

A Method for the Exploration of Hybrid Electric Powertrain Architectures with Two Planetary Gearsets

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ABSTRACT

The goal of this paper is to explore the complete set of single mode hybrid electric powertrain designs that can be generated with one and two planetary gearsets (PGs). Contrary to an automated design exploration approach, an analytically-based manual method is developed to identify all unique design modes for each hybrid electric powertrain architecture (parallel, series, power-split) that can be created with two planetary gearsets, one engine, one vehicle output shaft, two electric machines, and at most two brake clutches. Feasible design modes are generated according to a procedure that provably covers the entire design space. The procedure systematically creates all feasible combinations according to the number of connections between PGs; the number of brake clutches; whether brake clutches, vehicle output shaft, and engine are connected to the nodes that combine two PGs; whether engine and vehicle output shaft are connected to the same PG; whether they are connected to the PG with brake clutches and whether electric machines are collocated with the engine and/or vehicle output shaft. The results of this approach not only show the number of unique design modes for each hybrid electric vehicle architecture and their torque and speed relationships, but also provide a means to validate the results obtained through automated design mode exploration methods.

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INTRODUCTION

Fuel economy regulations in the U.S. will likely increase on average 4.9% per year from 2017 to 2021 and 2.6% per year from 2021 to 2025 [1]. The National Highway Traffic Safety Administration (NHTSA) projects that fuel economy regulations can be met with the application of new technologies to conventional engines and the introduction of a higher number of hybrid electric vehicles (HEVs), plug-in hybrid electric vehicles (PHEVs) and electric vehicles (EVs) into the market [2].

As of the end of 2014, the share of HEVs in the U.S. light-duty vehicle market is 2.8%, with 80% of these sales belonging to HEV architectures realized with planetary gearsets (PGs), similar to those of the Toyota Prius [3]. The remaining 20% of the HEVs belong primarily to simple parallel hybrid architectures, where a conventional transmission is supplemented with an electric machine. The key reason behind the high customer acceptance of PG-based powertrains is the design superiority and flexibility achieved with PGs. The price that comes with these advantages is the complications in the design and analysis of PG-based hybrid electric powertrains, which necessitate further research in this topic.

The feasible design modes with one PG are limited in terms of architecture variety and number of unique modes. Thus, the potential of PG-based architectures can be fully exploited with at least two PGs. However, the design space with multiple PGs consists of

thousands of modes that have to be generated and evaluated. Two approaches that exist in the literature for this analysis can be categorized as bottom-up and top-down methods.

In the bottom-up approach, all combinations that can be achieved with a given set of powertrain components are generated and evaluated [4, 5, 6, 7]. Each study in this type differs from each other in terms of the number of available powertrain components, the evaluation criteria, the methods to evaluate the design candidates, the ease of automation, and the assumptions applied during the design process. The commonality in this approach is automating the task of generating and evaluating all design combinations through software programs due to the very high number of combinations. The software assigns the powertrain components to the PG nodes, eliminates the infeasible and incapable design modes in the background, and gives the final results. The drawbacks in this approach are the inability to assess the characteristics of the feasible design space in terms of design topology and governing speed and torque equations, and the difficulty in validating the results of the computer-based search analytically, which may miss some design modes or may assign a design mode to a wrong category due to an undetected bug in the software or insufficiency in the search algorithm.

In the top-down approach, first a target HEV powertrain architecture is chosen and then its generalized representation is derived through a generic topology or generic set of equations [8, 9]. At the final step, the parameters of this generic model are optimized to meet the design

criteria. The drawbacks of this approach include the inability to cover all architectures in a single generic model or to investigate all possible design topologies in the design space that give the same set of equations.

The content of this paper is not an alternative to the top-down and bottom-up methods. Rather it complements these approaches by presenting the structure of all feasible unique modes for each HEV architecture (parallel, series, and power-split) that can be achieved with two PGs, one engine, one vehicle output shaft, two electric machines, and at most two brake clutches. The results of this study can be used not only to validate the results of top-down and bottom-up methods but also to understand the size and performance limits of the design space for a given set of components.

