

Reanalysis of Linear Dynamic Systems using Modified Combined Approximations with Frequency Shifts

Syed F. Haider
Oakland University

Zissimos Mourelatos
Oakland University

ABSTRACT

Weight reduction is very important in automotive design because of stringent demand on fuel economy. Structural optimization of dynamic systems using finite element (FE) analysis plays an important role in reducing weight while simultaneously delivering a product that meets all functional requirements for durability, crash and NVH. With advancing computer technology, the demand for solving large FE models has grown. Optimization is however costly due to repeated full-order analyses. Reanalysis methods can be used in structural vibrations to reduce the analysis cost from repeated eigenvalue analyses for both deterministic and probabilistic problems. Several reanalysis techniques have been introduced over the years including Parametric Reduced Order Modeling (PROM), Combined Approximations (CA) and the Epsilon algorithm, among others. It has been shown that the Modified Combined Approximations (MCA) method - an improvement over CA - is the most efficient reanalysis technique for problems with a large number of degrees of freedom. This paper proposes an improvement of the MCA method using frequency shifts. Numerical examples are presented and results are compared with existing methods.

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INTRODUCTION

Designs with minimum weight are sought while meeting all functional requirements in any industry. This is achieved using optimization. Finite element analysis is also a well-established numerical tool in the industry. It is frequently used for low frequency Noise, Vibration and Harshness (NVH) analysis. Reanalysis methods are used to estimate the natural frequencies and mode shapes of a modified structure without using a full eigenvalue analysis. This reduces the computational effort of an optimization method for NVH by eliminating repeated eigenvalue analyses. As real-life designs are becoming large and complex, the demand for efficient and accurate reanalysis methods has increased.

Reanalysis methods are based on local and global approximations. Local approximations are efficient but are effective only for small structural perturbations. On the other hand, global approximations are computationally expensive and preferable for larger structural perturbations. Rayleigh-Ritz is a popular local approximation method suited for small changes. Several reviews have been published on reanalysis methods [1, 2, 3, 4, 5, 6]. The first application of reanalysis methods was in linear static problems in early 1970s [1, 2]. Balmes [7, 8] proposed the Parametric Reduced Order Model (PROM) method by expanding the Rayleigh-Ritz method. He used the mode

shapes from a few selected design configurations to predict the response at any design point throughout the design domain. Although, PROM is a local approximation method, it can handle larger structural changes because it uses information from multiple design points. The Combined Approximations (CA) method developed by Kirsch [9, 10, 11, 12, 13] combines the strength of both local and global approximations and is shown to be accurate even for large structural modifications. It uses a combination of binomial series (local) approximation (Neumann expansion) and a global reduced basis approximation. The method was originally applied to linear static problems and later extended to linear dynamics problems [12]. An extended CA method was developed in [14] for large perturbations using the Rayleigh quotient. The use of the Epsilon algorithm for eigen-problem reanalysis using a Neumann series expansion was also demonstrated in [14].

A Modified Combined Approximation (MCA) method was introduced in [15, 16] for reanalysis of dynamic problems with many dominant modes. Recently, a Frequency Shift Combined Approximation (FSCA) method was proposed in [17] providing an improvement of the CA method based on frequency shifting. It was demonstrated that the use of the Epsilon algorithm [18] accelerates convergence. The authors in [17] claim that frequency shifting improves the accuracy and efficiency of the MCA method.

Reanalysis methods use a basis of eigenvectors from the baseline design, augmented sometimes by eigenvectors from other design points as in PROM [19], to estimate the eigenvalues and eigenvectors of a modified design without performing a new eigenvalue analysis. However, all basis eigenvectors are not usually linearly independent. To avoid numerical errors from this dependency, Gram-Schmidt orthogonalization must be performed. In general, this is very costly especially when the basis is large (many basis vectors) and/or the model size is large.

In this paper, we propose an improvement of the MCA method by introducing frequency shifting in order to eliminate the need for Gram Schmidt orthogonalization improving therefore, the efficiency of the MCA method. We present examples and demonstrate the superior efficiency and accuracy of the proposed method.

REVIEW OF EXISTING REANALYSIS METHODS

In this section, we briefly review the most popular reanalysis methods PROM, CA, MCA and FSCA.

