

Experimental Transfer Path Contribution Study with the Projected Operational Forces Estimated from the Responses

Hiromichi Tsuji and Kimihiko Nakano
University of Tokyo

ABSTRACT

This paper presents a study of experimental transfer path contribution with the estimation technique of the projected operational force under CVT clutch lockup operational condition. Since transfer path analysis is conducted with respect to the evaluation location, the forces applied onto the substructures are, therefore, also required to estimate the coherent operational forces. In order to estimate the forces, the coherent inertance matrix, which is the projection of the inertance matrix onto the subspace with respect to the evaluation location, is estimated without measuring it directly. The acceleration responses at the connections of the passive substructure are measured by the excitation at the evaluation location with reciprocity. The proposed technique decomposes the acceleration responses into the output and input element vectors on the subspace. With those vectors, the coherent full inertance matrix considering cross coupling effects is constructed. The coherent operational forces at the passive substructure can be estimated by the coherent operational accelerations, with respect to the evaluation location, pre-multiplied by the inverted coherent inertance matrix. As a result, the experimental transfer path contribution with respect to the evaluation location is calculated with these coherent estimated forces and the transfer functions of the passive substructure.

CITATION: Tsuji, H. and Nakano, K., "Experimental Transfer Path Contribution Study with the Projected Operational Forces Estimated from the Responses," *SAE Int. J. Passeng. Cars - Mech. Syst.* 9(2):2016, doi:10.4271/2016-01-1125.

1. INTRODUCTION

Recently, the noise and vibration performance tend to get worse by changing the structure due to the objective of weight reduction improving fuel consumption in the early stage of the product development phase. The change of the dynamic characteristic targets of the components is necessary frequently to be compatible with the performance enhancement to a competitiveness trend. In order to optimize the dynamic characteristics of the components, it is necessary to understand the mechanisms of the noise and vibration generated by the engine excitation and rough road excitation. As experimental analysis technique of this outbreak phenomenon, the identifications of experimental structural mode generating noise [1,2], experimental acoustic mode [3], the operational force on the components, and the most important paths of the vibration [4] have been conducted to optimize the dynamic characteristics of the subsystem components of the vehicle. When the finite element (FE) model accuracy is high enough, it is possible to optimize the dynamic characteristics of the vehicle to improve noise and vibration performance by using vibro-acoustic analysis. However, due to the lack of FE model accuracy, transfer path analysis and force identification are conducted only for the component design while validation of the FE model typical occurs with the prototype vehicle or production car.

In order to set component design requirements and validate FE model accuracy before the design phase for noise and vibration, and rough road excitation, the transfer path analysis (TPA) [5,6] is mainly

conducted using the frequency domain technique based on the theory of separating one structure into substructures. The widely used technique measures the acceleration responses at the connections of the substructures under the operational condition at first. Secondly, the full inertance matrix at all the connections of the substructures is measured by the unit force at each connection for the substructure including the evaluation location. Thirdly, the force is estimated with the inverse matrix of the inertance matrix multiplied by the accelerations of the connections under the operational condition. However, conducting this TPA technique is costly and requires a significant amount of time. Based on this technique, a new technique has been studied to have the same precision level with a shorter time period. The reference [7] showed that this new technique had those benefits with the case studies of a simple 4 beam system FE model and complicated vehicle FE model.

This paper presents a study of experimental transfer path contribution with the projected force estimation technique proposed in the reference [7]. Since the TPA is conducted with respect to the evaluation location, the forces for the substructures are, therefore, required only to estimate the projected forces with respect to the evaluation location. The review of theoretical development is explained and reformulated for experimental data analysis in the following chapter. Then, based on the theoretical validation in the reference [7], the technique is applied to experimental case study of

vibration generated by the engine excitation. In order to verify the accuracy of the proposed technique for experimental case, the classical TPA is also conducted to compare detail results.

2. THEORY

In this chapter, the theory of classical TPA is reviewed as an introduction of the proposed TPA method at first. The next section outlines the methodology of the proposed technique and explains the key theoretical developments. Some explanations and equations are reformulated for experimental data analysis from the reference [7].

