wave, there are two distinct regions: For time scales smaller than one period ($t_s < 1/f_1$), and for time scales longer than one period ($t_s > 1/f_1$). For time scales smaller than one period, the variance increases with time scale squared, which gives a slope of approximately 2 in a log-log diagram. This means that at these time scales, a high value is likely also followed by a high value (i.e. a positive autocorrelation for these scales), and that it looks unbounded when just looking at a small interval. For time scales larger than one period, on the other hand, the fluctuation function F^2 is independent of the time scale, which gives a slope of zero. This means that at these scales, the autocorrelation is negative, and therefore, the function seems bounded by an upper and lower value, respectively.

The root mean displacement of a Gaussian random walk $Y^b(t)$ grows with the square root of steps. Therefore, the fluctuation function of the normally distributed random variable $x^b(t)$, shown as black xmarkers in Figure 3, has a slope of 0.5 independent of time scale the log-log diagram.

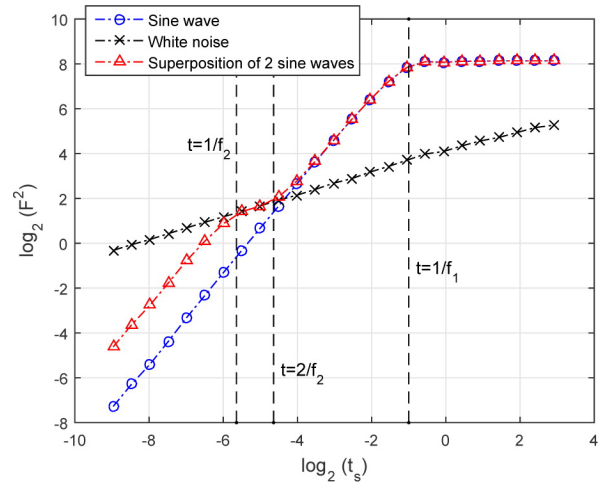


Figure 3. $\log_2 - \log_2$ plot of the fluctuation function F^2 over time scales t_s for a single sine wave, white noise, and a superposition of two sine waves.

For the superposition of two sine waves, shown as red triangle markers in Figure 3, the slopes are the same as for the single sine wave, with an exception for the time scales for the superimposed sine wave. For time scales from one short period to two short periods ($1/f_2 < t_s < 2/f_2$), the slope is equal to 0.5. This is since the superimposed sine wave is detrended using random segments of the larger original sine wave, which generates arbitrary waveforms. The signal is therefore effectively random at these scales. For even larger time scales, ($t_s \gg 1/f_2$), the original sine wave dominates the signal. Therefore, the fluctuation function shows the same behavior as the single sine wave with the same frequency.

These observations show that the slope of the fluctuation function over different time scales - the Hurst exponent - contains information about the frequencies at which oscillations are present in the time series. In the following section, this will be applied to real compressor pressure signals.

METHOD

Experimental Setup

The turbocharger used in these experiments is a Garrett GT1752. Its geometrical parameters are given in Table 1. It is suitable for e.g. a turbocharged SI engine with a displacement of 2 liters.

The experiments were carried out on a cold gas stand. The turbine is fed with compressed air, which is heated using an electric heater to avoid the formation of ice at the turbine outlet. The shaft speed is regulated by adjusting the turbine mass flow, using two parallel pneumatic valves located upstream of the electric heater.

The compressor air circuit is shown in Figure 4. The compressor induces air from the test room. Two different inlet geometries, which lead to differences in the compression system stability at high impeller speeds, are used. In the first inlet configuration, the pipe between the funnel and compressor inlet is straight, with a length of 18 inner diameters (D). In the second inlet configuration, a 90° bend

1. This is called DFA1; for a removal of a quadratic trend, the method would be called DFA2, and so on.