

An Unsupervised Machine-Learning Technique for the Definition of a Rule-Based Control Strategy in a Complex HEV

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ABSTRACT

An unsupervised machine-learning technique, aimed at the identification of the optimal rule-based control strategy, has been developed for parallel hybrid electric vehicles that feature a torque-coupling (TC) device, a speed-coupling (SC) device or a dual-mode system, which is able to realize both actions. The approach is based on the preliminary identification of the optimal control strategy, which is carried out by means of a benchmark optimizer, based on the deterministic dynamic programming technique, for different driving scenarios. The optimization is carried out by selecting the optimal values of the control variables (i.e., transmission gear and power flow) in order to minimize fuel consumption, while taking into account several constraints in terms of NO_x emissions, battery state of charge and battery life consumption. The results of the benchmark optimizer are then processed with the aim of extracting a set of optimal rule-based control strategies, which can be implemented onboard in real-time. The input variables of the rule-based strategy are the vehicle power demand, the vehicle speed and the state of charge of the battery. The method for the rule extraction can be summarized as follows. A clustering algorithm discretizes the input domain (in terms of vehicle power demand, vehicle speed and state of charge of the battery) into a mesh of clusters. The generic rule associated to a specific cluster (i.e., the combination of gear and power flow that has to be actuated) is identified by searching for the control strategy most frequently adopted by the benchmark optimizer within the considered cluster. The optimal mesh of clusters is generated using a genetic algorithm technique. Optimal sets of rules are identified for different driving scenarios. These strategies can then be implemented on-board, provided the mission features are known at the beginning of the trip. The main advantage of the proposed technique is that the definition of the rule-based strategy is derived from a machine learning method and is not based on heuristic techniques.

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INTRODUCTION

Powertrain electrification has increased over the last few years, due to the increasing need to improve the sustainability of the road transport sector, in terms of both energetic and pollutant emission aspects. So far, hybrid electric vehicles (HEV) have been considered as one of the most promising technologies. HEVs are provided with two energy sources, i.e., a battery and a fuel tank, and they are equipped with a conventional thermal engine and one or more electric machines. A significant advantage of HEVs, compared to battery electric vehicles (BEV), is related to their capability of increasing the driving range, as a consequence of the presence of the thermal engine. However, the driving range of HEVs running in pure electric mode is significantly lower than that of BEVs, due to their smaller battery packs. HEVs lead to a reduction in fuel consumption (FC) and CO₂ emissions, compared to conventional vehicles, mainly due to the reduction in engine size, to the kinetic energy recovery through regenerative braking, to the implementation of the Stop-Start mode and to the possibility of optimizing the power flow from the engine and electric machines [1]. Numerous studies have demonstrated the potential of the hybrid technology to reduce CO₂ emissions in developing countries [2, 3], which are expected to contribute significantly to

greenhouse gas emissions from the transport sector in the near future. A recent study has also shown that the life cycles of CO₂ emissions of modern HEVs are shorter than those of conventional vehicles [4].

An optimal control strategy to manage the thermal and electric machines is required to fully exploit the potential of hybrid vehicles in order to improve fuel economy and reduce pollutant emissions [5]. The control strategy consists of an algorithm that regulates the operation of the powertrain. It takes input data, such as the vehicle speed, the vehicle acceleration and the road grade, and makes decisions on turning certain components on/off or increasing/decreasing their power output [6]. The control strategy optimization tools that have been developed in recent years can roughly be classified in three categories [7]. The first one includes heuristic control techniques, such as control rules, fuzzy logic and neural networks. These algorithms usually provide an approximate solution to the given optimization problem, but they offer the advantage of requiring a low computational effort. An idea that is commonly employed in heuristic algorithms is the concept of loadleveling, which is based on the shifting of the engine operating conditions to a high efficiency map region and on the utilization of the battery as a load-leveling device, with reference to the model inputs (e.g., the

vehicle speed, state of charge of the battery, driver commands). The investigation of fuzzy logic controllers for the design, development and energy management of HEVs has been reported in several studies [8, 9, 10, 11].

The second category of control strategy optimization tools is constituted by static optimization methods. The main concept of these methods involves the identification of the optimal power split between the engine and the electric machine, on the basis of the instantaneous optimization of an equivalent fuel consumption.