The paper is organized as follows. In the first section, the basics and modeling procedure of PG-based HEV designs are explained. In the second section, the process that identifies all unique design modes is introduced. The fourth section explicitly shows the number, topology and governing steady-state speed and torque equations of all feasible unique design modes. The fifth section interprets the results of the proposed design process. The sixth section shows how the results of this paper can be used in HEV powertrain design through an example. The paper ends with the concluding remarks.

MODELING PROCEDURE OF PG-BASED HEV MODES

A HEV design mode studied in this paper consists of two planetary gearsets (PGs), one engine, one vehicle output shaft, two electric machines, and brake clutches. The planetary gearset is in the center of a design mode because all the remaining components are connected to its nodes and the characteristics of a design mode is determined by these connections.

A PG shown in Figure 1 is a mechanical device, which has four rotating elements called the sun gear, the planet gears along with their carrier, and the ring gear. It is represented as a lever with three nodes, each of which corresponds to ring, carrier and sun gears [10]. The lengths between ring-carrier nodes and carrier-sun nodes on the lever are taken as 1 and N_R/N_S , where N_R and N_S are the number of teeth on the ring and sun gears. In this paper, α and β represent N_R/N_S ratios of two PGs. The rotational motion of the gears is governed by equation 1, where ω_R , ω_C , and ω_S are the speed of ring, carrier, and sun gears, respectively. As observed from the equation 1, the degrees of freedom in a planetary gearset are two i.e. the speed of any two gears determine the speed of the third. The torque relationship between the gears of a PG is also determined by N_R/N_S as shown in equation 2, where T_R , T_C , and T_S are the torque at the ring, carrier, and sun gears, respectively, and F_1 is the tangential force between these gears.

$$(1 + \alpha)\omega_C - \alpha\omega_R - \omega_S = 0$$

(1)

$$\begin{aligned} T_R &= -\alpha F_1 \\ T_C &= (1 + \alpha)F_1 \\ T_S &= -F_1 \\ T_R + T_C + T_S &= 0 \end{aligned}$$

(2)

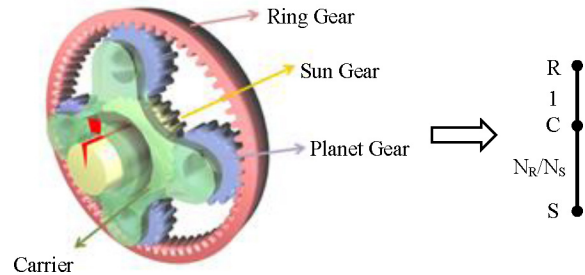


Figure 1. Planetary Gearset [11] and its Symbolic Representation.

In this study, the steady-state speed and torque equations of a design mode will be used to assess its characteristics and to draw conclusions. These equations will be derived leveraging the speed and torque equations of PGs in equations 1-2 and the constraints imposed by the connection of the powertrain components.

The derivation process of the equations will be shown through an example in Figure 2, where the engine (ICE), the vehicle output shaft (Vehicle) and the first electric machine (EM1) are connected to the ring (R1), carrier (C1), and sun (S1) gears of the first PG (PG1), respectively. Similarly, the brake clutch (B1) and the second electric machine (EM2) are connected to the ring (R2) and sun (S2) gears of the second PG (PG2). The connection between PG1 and PG2 is provided through their carrier gears (C1, C2). The torque exerted by the components is equal to the torque at the nodes of the PGs in the static equations.

First, the speed and torque equations of the PGs are written in equation 3, where TL1 is the transferred torque from PG1 to PG2 through their connection. Then, speed constraints due to the brake, and the connection between PGs are expressed in equation 4. Through variable substitutions, the speed and torque relationships between Vehicle, ICE, and electric machines are unveiled in equation 5. This final equation is used to determine the feasibility, uniqueness, and HEV architecture type of each design mode throughout this paper.