2.1. Theory of Classical Transfer Path Analysis

In this section, the theory of classical TPA in the frequency domain is reviewed as an introduction to the proposed TPA method because the newly developed technique is based on classical TPA. In the case study of the automotive structure as shown in Figure 1, the engine and suspension structure including the tires are defined as the active part. The cabin body structure is defined as the passive part. At first, the acceleration cross power spectra (CPS), $\{\ddot{U}^{op}\}$, with respect to the evaluation location, such as floor vibration in vertical direction, are measured at the connections, which are the input locations, of the substructures under driving condition. At the next step, the inertance matrix, $[G]$, is measured as the acceleration responses at all the connections on the separated passive substructure excited by the unit force at each connections. Finally, the operational force spectra, $\{F^{op}\}$, are estimated with the acceleration CPS, $\{\ddot{U}^{op}\}$, pre-multiplied by the inverse matrix of the inertance matrix at all the connections of the substructures. The two equations of the force estimation and the TPA are formulated as follows.

The operational force at the connections of the passive substructure in each frequency discretization “ ω ” is

$$\{F^{op}\}_n = [G]_{mn}^{-1} \cdot \{\ddot{U}^{op}\}_m \quad (1)$$

where the superscript “op” shows the operational data. The subscript “n” is the number of excitation, which is the number of connections multiplied by the number of directions. The subscript “m” is the number of measured responses. $\{F^{op}\}_n$, which is $n \times 1$ vector size, is the force spectra at the connections of the passive substructure. $[G]_{mn}$ is $m \times n$ matrix size and is the inertance matrix at the connections of the passive substructure. $\{\ddot{U}^{op}\}_m$ is $m \times 1$ vector size and is the acceleration CPS of the measured responses, typically at all the connections, for the passive substructure in the driving condition at the proving ground.

The sum of the transfer functions (TF) multiplied by the estimated forces at each connection of the passive substructure in each frequency discretization “ ω ” gives the operational vibration spectra.

$$\{\ddot{X}\}_s = [H]_{sn} \cdot \{F^{op}\}_n \quad (2)$$

where the subscript “s” is the number of the evaluation locations, which in this study are the passenger’s floor locations. $\{\ddot{X}\}_s$ is $s \times 1$ vector size and is the acceleration CPS with respect to the evaluation location, which is at the passenger’s floor location itself. $[H]_{sn}$ is $s \times n$ matrix size and is a TF from the connections of the disassembled passive substructure to the evaluation locations, such as the driver’s floor locations.

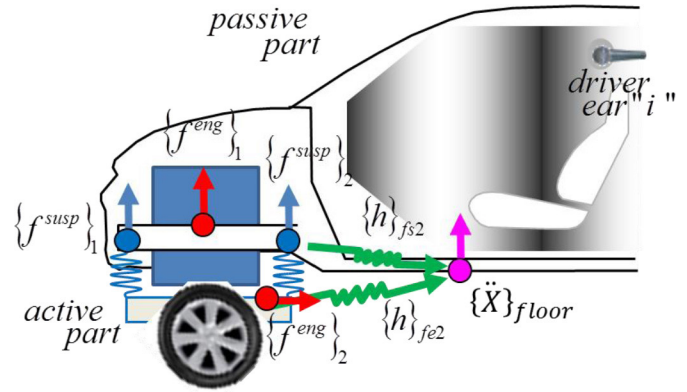


Figure 1. Image of the vehicle structures and the paths

2.2. Theoretical Development for Proposed Technique

In this section, the newly developed technique for TPA is explained with the concept of the subspace. The theoretical background is briefly described as follows. The TPA is conducted with respect to the evaluation location, “i”. The result, $\{\ddot{X}\}_i$, is the inner product of the row vector of $[H]_{in}$ and the column vector of $\{F^{op}\}_n$ from the equation (2). Therefore, only the projected force vector, $\{F^{i-op}\}_n$, on the subspace, $[H]_{in}$, is required for the TPA calculation. The superscript “i” of $\{F^{i-op}\}_n$ shows the projection of the operational force, $\{F^{op}\}_n$, onto the subspace, $[H]_{in}$. The TPA equation (2) with respect to the location “i” in each frequency discretization “ ω ” can be reformulated with the projected force, $\{F^{i-op}\}_n$, as