The third category of control strategy optimization tools is constituted by dynamic optimization methods, which are based on the modeling of the system dynamics over a defined vehicle mission, and on the minimization of a multi-objective function, which usually includes the cumulated fuel consumption over the entire mission, according to a global optimization approach [12]. The deterministic dynamic programming approach is a well-known dynamic optimization technique that has been widely used in many applications, including those pertaining to the identification of the optimal control strategy in hybrid vehicles. This methodology, which was first presented by Bellman [13, 14], is a multi-stage decision-making process, based on the principle of optimality, which involves a dynamic system, a cost function and control/state grids. Applications based on this technique have been investigated in several papers [14, 15, 16, 17, 18, 19].

The main drawback of static optimization methods is that the optimization is performed at each time instant and not globally over a defined vehicle mission. However, the required computational time is limited, and these methods can be suitable for on-board applications. On the other hand, algorithms resulting from dynamic optimization approaches are more accurate under transient conditions, but are also computationally more demanding.

Contribution of the Present Study

The present paper is mainly focused on the development of an unsupervised machine-learning method aimed at the identification of an optimal rule-based control strategy for HEVs. A rule-based control strategy in fact has the advantage of being easily implementable in the vehicle control unit for the real-time optimization of the control strategy; however, traditional rule-based controllers are generally developed and tuned on the basis of engineering experience or heuristic approaches, which might be lacking in performance over a broad range of applications. This study is therefore focused on adopting machine-learning techniques for the extraction of the rules that need to be implemented in the controller.

With reference to the choice of the HEV architecture investigated in this paper, to the best of the authors' knowledge, there is a lack of studies in the literature on dual-mode parallel hybrid electric vehicles, which feature both torque coupling and speed coupling systems, equipped with a compression ignition engine. The hybrid vehicles have been equipped with a diesel engine and an electric machine, which are coupled either via a gearbox (torque coupling) or via a planetary-gear set (speed coupling). The two mechanical connections

enable the engine operating point to be shifted either horizontally (speed) or vertically (torque) in the engine map during hybrid operation, and this allows more degrees of freedom to be included for the control strategy optimization. The performance of this vehicle layout has been compared with two parallel configurations that feature either the torque coupling or speed coupling functions separately.

Plug-in HEVs have the potential of further reducing fuel consumption, as well as pollutant and CO₂ emissions, compared with non-plug-in HEVs [20, 21, 22]. However, the incremental battery cost of plug-in HEVs might not be counterbalanced by the fuel cost savings, as shown in [23]. A non-plug-in HEV has therefore been analyzed in this paper. Two different engines have been considered, in order to investigate the impact of the degree of hybridization on such a vehicle architecture.

In general, the identification of an optimal control strategy in hybrid vehicles is of fundamental importance to exploit their potential in order to reduce fuel consumption and pollutant emissions. The control variables that have been selected for this study are the power flow, which is related to the management of the traction power between the thermal and the electric machine, and the gear number of the transmission; the control strategy optimization has the aim of minimizing an objective function, which takes into account the cumulated fuel consumption, the NO_x emissions and the battery aging effect. NO_x emissions are in fact a major issue in diesel engines, and taking into account battery aging is crucial in order to avoid too aggressive strategies that could lead to an excessive battery depletion, with a consequent increase in the operating costs of the HEV related to the battery replacement. A zero-dimensional model of the vehicle has been developed in the Matlab environment [24] in order to evaluate the evolution of the fuel consumption and of the state of charge (SOC) of the battery, as a function of the vehicle velocity, of the road slope and of the control variables. A global optimizer, which is based on the deterministic dynamic programming theory, has been used to identify the benchmark solution of the optimal control strategy. The outcomes of the proposed rule-based controller, which is based on machine-learning tuning, have been compared with the benchmark optimizer.

This method can be summarized as follows. First, for a given vehicle mission, the optimal control strategy is identified using the benchmark optimizer. A 3D input domain, constituted by the vehicle velocity, vehicle acceleration and state of charge of the battery, is then generated and discretized, so that a grid of input clusters (whose geometrical shape is of parallelepiped type) is obtained. The most frequent action taken by the benchmark optimizer is then selected for each cluster and stored as the rule of the controller for that specific cluster. The optimal discretization grid for the 3D input domain is selected using a genetic algorithm technique. The proposed tool has therefore been referred to as CORE (clustering optimization rule extraction). The resulting rule-based controller can easily be implemented in the vehicle control unit, in order to perform an onboard optimization of the control strategy.