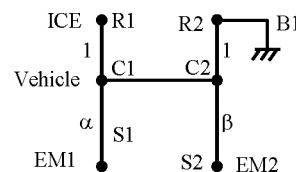


Figure 2. Example Design Mode.

$$\begin{aligned}
(1+\alpha)\omega_{Vehicle} - \alpha\omega_{ICE} - \omega_{EM1} &= 0 \\
(1+\beta)\omega_{C2} - \beta\omega_{R2} - \omega_{EM2} &= 0 \\
T_{ICE} &= -\alpha F_1 \\
T_{Vehicle} + T_{L1} &= (1+\alpha)F_1 \\
T_{EM1} &= -F_1 \\
T_{B1} &= -\beta F_2 \\
-T_{L1} &= (1+\beta)F_2 \\
T_{EM2} &= -F_2
\end{aligned} \tag{3}$$

$$\begin{aligned}
\omega_{R2} &= 0 \\
\omega_{Vehicle} &= \omega_{C2}
\end{aligned} \tag{4}$$

$$\begin{aligned}
\omega_{ICE} &= \frac{(1+\alpha)}{\alpha}\omega_{Vehicle} - \frac{1}{\alpha}\omega_{EM1} \\
T_{Vehicle} &= -\frac{(1+\alpha)}{\alpha}T_{ICE} - (1+\beta)T_{EM2}
\end{aligned} \tag{5}$$

DESIGN IDENTIFICATION PROCESS

One way of identifying the design space is to generate and evaluate all combinations with two PGs, one ICE, one vehicle output shaft, two electric machines, and brake clutches. The combinations can be categorized according to the number of connections between two PGs that varies between one and three. In the case of three connections between two PGs, the speed and torque are the same at each PG node. Thus, the investigation of the combinations in this category can be omitted. In the case of a single connection, any node of the first PG can be connected to any node of the second PG. Hence, 9 connection options are possible. There are five nodes, to which the components (ICE, two EMs, Vehicle, Brake Clutches) can be assigned. The ICE and Vehicle cannot be assigned to the same node but EMs can be assigned to any node. Brake clutches can also be assigned to any node, except the ones for the ICE and Vehicle. As a result, the total number of design modes that needs to be evaluated for the case of a single connection becomes $9 \times 5 \times 4 \times 5 \times 5 \times 2^3 = 36,000$. Conducting a similar analysis for the case of two connections, $18 \times 4 \times 3 \times 4 \times 4 \times 2^2 = 13,824$ design modes must be evaluated. As seen from this analysis, approximately 50,000 design modes should be generated and investigated in order to cover the entire design space. This approach is impractical if the analysis is performed manually. Thus, the feasible design space should be analyzed using a different approach, which will be introduced in this paper.

The first step in the proposed approach is to represent a group of design combinations with a single set of speed and torque equations, and graphical representation. The steady-state speed and torque equations are written in the form of [equation 6](#), where ω_i , $i=1,2,3$ can be the speed of any PG node and Ψ_i , $i=1,2,3$ can take any of the values $(1+\alpha)$, $-\alpha$, and -1 . For example, all 1-PG design mode combinations, where ICE, Vehicle, and an electric machine are connected to the different nodes of the PG can be represented with a single set of equations in [equation 7](#). This set of combinations can also be represented with a single lever diagram in [Figure 3](#). The interpretation of the lever diagram in [Figure 3](#) is different from the one in [Figure 1](#). In [Figure 1](#), the top, middle, and bottom nodes are

always the ring, carrier, and sun gears of the PG, respectively. However, the three nodes in [Figure 3](#) can be any gear of the PG depending on to which node ICE, Vehicle, and EM are connected. If ICE, Vehicle, and EM are connected to the carrier, ring, and sun gears of the PG, Ψ_1 , Ψ_2 , and Ψ_3 would take $1+\alpha$, $-\alpha$, and -1 , respectively.