$$\{\ddot{X}\}_i = [H]_{in} \cdot \{F^{i-op}\}_n \quad (3)$$

where the subscript “i” is the evaluation location, which in this study is the passenger’s floor locations. $\{\ddot{X}\}_i$ is the acceleration cross power spectra at the passenger’s floor location itself. $[H]_{in}$ is the TF from the connections of the passive substructure to the evaluation locations, such as the driver’s floor locations.

In order to estimate the force spectra, $\{F^{op}\}_n$, the inertance matrix, $[G]_{mn}$, is normally measured as shown in the equation (1). However, the proposed technique estimates the projected force spectra vector, $\{F^{i-op}\}_n$, on the subspace, $[H]_{in}$. Therefore, the projected inertance matrix, $[G^i]_{mn}$, on the subspace, $[H]_{in}$, is required for this projected force estimation. The projected force, $\{F^{i-op}\}_n$, is the inner product of the row vector of $[G^i]_{mn}^{-1}$ and the column vector of $\{\ddot{U}^{op}\}_m$. The force estimation equation (1) with respect to the location “i” in each frequency discretization “ ω ” can be reformulated with the projected inertance matrix, $[G^i]_{mn}$, as

$$\{F^{i-op}\}_n = [G^i]_{mn}^{-1} \cdot \{\ddot{U}^{op}\}_m \quad (4)$$

The projection, $[G^i]_{mn}$, of the inertance matrix, $[G]_{mn}$, onto the subspace, $[H]_{in}$, is constructed with the output and input element vectors on the subspace, $[H]_{in}$. Those element vectors are estimated by decomposing the acceleration FRFs measured at the connections of the passive substructure by the reciprocal excitation at the evaluation location “i”.

2.3. Procedure of Proposed Technique

In The methodology of the proposed technique was explained above. Each steps of the methodology from 1) to 8) is explained with the detailed theoretical development as follows.

1. Measure FRFs, $\{h\}_{ii}$ & $\{h\}_{ei}$, by the Reciprocal Excitation

The reciprocal excitation, the shaker excitation, is set at the evaluation location, such as floor vibration. The acceleration FRF, $\{h\}_{ii}$, at the driving point and the other acceleration FRFs, $\{h\}_{ei}$, at the joints of the passive substructures are measured with the reciprocal excitation of the shaker unit force, $\{f\}_i$. These FRFs are measured as the relationship between output and input as follows.

$$\{\ddot{u}\}_i = \{h\}_{ii} \cdot \{f\}_i \quad (5)$$

$$\{\ddot{u}\}_e = \{h\}_{ei} \cdot \{f\}_i \quad (6)$$

From [equation \(5\)](#), the acceleration FRF, $\{\ddot{u}\}_i$, is the inner product of the row vector of $[h]_{ii}$ and the column vector of unit force $\{f\}_i$. In order to explain the concept, the FRFs, $\{h\}_{ii}$ and $\{h\}_{ei}$, are assumed to be the inner product of the output element vector and the input element vector though the theoretical background is explained in the next section. Since [equation \(5\)](#) is the driving point measurement that has the same location for the output and the input, the output element vector and the input element vector become the same element vector. Therefore, there are no phase differences between the output element vector and the input element vector. On the other hand, as presented in the [equation \(6\)](#), the acceleration FRF, $\{\ddot{u}\}_e$, is the inner product of the row vector of $[h]_{ei}$ and the column vector of unit force $\{f\}_i$. Based on this concept, the $[h]_{ei}$ can be decomposed into the two vectors, the output element vector and the input element vector. This $[h]_{ei}$ can be also considered as the subspace created by those element vectors at the output and input locations in each frequency discretization “ ω ”.