A previously developed mathematical technique has been used in this study in order to decrease the computational time of the benchmark optimizer and of the rule extraction process. In short, the control variables of the problem were discretized, and the values of the main model variables (i.e., instantaneous fuel consumption, battery electric power, ...) were evaluated for all of the possible combinations of the control variables and for each time node, and stored in a separate matrix (referred to as 'configuration matrix') for each variable, before the optimization process was started. In this way, the optimizers are able to directly read the actual values of the relevant variables from the preprocessed data, i.e. the configuration matrices, instead of having to calculate them iteratively during the optimization stage. This approach has led to a great computational time saving. The simulations were carried out over a set of driving cycles that includes the New European Driving Cycle (NEDC), three Artemis Driving Cycles and an internally-developed test cycle (TC4).

POWERTRAIN DESIGN

The layout of the hybrid vehicle considered in the present study is reported in Fig. 1; this architecture is here referred to as Dual-Mode Vehicle (DMV). A planetary gear set (PG) and a gearbox (GB) have been installed as speed-coupling and torque-coupling devices, respectively. The engine (E) is connected to the first shaft of the GB and to the PG ring, the electric machine (EM) is connected to the second GB shaft and to the PG sun, the driveline is connected to the PG carrier. Two different clutches have been inserted in order to enable either the speed-coupling mode, if the clutch C1 is engaged and clutch C2 is released, or the torque-coupling mode otherwise. The driveline is depicted in yellow and consists of the transmission (TR) and of the final drive (FD). The battery is connected to the electric machine through the inverter (Inv). This architecture features both torque coupling and speed coupling systems.

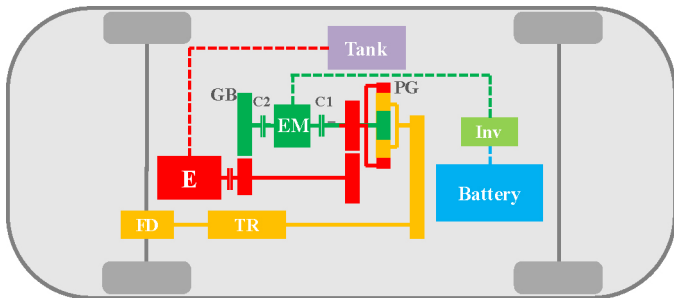


Figure 1. Scheme of the Dual-Mode Vehicle.

In addition to the layout shown in Fig. 1, two additional vehicle architectures have been considered, i.e., the torque-coupling vehicle (TCV) and the speed-coupling vehicle (SCV). They are particular cases of the DMV, in which only the gearbox or the planetary-gear set are present, respectively.

VEHICLE MODEL

Components

Engine

Two different engines have been employed in this study. The first engine, which is here referred to as engine D1, is a 1.7 L Euro 5 GM prototype diesel engine that is capable of providing a maximum power of 97 kW at 3700 rpm and a maximum torque of 304 Nm at 2400 rpm. The second engine, which is here referred to as engine D2, is a 1.3 L Euro 5 GM prototype diesel engine that is capable of providing a maximum power of 70 kW at 4000 rpm and a maximum torque of 206 Nm at 1800 rpm. The adoption of engines that feature different power outputs is of fundamental importance in order to investigate the impact of the degree of hybridization on the performance of the hybrid vehicle.

The performance of the engine was modeled using experimentally-derived look-up tables. The mass flow rate of both fuel and NO_x emissions were evaluated by interpolating a 2D map, which is a function of the engine power and speed. The corresponding CO₂ emissions have instead been linearly determined as follows:

$$\dot{m}_{\text{CO}_2} = 2.65 \cdot \frac{\dot{m}_{\text{fc}}}{\rho} \quad (1)$$

where ρ is the fuel density, $\dot{m}_{\text{fc}}$ is the fuel mass flow rate and 2.65 represents a conversion factor. The latter was obtained considering a combustion reaction of diesel oil with air, such as the one presented in [25], and taking into account an average composition of the fuel from literature data [26].