Parametric Reduced Order Modeling (PROM)

The PROM method approximates the mode shapes of a new design in the subspace spanned by the dominant mode shapes of some representative designs, which are selected so as the formed basis captures the dynamic characteristics in each dimension of the parameter space. Balmes [7, 8] suggested that each of these representative designs should be at the mid-range of each design parameter in the parameter space.

For a structure with m design variables, Zhang [19] suggested that the representative designs include a *baseline* design for which all parameters are at their lower limits plus m designs obtained by perturbing the design variables from their lower limits to their upper limits, one at a time. The points representing these designs in the space of the design parameters are called *corner points*. This resulted in a more accurate PROM algorithm compared with the original Balmes [7, 8] algorithm.

In PROM, the mode shapes of a new design are approximated in the space of the mode shapes of the corner points as

$$\Phi \approx \tilde{\Phi}_p = P\Theta \quad (1)$$

where the modal matrix P contains the basis vectors as in Equation (2)

$$P = [\Phi_0 \quad \Phi_1 \quad \dots \quad \Phi_m] \quad (2)$$

and Θ contains the coefficients of these vectors. The columns of P are the dominant mode shapes of the above $(m + 1)$ designs. Φ_0 is the modal matrix composed of the dominant mode shapes of the baseline design, and Φ_i is the modal matrix of the i^{th} corner point. The mode shapes of the new design satisfy the following eigenvalue problem,

$$K\tilde{\Phi}_p = \Lambda M\tilde{\Phi}_p \Leftrightarrow KP\Theta = \Lambda MP\Theta \quad (3)$$

where Λ is a diagonal matrix of the first s eigenvalues.

A reduced eigenvalue problem is obtained by pre-multiplying both sides of Equation (3) by P^T to obtain

$$K_R\Theta = \Lambda M_R\Theta \quad (4)$$

where the reduced stiffness and mass matrices are

$$K_R = P^T K P \text{ and } M_R = P^T M P. \quad (5)$$

The matrix Θ in Equation (4) contains the eigenvectors of the reduced stiffness and mass matrices K_R and M_R .

For m design variables, $(m + 1)$ eigenvalue problems must be solved in order to form the basis P in Equation (2). Therefore, both the cost of obtaining the modal matrices Φ_i and the size of matrix P increase linearly with m . The PROM approach uses the following algorithm to compute the mode shapes of a new design:

1. Find the mode shapes of the baseline design and the designs corresponding to the m corner points in the design space, and form subspace basis P .
2. Find the reduced stiffness and mass matrices K_R and M_R from Equation (5).
3. Solve eigen-problem (4) for matrix Θ .
4. Reconstruct the approximated eigenvectors in $\tilde{\Phi}_p$ using Equation (1).

In the above procedure, step 1 is performed only once. A reanalysis requires only steps 2 to 4. For a small number of mode shapes and a small number of design parameters, the cost of steps 2 to 4 is much smaller compared to the cost of a full analysis.

The computational cost of PROM consists of a) the cost of performing $(m + 1)$ full eigen-analyses to form subspace basis P in Equation (7), and b) the cost of reanalysis of each new design in steps 2 to 4. The former is considered the fixed cost of PROM because it does not depend on the number of reanalyses and the latter represents the variable cost of PROM because it is proportional to the number of reanalyses. The fixed cost is not attributed to the calculation of the response of a particular design. Simply, it is required to obtain the information needed to apply PROM for a given problem. The variable cost (cost of reanalysis of a new design in part b) is small compared to the fixed cost.

The fixed cost of PROM is proportional to the number of design variables m since the basis $\mathbf{P}$ consists of the dominant eigenvectors Φ_0 of the baseline design, and the dominant eigenvectors Φ_p , $i = 1, \dots, m$ of the m corner design points as shown in Equation (2). As the size of $\mathbf{P}$ increases, so does the fixed cost because more eigenvalue problems and mode shapes must be calculated. The PROM method results in significant cost savings when applied to problems that involve few design variables (less than 20) and require many analyses (e.g. Monte Carlo simulation or gradient-free optimization using genetic algorithms).