$$\begin{aligned}
\Psi_1\omega_1 + \Psi_2\omega_2 + \Psi_3\omega_3 &= 0 \\
\Psi_1 + \Psi_2 + \Psi_3 &= 0 \\
T_1 &= \Psi_1 F_1 \\
T_2 &= \Psi_2 F_1 \\
T_3 &= \Psi_3 F_1
\end{aligned} \tag{6}$$

$$\begin{aligned}
\Psi_1\omega_{ICE} + \Psi_2\omega_{Vehicle} + \Psi_3\omega_{EM1} &= 0 \\
T_{ICE} &= \Psi_1 F_1 \\
T_{Vehicle} &= \Psi_2 F_1 \\
T_{EM1} &= \Psi_3 F_1
\end{aligned} \tag{7}$$

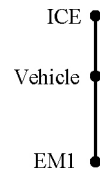


Figure 3. Generic Representation of a Subset of Design Mode Combinations.

Once the design modes are assigned to groups according to the characteristics of their steady-state speed and torque equations, the design analysis can be conducted over these groups instead of the individual design modes. However, these groups should be created methodically in order not to miss any group in the design space. To construct the entire set of design groups, the following rules that take into account all feasible combinations of connections are evaluated in the given order:

1. How many connections are there between two PGs?
2. How many brake clutches exist in the design group?
3. Is any brake clutch assigned to any connection between two PGs?
4. Is any component other than the brake clutches assigned to any connection between two PGs?
5. Is ICE or Vehicle assigned to the connection between two PGs?
6. Are both ICE and Vehicle connected to the nodes of the same PG?
7. Are both ICE and Vehicle connected to the nodes of the PG to which a brake clutch is also assigned?
8. How many ways can EM1 and/or EM2 be collocated with the ICE and Vehicle?

Once the design mode groups are generated, their HEV mode types are determined using the steady-state speed and torque equations that are evaluated according to the following criteria where f_{ij} 's $i=1,2$ and $j=1,2,3$; g_{mn} 's $m=1$ and $n=1,2,3$; and h_{pq} 's $p=1,2$ and $q=1,2,3$ are the coefficients of the speed and torque variables in the equations, and are the functions of $\Psi_1, \dots, \Psi_6$.

Parallel HEV Mode:

$$\begin{aligned}
\omega_{ICE} &= f_{11}\omega_{Vehicle} \\
T_{Vehicle} &= h_{11}T_{ICE} + h_{12}T_{EM1} + h_{13}T_{EM2} \\
f_{11} &\neq 0 \text{ and } h_{11} \neq 0 \text{ and } [h_{12} \neq 0 \text{ or } h_{13} \neq 0]
\end{aligned}
\quad (8)$$

Series HEV Mode:

$$\begin{aligned}
\omega_{ICE} &= f_{12}\omega_{EM1} \\
\omega_{Vehicle} &= f_{23}\omega_{EM2} \\
T_{ICE} &= h_{21}T_{EM1} \\
T_{Vehicle} &= h_{13}T_{EM2} \\
f_{12} &\neq 0 \text{ and } f_{23} \neq 0 \text{ and } h_{21} \neq 0 \text{ and } h_{13} \neq 0
\end{aligned}
\quad (9)$$

Power-Split HEV Mode:

$$\begin{aligned}
\omega_{ICE} &= f_{11}\omega_{Vehicle} + f_{12}\omega_{EM1} + f_{13}\omega_{EM2} \\
f_{11} &\neq 0 \text{ and } [f_{12} \neq 0 \text{ or } f_{13} \neq 0]
\end{aligned}
\quad (10)$$

Power-split modes can be subcategorized according to the following equations in addition to [equation 10](#):

Output-Split:

$$\begin{aligned}
\omega_{ICE} &= f_{22}\omega_{EM1} + f_{23}\omega_{EM2} \\
[f_{22} \neq 0 \text{ and } f_{23} = 0] \text{ or } [f_{22} = 0 \text{ and } f_{23} \neq 0]
\end{aligned}
\quad (11)$$

Input-Split:

$$\begin{aligned}
\omega_{Vehicle} &= g_{12}\omega_{EM1} + g_{13}\omega_{EM2} \\
[g_{12} \neq 0 \text{ and } g_{13} = 0] \text{ or } [g_{12} = 0 \text{ and } g_{13} \neq 0]
\end{aligned}
\quad (12)$$