Electric Machine

The electric machine model simulates the power conversion from the electric to the mechanical form, and vice-versa, taking into account the energy losses by means of efficiency maps, which are functions of the machine power and speed. The electric machines that have been considered in the present study belong to the brushless permanent magnet electric motor/generator family, and all the data were taken from the official website of UQM Technologies [27].

Battery

Lithium-ion batteries have been adopted in this study as the secondary power storage device. The battery model is represented by an equivalent resistance circuit, in which the resistance and the open-circuit voltage of the battery are SOC-dependent. This has allowed a lumped representation of more complex chemical processes to be realized. The battery temperature was assumed to be constant and the temperature effect was therefore disregarded. The SOC represents the electrical status of the battery and depends on the equivalent battery capacity C_{bat} and on the flowing current I_{bat} , as follows:

$$\text{SOC} = \text{SOC}_0 - \int \frac{I_{\text{bat}}}{C_{\text{bat}}} \cdot dt$$

The battery SOC has been limited within the [0.4-0.8] range, while the maximum current that may be transferred through a cell was established as 120A.

A phenomenological damage accumulation model, based on the concept of “accumulated Ah-throughput” (i.e. the total amount of electrical charge, in both charging and discharging modes, which can flow to or from the battery before it reaches the end of life) has been adopted to estimate the depletion of the battery life. The parameter λ , which represents the effective Ah-throughput, can be computed as follows:

$$\lambda = \int \sigma \cdot |I_{bat}| \cdot dt \quad (3)$$

This parameter represents the amount of charge that would have to be exchanged when a nominal cycle is adopted in order to have the same aging effect as the actual cycle the battery has undergone. The severity factor σ represents the relative aging effect, with respect to the nominal cycle, and it is higher than 1 for severe aging conditions (i.e. which would lead to a shorter life). The severity factor is estimated as follows:

$$\sigma = 1.5 \cdot \left(\frac{I_{bat}}{C_{bat}} \right)^2 + 1 \quad (4)$$

The residual battery life λ_r , which represents the fraction of charge that could still flow through the battery before the end of its life, is determined as follows:

$$\lambda_r = 1 - \frac{\lambda}{\Lambda} \quad (5)$$

where Λ is the battery life, which has here been set to 20000 Ah. The end of life condition is defined as the instant at which $\lambda_r = 0$.

Torque-Coupling and Speed-Coupling Devices

A mechanical torque-coupling device has been employed to couple the torque that is obtained from two different power sources (i.e., the engine and the electric machine) and to provide the resulting torque to the output shaft. A two-shaft type gearbox has been used in this study, as shown in Fig. 2 (top). The speed ratio is defined as $\tau_{tc} = \frac{z_1}{z_2} = \frac{R_1}{R_2}$, where z_1 and z_2 are the number of teeth of the corresponding gear, while R_1 and R_2 are the radii of the corresponding gear. The speed correlation is written as follows:

$$\omega_{out} = \omega_{in,1} = \frac{\omega_{in,2}}{\tau_{tc}} \quad (6)$$

With reference to the free body diagram, the torque correlation between the shafts is obtained as follows (if losses are ignored):

$$T_{out} = T_{in,1} + F_c \cdot R_1 = T_{in,1} + \frac{T_{in,2}}{R_2} \cdot R_1 = T_{in,1} + T_{in,2} \cdot \tau_{tc} \quad (7)$$

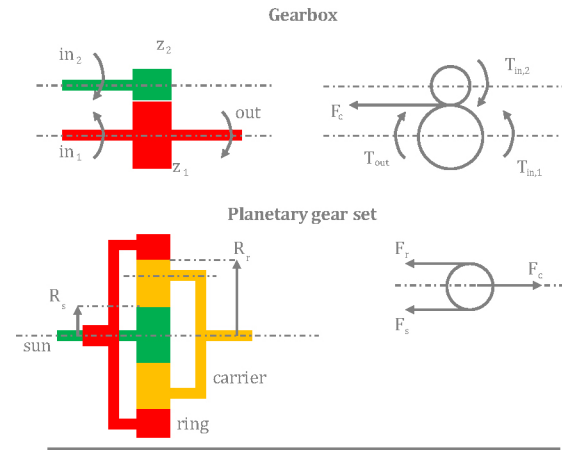


Figure 2. Scheme of the torque-coupling device (top-left) and its free diagram scheme (top-right); scheme of the speed-coupling device (bottom-left) and its free diagram scheme (bottom-right).