Combined Approximations (CA)

In the CA method [9, 10, 11, 12, 13], the eigenvectors at a new design are approximated using a linear combination of basis vectors. A subspace basis is formed as

$$\mathbf{R}_{n \times s} = [\mathbf{R}_{n \times m}^1 \quad \mathbf{R}_{n \times m}^2 \quad \dots \quad \mathbf{R}_{n \times m}^s] \quad (6)$$

where s is the number of basis vectors normally in the range of 3-6 depending on accuracy and efficiency [10, 20].

For a modified design, the stiffness and mass matrices are given by

$$\mathbf{K}_p = \mathbf{K}_0 + \Delta\mathbf{K}, \quad \mathbf{M}_p = \mathbf{M}_0 + \Delta\mathbf{M} \quad (7)$$

where $\mathbf{K}_0$ and $\mathbf{M}_0$ are the stiffness and mass matrices of the baseline (original) design and $\Delta\mathbf{K}$ and $\Delta\mathbf{M}$ represent large perturbations. The basis vectors in Equation (6) are given by Equations (8) and (9) [6] as

$$\mathbf{R}_{n \times m}^1 = \mathbf{K}_0^{-1} \mathbf{M}_p \Phi_0 \quad (8)$$

$$\mathbf{R}_{n \times m}^j = -\mathbf{K}_0^{-1} \Delta\mathbf{K} \mathbf{R}_{n \times m}^{j-1}, j = 2, 3, \dots, s \quad (9)$$

where Φ_0 are the eigenvectors of the baseline design. For the perturbed (new) design, a reduced model is obtained with the following stiffness and mass matrices

$$\mathbf{K}_{n \times n}^R = \mathbf{R}_{n \times m}^T \mathbf{K}_p \mathbf{R}_{m \times n}, \quad \mathbf{M}_{n \times n}^R = \mathbf{R}_{n \times m}^T \mathbf{M}_p \mathbf{R}_{m \times n} \quad (10)$$

and an eigenvalue problem is solved using $\mathbf{K}_{n \times n}^R$ and $\mathbf{M}_{n \times n}^R$ of Equation (10) to calculate the eigenvectors Θ of the reduced model. These eigenvectors are then projected back to the physical domain to obtain the following approximate eigenvectors Φ_p of the new design as

$$\Phi_p = \mathbf{R}_{n \times s} \Theta. \quad (11)$$

Because the basis vectors are linearly dependent, Gram Schmidt orthogonalization is performed to avoid numerical errors. The CA method is efficient for problems where the number of desired modes is low. In such a case, the computational effort of stiffness matrix decomposition (Equations 8 and 9) dominates.

The CA method has the following three main advantages:

1. It only requires a single matrix decomposition of stiffness matrix $\mathbf{K}_0$ to calculate the subspace basis $\mathbf{R}$,
2. It is accurate because the basis is updated for every new design, and
3. The eigenvectors of a new design are efficiently approximated using Equation (11) where the eigenvectors Θ correspond to a much smaller eigen-problem.

Kirsch [13] has suggested using frequency shifts to improve the estimation accuracy of higher modes. By introducing a frequency shift μ as

$$\bar{\lambda} = \lambda - \mu \quad (12)$$

the modified stiffness becomes

$$\bar{\mathbf{K}} = \mathbf{K} - \mu \mathbf{M} = \mathbf{K}_0 + (\Delta\mathbf{K} - \mu \mathbf{M}) \quad (13)$$

The shift μ can be calculated as

$$\mu^{(k+1)} = \frac{\Phi^{(k)T} \mathbf{K} \Phi^{(k)}}{\Phi^{(k)T} \mathbf{M} \Phi^{(k)}} \quad (14)$$

Due to the increased computational cost however, the Rayleigh quotient formula is used instead

$$\mu = \frac{\Phi_0^T \mathbf{K}_0 \Phi_0}{\Phi_0^T \mathbf{M}_0 \Phi_0}. \quad (15)$$

The concept of shifts is utilized to improve the quality of basis vectors in order to improve the estimation of the eigenvectors of the modified design. The eigenvectors computed using the stiffness with shifting (Equation 13) are the same without shifting (Equation 4).

It should be noted that the CA method is more suitable than PROM, if the number of reanalyses is less than the number of design parameters. However, it is efficient only for problems where the number of retained modes is small.