Compound-Split:

$$\begin{aligned}
\omega_{ICE} &= f_{21}\omega_{Vehicle} + f_{22}\omega_{EM1} + f_{23}\omega_{EM2} \\
f_{21} &\neq 0 \text{ and } [f_{22} \neq 0 \text{ or } f_{23} \neq 0]
\end{aligned}
\quad (13)$$

Special Type Power-Split:

$$T_{Vehicle} = h_{11}T_{ICE} \quad (14)$$

At the last stage of the process, all combinations of $(1+\alpha)$, $-\alpha$, -1 and $(1+\beta)$, $-\beta$, -1 that are assigned to Ψ_1 , Ψ_2 , Ψ_3 and Ψ_4 , Ψ_5 , Ψ_6 are analyzed. The combinations, which make h_{11} coefficient negative in the static torque equations are counted as feasible design modes because $T_{Vehicle}$ should be negative for forward motion and positive T_{ICE} is desired to contribute to forward motion.

IDENTIFIED HEV POWERTRAIN DESIGN MODES

Using the eight rules described in the previous section, the charts in [Appendices A](#) and [B](#) are created. [Appendix A](#) covers the design groups that have one connection between two PGs, whereas [Appendix B](#) consists of design groups with two connections between two PGs. Each item in these charts is going to be analyzed in this section.

Although the focus of this paper is two-PG HEV designs, some design groups in [Appendices A](#) and [B](#) have the same characteristics as a one-PG HEV design. Thus, the identification of HEV powertrain design modes should start first with the one-PG case.

One-PG HEV Design Modes

In a one-PG HEV design, there are three PG nodes. Since ICE and Vehicle are assigned to different nodes, there is only one node that can be assigned to either a brake clutch or an electric machine.

Brake Clutch is Assigned to the Third Node

EM1 and EM2 can be collocated with ICE and Vehicle as seen in [Figure 4](#). After the speed and torque equations in [equation 15](#) are derived, the HEV powertrain type and the number of feasible design modes are determined according to the process described in the previous section.

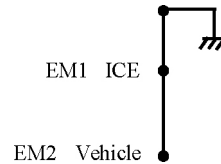


Figure 4. 1-PG Design Group, where Brake is assigned to the first node.

$$\begin{aligned}
\omega_{ICE} &= -\frac{\Psi_3}{\Psi_2}\omega_{Vehicle} \\
T_{Vehicle} &= \frac{\Psi_3}{\Psi_2}(T_{ICE} + T_{EM1}) - T_{EM2}
\end{aligned}
\quad (15)$$

Since the structure of [equation 15](#) is the same as the one in [equation 8](#), this design mode group is a parallel HEV. Ψ_3/Ψ_2 should be negative for the forward motion supported by T_{ICE} . Thus, the number of feasible parallel HEV design modes in this group is four. [Table 1](#) shows these four combinations.

Table 1. Feasible Ψ_2 and Ψ_3 Combinations.

Ψ_2	Ψ_3
$1+\alpha$	$-\alpha$
$-\alpha$	$1+\alpha$
$1+\alpha$	-1
-1	$1+\alpha$

EM is Assigned to the Third Node

In this case, EM2 can be collocated with either ICE or Vehicle as seen in [Figure 5](#). The components in the parentheses in the figure mean that their assignment is optional.

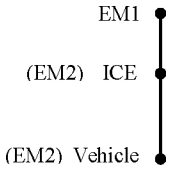


Figure 5. 1-PG Design Group, where EM1 is assigned to the first node.

EM2 on ICE

$$\begin{aligned}\omega_{ICE} &= -\frac{\Psi_3}{\Psi_2} \omega_{Vehicle} - \frac{\Psi_1}{\Psi_2} \omega_{Vehicle} \\ \omega_{ICE} &= \omega_{EM1} \\ T_{Vehicle} &= \frac{\Psi_3}{\Psi_2} (T_{ICE} + T_{EM2})\end{aligned}\quad (16)$$

The structure of speed equations in [equation 16](#) is the same as the one in [equations 10 and 11](#). Thus, this design mode group is an output-split HEV. Since Ψ_3/Ψ_2 should be negative for forward motion, the number of feasible output-split design modes in this group is four.