A mechanical speed-coupling device has instead been employed to couple the speeds of two different power sources that enter the device separately (through two distinct shafts) and to provide the resulting torque to the output shaft. A planetary gear unit, which mechanically connects the power derived from all the power sources, has been used in this study. This device consists of three rotating nodes, namely a sun gear, a ring gear, and a carrier gear, as shown in Fig. 2 (bottom). These nodes are linked by a few small pinion gears. The speed ratio is defined as $\tau_{sc} = \frac{R_r}{R_s}$, where R_s and R_r are the radii of the sun gear and the ring gear, respectively. With reference to the Willis method, the speed correlation between the ring gear ω_r , the sun gear ω_s the carrier gear ω_c is obtained as follows:

$$\omega_s + \tau_{sc} \cdot \omega_r = (1 + \tau_{sc}) \cdot \omega_c \quad (8)$$

With reference to the free body diagram, the torque correlations are then determined taking into account the geometry of the device:

$$T_s = \frac{T_r}{\tau_{sc}} = \frac{T_c}{1 + \tau_{sc}} \quad (9)$$

where T_s , T_c , T_r are the torque values of the sun gear shaft, the carrier shaft and the ring gear shaft, respectively.

Transmission

Table 1. Transmission speed ratio specifications.

GN	1	2	3	4	5	6
$\tau [-]$	4.17	2.13	1.32	0.95	0.75	0.62

The driveline consists of a 6-speed gearbox (which will be referred to as “transmission” in this study) and of the front final drive. The power losses of the transmission have been estimated by means of an efficiency map, which is a function of the output shaft speed and torque and of the selected gear number. The transmission inertia has been also taken into account. The final drive has instead been

modeled as a torque multiplier, while power losses and inertia contributions have been neglected. The transmission speed ratio values are reported in [Table 1](#) for each gear.

Determination of the Power and Speed of the Engine and Electric Machine

The total vehicle power demand P_v at the wheel level is the sum of several contributions: the rolling resistance, the grade resistance, the drag resistance and the inertia. The total power demand of the vehicle can be expressed as follows:

$$P_v = \left(m_v \cdot g \cdot r_v \cdot \cos(\alpha_r) + m_v \cdot g \cdot \sin(\alpha_r) + \frac{1}{2} \rho_{\text{air}} \cdot c_x \cdot A_v \cdot V_v^2 + \left(m_v + \frac{I_{\text{wh}}}{R_{\text{wh}}} \right) \cdot \dot{V}_v \right) \cdot V_v \quad (10)$$

where m_v is the vehicle mass, g is the gravitational acceleration, r_v is the rolling resistance coefficient of the vehicle, α_r is the slope of the road, V_v is the vehicle velocity, A_v is the front area of the vehicle, ρ_{air} is the air density, c_x is the aerodynamic drag coefficient, R_{wh} is the dynamic wheel radius and I_{wh} is the wheel inertia.

The vehicle working condition may be either of a traction type, if the power is positive, or of a braking type, if the power is negative. The power at the final drive level is then determined as follows:

$$P_{\text{fd}} = \begin{cases} P_v, & \text{traction} \\ \gamma_{\text{fr}} \cdot (1 - \gamma_{\text{br}}) \cdot P_v, & \text{braking} \end{cases} \quad (11)$$

where the factor γ_{fr} represents the power split between the front and the rear wheels of the vehicle during the braking phase, while γ_{br} represents the power share that has to be managed by the mechanical frictional brakes during braking. The γ_{fr} value has been kept constant and equal to 0.75 for all of the simulations, while the value of γ_{br} has been set equal to 0.1.