2. Decompose FRFs into the Modal Summations, $\Sigma\{h_k\}_{ee}$

For the exact FRF estimation with the proposed technique, all the reciprocal FRFs measured by the reciprocal shaker in the previous section need to be decomposed into the modal summations by experimental modal parameter estimation as follows.

$$\{h(\omega)\}_{ee} = \sum_{k=1}^n \left[\frac{\{\phi_k\} \{\xi_k\}^T}{\sigma_k + j(\omega - \omega_{dk})} + \frac{\{\phi_k^*\} \{\xi_k^*\}^T}{\sigma_k + j(\omega + \omega_{dk})} \right] + \sum_r [Z_r](j\omega) \quad (7)$$

where “k” is mode number, ω_{dk} is damped natural frequency, σ_k is modal damping, ξ_k is modal participation, $\{\phi_k\}$ is mode shape, and $[Z_r]$ is residual term.

As presented in [equation \(7\)](#), the FRFs are decomposed into the first term in the form of modal “k” representation and the second term in the form of residual “r” representation. Each FRF representation of the right hand side in [equation \(7\)](#) is reformulated as [equation \(8\)](#). The representation of [equation \(8\)](#) is kept until the construction of the coherent inertance matrix at [section 6](#).

$$\{h(\omega)\}_{ee} = \sum_{k=1}^{n+r} \{h_k(\omega)\}_{ee} \quad (8)$$

3. Estimate the Projected FRFs, $\Sigma\{h_k^i\}_{ee}$, for Diagonal Term

Since the modal representation of the measured FRFs, $\Sigma\{h_k\}_{ii}$ and $\Sigma\{h_k\}_{ei}$, are excited by the unit force at the same location “i”, those FRFs exist on the subspace, $[H]_{in}$. Therefore, with the superscript, “i”, those FRFs can be expressed as $\Sigma\{h_k^i\}_{ii}$ and $\Sigma\{h_k^i\}_{ei}$. In addition, the phase reference for the inner product of the FRF calculation can be either of the input location and the output location. Therefore, based on the phase reference on the input location “i”, these modal summation FRFs, $\Sigma\{h_k^i\}_{ii}$ and $\Sigma\{h_k^i\}_{ei}$, can be decomposed into the two vectors, which are the output element vector, $\Sigma\{\phi_k^i\}_i$, for $\Sigma\{h_k^i\}_{ii}$ and $\Sigma\{\phi_k^i\}_e$ for $\Sigma\{h_k^i\}_{ei}$, and the input element vector, $\Sigma\{\phi_k^i\}_i$, on the subspace, $[H]_{in}$, in each frequency discretization “ ω ”.

The driving point measurement, the FRF, $\Sigma\{h_k\}_{ii}$, on the subspace, $[H]_{in}$, is expressed as

$$\sum_{k=1}^n \{h_k(\omega)\}_{ii} = \sum_{k=1}^n \{h_k^i(\omega)\}_{ii} = \sum_{k=1}^n (\{\phi_k^i(\omega)\}_i \cdot \{\phi_k^i(\omega)\}_i) \quad (9)$$

where $\{h_k^i(\omega)\}_{ii}$ is the FRF of the modal representation. The subscript “i” is both output location and input location. The subscript “ ω ” is an increment in frequency domain. $\{\phi_k^i(\omega)\}_i$ is both output and input element vectors on the subspace, $[H]_{in}$, at each frequency.

The other FRFs, $\{h_k(\omega)\}_{ei}$, on the subspace, H_{in} , for the case that the output location is different from the input location are also decomposed into two vectors in the same manner of the [equation \(9\)](#).

$$\sum_{k=1}^n \{h_k(\omega)\}_{ei} = \sum_{k=1}^n \{h_k^i(\omega)\}_{ei} = \sum_{k=1}^n (\{\phi_k^i(\omega)\}_e \cdot \{\phi_k^i(\omega)\}_i) \quad (10)$$

where $\{h_k^i(\omega)\}_{ei}$ is the FRF of the modal representation. The subscript “e” is the output location and “i” is the input location. $\{\phi_k^i(\omega)\}_i$ is the element input vector.