EM2 on Vehicle

$$\begin{aligned}\omega_{ICE} &= -\frac{\Psi_3}{\Psi_2} \omega_{Vehicle} - \frac{\Psi_1}{\Psi_2} \omega_{Vehicle} \\ \omega_{Vehicle} &= \omega_{EM1} \\ T_{Vehicle} &= \frac{\Psi_3}{\Psi_2} T_{ICE} - T_{EM2}\end{aligned}\quad (17)$$

The structure of speed equations in [equation 17](#) is the same as the one in [equations 10 and 12](#). Thus, this design mode group is an input-split HEV. The number of feasible input-split design modes in this group is four.

As a result of this analysis, four parallel and eight power-split feasible HEV design modes can be generated with one PG.

Two-PG HEV Design Modes

Although the focus of this paper is HEV design modes with two PGs, all HEV design modes with one PG were explored first in the previous subsection since some 2-PG combinations end up to have the functionality of a one-PG mode. In this subsection, the design mode groups that are categorized according to the set of rules described in the previous section are analyzed. The first condition of these rules is the number of connections between two PGs. The next condition is the number of brake clutches in the design mode group. In the following subsections, each design mode group is identified by two numbers and one letter, each of which is separated by underscores. For example, 1_0_A represents the first (A) design mode

group under the category of one connection between PGs (1_) and no brake clutch (0_). These codes are also written under each design mode group in [Appendices A and B](#).

One Connection between Two PGs

1_0_A

In this design mode group, there is not any component assigned to the connection between PGs. Furthermore, ICE and Vehicle are connected to the nodes of the same PG as shown in [Figure 6](#). EM1 and EM2 should be connected to the remaining two nodes of the second PG. Otherwise, this design mode group becomes identical to a 1-PG design.

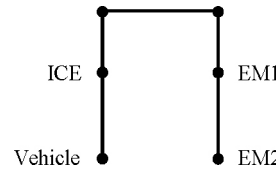


Figure 6. 1_0_A Design Mode Group.

After the manipulation of the steady-state speed and torque equations, [equation 18](#) is obtained, where $\Psi_i, i=1,2,3$ and $\Psi_j, j=4,5,6$ can take any value from the sets $\{1+\alpha, -\alpha, -1\}$ and $\{1+\beta, -\beta, -1\}$, respectively. The structure of [equation 18](#) is the same as the one in [equations 10 and 14](#). Thus, this design mode group is a special type power-split HEV. Since Ψ_3/Ψ_2 should be negative for forward motion, and there is no restriction on the selection of Ψ_4, Ψ_5 , and Ψ_6 , the number of feasible design modes in this group is $4 \times 6=24$.

$$\begin{aligned}\omega_{ICE} &= -\frac{\Psi_3}{\Psi_2} \omega_{Vehicle} + \frac{\Psi_1\Psi_5}{\Psi_2\Psi_4} \omega_{EM1} + \frac{\Psi_1\Psi_6}{\Psi_2\Psi_4} \omega_{EM2} \\ T_{Vehicle} &= \frac{\Psi_3}{\Psi_2} T_{ICE}\end{aligned}\quad (18)$$

1_0_B

In this design mode group, there is not any component assigned to the connection between PGs. Furthermore, ICE and Vehicle are connected to the nodes of the different PGs as shown in [Figure 7](#). EM1 and EM2 should be connected to the remaining two nodes of the two PGs.

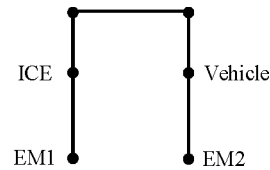


Figure 7. 1_0_B Design Mode Group.

After the manipulation of the steady-state speed and torque equations, [equation 19](#) is obtained. This design mode group is a special power-split HEV. $\Psi_1\Psi_5/\Psi_2\Psi_4$ should be positive for forward motion and there are 20 combinations that can meet this requirement.