The power required at the front powertrain level is obtained as follows:

$$P_f = (P_{\text{fd}} + P_{\text{tr,in}}) \cdot \eta_{\text{tr}}^k + P_{\text{e,in}} + P_{\text{em,in}} \quad (12)$$

where $P_{\text{tr,in}}$, $P_{\text{e,in}}$ and $P_{\text{em,in}}$ are the inertial power of the transmission, of the engine and of the electric machine, respectively, η_{tr} is the transmission efficiency, and exponent k is equal to 1, if the vehicle is braking, or equal to -1 otherwise.

The angular velocity ω_f at the front powertrain level is a function of the control variable τ as follows:

$$\omega_f = \tau \cdot \tau_{\text{fd}} \cdot \frac{V_v}{R_{\text{wh}}} \quad (13)$$

where τ_{fd} is the speed ratio of the final drive.

The formulation used to determine the power of the engine P_e and of the electric machine P_{em} is obtained considering the value of the sub-control variable α :

$$P_e = (1 - \alpha) \cdot P_f \quad (14)$$

$$P_{\text{em}} = \alpha \cdot P_f \quad (15)$$

As far as the formulation of the speed of the engine and of the electric machine is concerned, two different modes have to be considered.

Mode 1 - Torque Coupling

With reference to [Eq. \(8\)](#), the engine speed is obtained if the sun shaft is blocked:

$$\omega_{e,t} = \frac{1 + \tau_{\text{sc}}}{\tau_{\text{sc}}} \cdot \omega_f \quad (16)$$

where $\omega_{e,t}$ and ω_f are the speed of the engine and of the front powertrain, while τ_{sc} is the PG speed ratio. The electric machine is now connected to the second GB shaft, and its speed can be defined as follows:

$$\omega_{\text{em},t} = \tau_{\text{tc}} \frac{1 + \tau_{\text{sc}}}{\tau_{\text{sc}}} \cdot \omega_f \quad (17)$$

where $\omega_{\text{em},t}$ is the electric machine speed and τ_{tc} is the GB speed ratio.

Mode 2 - Speed Coupling

With reference to [Eq. \(9\)](#), the engine power can be expressed as follows:

$$P_e = (1 - \alpha) \cdot P_f = (1 - \alpha) \cdot T_f \cdot \omega_f = (1 - \alpha) \cdot \frac{1 + \tau_{\text{sc}}}{\tau_{\text{sc}}} \cdot T_e \cdot \omega_f = T_e \cdot \omega_e \quad (18)$$

where T_e and T_f are the torque at the engine and at the front powertrain level, respectively, while ω_e and ω_f are the speed of the engine and of the front powertrain and τ_{sc} is the PG speed ratio. The engine speed is therefore obtained as:

$$\omega_{e,s} = (1 - \alpha) \cdot \frac{1 + \tau_{\text{sc}}}{\tau_{\text{sc}}} \cdot \omega_f \quad (19)$$

In a similar way, the power of the electric machine can be written as follows:

$$P_{\text{em}} = \alpha \cdot P_f = \alpha \cdot T_f \cdot \omega_f = \alpha \cdot (1 + \tau_{\text{sc}}) \cdot T_{\text{em}} \cdot \omega_f = T_{\text{em}} \cdot \omega_{\text{em}} \quad (20)$$

where T_{em} , ω_{em} are the torque and the speed of the electric machine, whose speed is calculated as follows:

$$\omega_{em,s} = \alpha \cdot (1 + \tau_{sc}) \cdot \omega_f \quad (21)$$

Determination of the Maximum Power Curve of the DMV

The maximum power curve of the DMV, as a function of the vehicle speed, can be derived as follows. First, it should be noted that the maximum power can be obtained either in torque-coupling mode or in speed-coupling mode. The maximum power that can be delivered in torque-coupling mode is evaluated as follows:

$$P_{pt,tc} = P_{em,max}(\omega_{em}) + P_{e,max}(\omega_e) \quad (22)$$

where the values of the speed of the electric machine and of the engine are obtained from Eq. (17) and Eq. (16), respectively. The maximum power which can be delivered in speed-coupling mode is obtained by selecting the engine speed ω_e^* that leads to the maximum power value of $P_{pt,sc}$ for a specific combination of vehicle velocity V_v and of gear number GN, as follows:

$$P_{pt,sc} = P_{em,max}(\omega_{em}) + P_{e,max}(\omega_e^*), \text{ if } \omega_{em} \geq 0 \quad (23)$$

where the electric machine speed stems from Eq. (8). Finally, the GN which maximizes the power at a given vehicle velocity is selected, so that:

$$P_{pt,max} = \max(P_{pt,tc}, P_{pt,sc}) \quad (24)$$

The TC and SC Vehicles

The model equations of the TCV can be obtained from the DMV model if the speed ratio $\tau_{sc} \rightarrow \infty$. The model equations of the SCV can instead be found from those of the DMV, if the speed ratio τ_{tc} is set to 1.

Control/State Variables and their Discretization

The gear number GN and the power flow PF have been chosen as the control variables in this study. The GN variable determines the speed ratio of the transmission, τ (see Table 1). The GN domain is defined by the $S_{gn} = \{1, 2, \dots, 6\}$ set, since a 6-speed transmission has been adopted on the front axle. The PF variable is related to the management of the power of the engine and the electric machine. In particular, it controls the sub-control variable α , which quantifies the power contribution provided by the battery through the front axle, with respect to the front vehicle power demand (see Eqs. (14-15)). The PF domain is defined by the $S_{pf} = \{1, 2, \dots, N_{pf}\}$ set, where N_{pf} stands for the number of different power flow types. The default size of S_{pf} has been set equal to 7 for the dual-mode vehicle.

All the different values of the PF control variable are listed in Table 2, where **pe** stands for pure electric mode, **pt** for pure thermal mode, **ps** for power split mode and **bc** for battery charge mode, while **tc** stands for torque-coupling and **sc** for speed-coupling.

Two different pure-electric modes are possible for the dual-mode vehicle: the electric machine can in fact drive the vehicle either in torque-coupling mode (**pe.tc**) or in speed-coupling mode (**pe.sc**). Only one pure-thermal mode is possible for this vehicle layout, where both the electric machine clutches are disengaged and the engine is connected to the driveline in the torque-coupling mode.

Table 2. Working modes of the Dual-mode vehicle.

	PF	α	ω_e	ω_{em}	P_e	P_{em}
1	pe.tc	1	0	$\omega_{em,t}$	0	P_f
2	pe.sc	1	0	$\omega_{em,s}$	0	P_f
3	pt.tc	0	$\omega_{e,t}$	0	P_f	0
4	ps.tc	$\frac{1}{2}$	$\omega_{e,t}$	$\omega_{em,t}$	P_e	P_{em}
5	bc.tc	$-\frac{1}{2}$	$\omega_{e,t}$	$\omega_{em,t}$	P_e	P_{em}
6	ps.sc	$\frac{1}{2}$	$\omega_{e,s}$	$\omega_{em,s}$	P_e	P_{em}
7	bc.sc	$-\frac{1}{2}$	$\omega_{e,s}$	$\omega_{em,s}$	P_e	P_{em}

Finally, the electric machine can work either to assist the engine or to charge the battery, and it can be connected either in speed-coupling mode or in torque-coupling mode. The power of each component is identified according to the value of the sub-control variable α (see Eqs. (14-15)). The speeds of the engine and of the electric machine are obtained by means of Eq. (19) and Eq. (21), respectively, if the speed-coupling mode is enabled, otherwise by means of Eq. (16) and Eq. (17), respectively, if the torque-coupling mode is selected.

With reference to the state variables, the state of the engine, which can be equal to 0 if the engine is off, or equal to 1 otherwise (the related discrete set is therefore $S_{es} = \{0, 1\}$), was chosen as the first variable. The battery SOC was chosen as the second state variable. It can vary in the $S_{soc} = \{0.4, 0.4001, \dots, 0.8\}$ discrete set, since 4000 discretization steps have been considered. While the evolution of the battery SOC depends on the time-history of the two control variables, the engine state is obtained directly as a function of the instantaneous value of the power flow.

Time Grid Discretization

The time domain has been discretized uniformly with a time step of 3 s in order to create the corresponding time grid. To this end, the vehicle model equations were written considering separate formulations for the left and right sides of each time node, in order to reduce the errors in the evaluation of the cumulated quantities (such as FC and SOC). The introduced procedure leads to an interval-grid domain, which replaces a node-grid approach. This approach was introduced in [28] and has resulted in a significant reduction in the simulation time and memory requirement, as a refined time domain discretization is not required.